The Radius Of A Sphere Is Decreasing At A Constant Rate Of 6 Feet Per Second. At The Instant When The

The Radius Of A Sphere Is Decreasing At A Constant Rate Of 6 Feet Per Second. At The Instant When The we examine this scenario, it presents an interesting problem in the realm of related rates, a fundamental concept in calculus. Understanding how the radius changes over time and how this affects other properties of the sphere, such as its volume and surface area, can provide valuable insights into dynamic systems involving spherical objects. Whether analyzing physical phenomena like bubbles shrinking or engineering applications involving spherical tanks, grasping the principles behind the rate at which the radius decreases is crucial for accurate modeling and problem-solving.

Understanding Related Rates in Calculus

Related rates problems are a key part of differential calculus that deal with understanding how one quantity changes concerning another over time. These problems typically involve differentiating equations that relate different variables, all functions of time.

What Are Related Rates?

Related rates problems involve:

- Variables that depend on time (e.g., radius, volume, surface area)
- Differentiation with respect to time (usually denoted as t)
- Using the chain rule to find the rate of change of one variable given the rate of change of another

Typical Steps to Solve Related Rates Problems

To solve such problems, follow these general steps:

1. Identify the variables involved and assign symbols (e.g., r for radius, V for volume)
2. Write down the formula that relates these variables (e.g., volume of a sphere: $V = (4/3)\pi r^3$)
3. Differentiate the formula with respect to time t, applying the chain rule
4. Plug in the known values for the variables and their rates
5. Solve for the unknown rate of change

Applying Related Rates to the Sphere Problem

In the context of a sphere, several properties depend on the radius, and their rates of change can be analyzed similarly.

Given Data

- The radius of the sphere is decreasing at a rate of 6 feet per second: $\frac{dr}{dt} = -6 \text{ ft/sec}$
- The negative sign indicates the radius is decreasing

Objectives

- Find the rate of change of the volume of the sphere at a specific instant
- Determine the rate at which the surface area is changing
- Explore how these rates relate to the decreasing radius

Mathematical Background: Sphere Properties

Understanding the formulas for the volume and surface area of a sphere is essential.

Volume of a Sphere

$$
V = \frac{4}{3} \pi r^3
$$

Surface Area of a Sphere

$$
A = 4 \pi r^2
$$

These formulas show how the volume and surface area depend on the radius, allowing us to differentiate them with respect to time.

Differentiating Sphere Properties Over Time

Rate of Change of Volume

Differentiating the volume formula:
$$
\frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt}
$$
This relation shows that the rate at which volume changes depends on the current radius and the rate of change of the radius.

Rate of Change of Surface Area

Differentiating the surface area formula:
$$
\frac{dA}{dt} = 8 \pi r \frac{dr}{dt}
$$
Similarly, this indicates how the surface area decreases over time as the radius shrinks.

Calculating the Rates at a Specific Instant

Suppose at a particular moment, the radius of the sphere is r feet. Using the given rate $\frac{dr}{dt} = -6 \text{ ft/sec}$, we can compute the rates of change of volume and surface area.

Example Calculation

Let's consider when the radius $r = 10$ feet:

- Rate of change of volume:
$$
\frac{dV}{dt} = 4 \pi (10)^2 \times (-6) = 4 \pi \times 100 \times (-6) = -2400 \pi \text{ ft}^3/\text{sec}
$$
This means the volume is decreasing at a rate of approximately $-7,539.8$ cubic feet per second.

- Rate of change of surface area:
$$
\frac{dA}{dt} = 8 \pi \times 10 \times (-6) = -480 \pi \text{ ft}^2/\text{sec}
$$
Approximately, the surface area decreases at about $-1,507.96$ square feet per second.

Implications of the Radius Decrease Rate

The fact that the radius is decreasing at a constant rate simplifies analysis but also prompts questions about physical constraints and real-world applications.

Physical Interpretation

- The shrinking radius could represent a deflating balloon, a melting sphere, or a shrinking bubble.
- The constant rate of decrease suggests a uniform process, such as controlled drainage or uniform melting.

Effects on Volume and Surface Area

Since both volume and surface area depend on the radius, their rates of change are proportionally related to the current radius:

- As the radius decreases, the volume shrinks rapidly, especially when the radius is large
- The surface area also decreases, affecting how the sphere interacts with its environment (e.g., heat transfer)

Real-World Applications and Examples

Understanding how the radius decreases and how that affects other properties has numerous practical applications:

Engineering and Design

- Designing spherical tanks that shrink over time due to material fatigue or leakage
- Calculating how quickly a spherical object, such as a ball bearing, loses volume when deformed

Physics and Natural Phenomena

- Modeling the melting of ice spheres or bubbles
- Analyzing the shrinking of celestial bodies due to mass loss

Medical Applications

- Monitoring the reduction in size of spherical tumors during treatment
- Understanding the dynamics of cell volume changes

Conclusion

The problem of a sphere's radius decreasing at a constant rate offers a rich context for exploring related rates in calculus. By differentiating the fundamental formulas for volume and surface area, we gain insights into how these properties evolve over time. The constant rate of radius decrease not only simplifies calculations but also provides a model for various real-world phenomena. Recognizing the relationships between the radius and other geometric properties enables engineers, scientists, and mathematicians to predict behavior, optimize designs, and understand natural processes more accurately. Ultimately, mastering related rates problems enhances our ability to analyze dynamic systems involving spherical objects, which are prevalent across numerous fields of science and engineering.

Frequently Asked Questions

The radius of a sphere is decreasing at a rate of 6 feet per second. How do you express the rate of change of the volume of the sphere with respect to time?

The volume V of a sphere is given by V = (4/3)πr^3. Differentiating with respect to time t, we get dV/dt = 4πr^2 dr/dt. Since dr/dt = -6 ft/sec (decreasing), the rate of change of volume is dV/dt = 4πr^2 (-6).

At the instant when the radius of the sphere is 10 feet, what is the rate at which the volume is decreasing?

Using the formula dV/dt = 4πr^2 dr/dt, substitute r = 10 ft and dr/dt = -6 ft/sec: dV/dt = 4π(10)^2 (-6) = 4π 100 (-6) = -2400π cubic feet per second. The volume is decreasing at a rate of 2400π cubic feet per second.

How does the rate of change of surface area relate to the radius decreasing at 6 feet per second?

The surface area S of a sphere is S = 4πr^2. Differentiating, dS/dt = 8πr dr/dt. With dr/dt = -6 ft/sec, at radius r, dS/dt = 8πr (-6) = -48πr ft^2/sec. The surface area decreases at a rate proportional to the current radius.

What is the significance of the negative sign in the rate of change of the radius?

The negative sign indicates that the radius is decreasing over time, meaning the sphere is shrinking.

If the radius continues to decrease at 6 feet per second, how long will it take for the sphere to reach a radius of 0 feet?

Assuming the initial radius is R0, time t to reach radius 0 is t = R0 / 6 seconds. For example, if R0 = 30 ft, then t = 30 / 6 = 5 seconds.

Can you find the instantaneous rate of change of the sphere's surface area when the radius is 15 feet?

Yes. Using dS/dt = 8πr dr/dt, substitute r = 15 ft and dr/dt = -6 ft/sec: dS/dt = 8π 15 (-6) = -720π ft^2/sec. The surface area decreases at this rate when the radius is 15 feet.

Additional Resources

The Radius Of A Sphere Is Decreasing At A Constant Rate Of 6 Feet Per Second. At The Instant When The

Understanding how the radius of a sphere changes over time is a classic problem in calculus, particularly within the realm of related rates. When a sphere's radius is decreasing at a constant rate of 6 feet per second, it prompts intriguing questions about how other properties of the sphere—such as its volume and surface area—are affected at that precise moment. This article offers a comprehensive guide to dissecting this problem, walking through the principles involved, the mathematical techniques needed, and step-by-step solutions to related questions.

Introduction: The Significance of Related Rates in Calculus

Related rates problems are a fundamental part of calculus, involving the calculation of how one quantity changes with respect to another, often over time. These problems are essential because they model real-world scenarios: objects shrinking, expanding, moving, or changing shape dynamically. When the radius of a sphere is decreasing at a constant rate, it exemplifies a classic related rates problem, where the key is to connect the rate of change of the radius to other quantities like volume or surface area.

Understanding the Basic Concepts

Before diving into calculations, it’s important to understand the core properties of a sphere:

The Sphere and Its Geometric Properties

- Radius (r): The distance from the center of the sphere to any point on its surface.
- Volume (V): The three-dimensional space enclosed by the sphere, given by:

\[
V = \frac{4}{3} \pi r^3
\]
- Surface Area (A): The total area covered by the sphere’s surface, given by:

\[
A = 4 \pi r^2
\]

How These Quantities Change with Radius

Since both volume and surface area depend on the radius, their rates of change are directly related to how quickly the radius is shrinking or growing.

Setting Up the Problem

Suppose at a specific instant, the radius of a sphere is decreasing at a constant rate of 6 feet per second. Mathematically, this is expressed as:

$$
\frac{dr}{dt} = -6 \text{ ft/sec}
$$

The negative sign indicates that the radius is decreasing.

The primary questions typically asked in such a problem are:

- At that instant, what is the rate of change of the volume?
- At that instant, what is the rate of change of the surface area?
- How do these rates relate to each other?

Step-by-Step Solution Approach

1. Identify Known and Unknown Quantities

- Known:
- $\frac{dr}{dt} = -6 \text{ ft/sec}$
- Unknown:
- $\frac{dV}{dt}$ (rate of change of volume)
- $\frac{dA}{dt}$ (rate of change of surface area)

2. Write the Formulas for Volume and Surface Area

$$
V = \frac{4}{3} \pi r^3
$$
$$
A = 4 \pi r^2
$$

3. Differentiate Each with Respect to Time (t)

Using the chain rule:

- For volume:

$$
\frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt}
$$

- For surface area:

$$
\frac{dA}{dt} = 8 \pi r \frac{dr}{dt}
$$

4. Plug in the Known Rate and the Current Radius

To compute the rates at a specific instant, we need the current radius r. If a particular radius is given, we can substitute directly. If not, we can analyze the general case or pick a value for illustration.

Assuming at the instant we're analyzing, the radius is $r = R$ feet (a known specific value), then:

$$
\frac{dV}{dt} = 4 \pi R^2 \times (-6) = -24 \pi R^2 \text{ ft}^3/\text{sec}
$$

$$
\frac{dA}{dt} = 8 \pi R \times (-6) = -48 \pi R \text{ ft}^2/\text{sec}
$$

The negative signs indicate the volume and surface area are decreasing as the radius shrinks.

Exploring the Relationship Between the Rates

How Are the Rate of Change of Volume and Surface Area Related?

From the formulas, observe:

$$
\frac{dV}{dt} = \frac{r}{2} \times \frac{dA}{dt}
$$

because:

$$
\frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt}
$$
$$
\frac{dA}{dt} = 8 \pi r \frac{dr}{dt}
$$

Dividing $\frac{dV}{dt}$ by $\frac{dA}{dt}$:

$$
\frac{\frac{dV}{dt}}{\frac{dA}{dt}} = \frac{4 \pi r^2 \frac{dr}{dt}}{8 \pi r \frac{dr}{dt}} = \frac{r}{2}
$$

This indicates that at any instant, the rate of volume decrease is proportional to the radius times half.

Practical Applications and Real-World Contexts

Related rates problems like this appear in numerous real-world scenarios:

- Manufacturing: When a spherical object cools or shrinks over time.
- Biology: Cells or bubbles changing size dynamically.
- Physics: Spheres in motion with changing radii, such as in certain astrophysical applications.

Understanding how the radius affects other properties is crucial for engineers, physicists, and mathematicians.

Advanced Considerations

When the Radius is Not Constant

Suppose the radius's rate of change varies over time. The same differentiation principles apply, but $\frac{dr}{dt}$ becomes a function of t. Calculating the instantaneous rates then involves substituting the current value of r and $\frac{dr}{dt}$.

When the Radius Approaches Zero

As the sphere shrinks, both volume and surface area approach zero. The rates of change also diminish proportionally, highlighting how properties vanish smoothly as the sphere diminishes.

Summary of Key Points

- The rate of change of the volume of a sphere with respect to time is:

$$
\frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt}
$$

- The rate of change of the surface area is:

$$
\frac{dA}{dt} = 8 \pi r \frac{dr}{dt}
$$

- When the radius decreases at a constant rate ($\frac{dr}{dt} = -6$ ft/sec), the volume and surface area decrease proportionally to the radius at that instant.

- The relationship between $\frac{dV}{dt}$ and $\frac{dA}{dt}$ is:

$$
\frac{dV}{dt} = \frac{r}{2} \times \frac{dA}{dt}
$$

Final Thoughts

Analyzing how the radius of a sphere decreasing at a constant rate influences its volume and surface area offers invaluable insights into related rates problems. These problems elegantly demonstrate the interconnectedness of geometric properties and their derivatives, reinforcing core calculus concepts. Whether in theoretical mathematics or practical applications, mastering such techniques equips students and professionals with tools to model dynamic systems effectively.

Additional Practice Problems

To deepen understanding, consider tackling these related problems:

- If the radius of a sphere is decreasing at 6 ft/sec, how fast is the volume decreasing when the radius is 10 ft?
- Calculate the rate at which the surface area is decreasing when the radius is 5 ft and decreasing at 6 ft/sec.
- Determine the time it takes for the sphere to shrink from radius 20 ft to radius 10 ft if the radius decreases at 6 ft/sec constantly.

Exploring these questions will enhance your grasp of related rates and the fascinating dynamics of geometric objects.

Remember: Calculus is about understanding change—both in the abstract and in real-world phenomena. With practice, problems like these become intuitive tools for modeling and solving complex, ever-changing systems.

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