

The Ratio Of The Measures Of The Three Sides Of A Triangle Is $\frac{1}{4} : \frac{1}{8} : \frac{1}{6}$. Its Perimeter Is 4.75

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Understanding the relationships between the sides of a triangle is fundamental in geometry. When given the ratio of the sides, along with the perimeter, it becomes possible to determine the actual lengths of each side. In this article, we explore a specific problem: the ratio of the measures of the three sides of a triangle is $\frac{1}{4} : \frac{1}{8} : \frac{1}{6}$, and the perimeter of the triangle is 4.75 units. We will analyze how to find the lengths of each side, interpret the ratio, and understand the properties of this triangle.

Deciphering the Ratio of the Triangle's Sides

Understanding Ratios in Triangles

Ratios describe the proportional relationships between different quantities. When applied to the sides of a triangle, they indicate how long each side is relative to the others. The given ratio:

- Side 1 : $\frac{1}{4}$
- Side 2 : $\frac{1}{8}$
- Side 3 : $\frac{1}{6}$

means that if we assign a common variable to represent the scale factor, the actual lengths can be expressed in terms of this variable.

Expressing Side Lengths in Terms of a Variable

To work effectively with the ratios, it's helpful to convert all ratios to a common basis or to eliminate the fractions:

- The ratios are $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{6}$.
- Find a common denominator to express these ratios as whole numbers.

The denominators are 4, 8, and 6. The least common multiple (LCM) of these is 24.

Converting each ratio:

- $\frac{1}{4} = \frac{6}{24}$
- $\frac{1}{8} = \frac{3}{24}$
- $\frac{1}{6} = \frac{4}{24}$

Thus, the ratio of the sides can be written as:

$$6 : 3 : 4$$

Let the common scale factor be (k) . Then, the actual lengths of the sides are:

- Side 1: $(6k)$
- Side 2: $(3k)$
- Side 3: $(4k)$

Calculating the Actual Lengths Using the Perimeter

Using the Given Perimeter

The perimeter (P) of the triangle is the sum of its sides:

$$P = 6k + 3k + 4k = (6 + 3 + 4)k = 13k$$

Given that the perimeter is 4.75 units, we can set up the equation:

$$13k = 4.75$$

Solving for (k) :

$$k = \frac{4.75}{13}$$

Calculating:

$$k \approx 0.36538$$

Note: To maintain precision, keep the fractional form for exact calculations.

Finding the Exact Lengths of Each Side

Now, multiply each ratio coefficient by (k) :

- Side 1: $6k = 6 \times \frac{4.75}{13} = \frac{6 \times 4.75}{13}$
- Side 2: $3k = \frac{3 \times 4.75}{13}$
- Side 3: $4k = \frac{4 \times 4.75}{13}$

Calculations:

1. Side 1:

$$6k = \frac{6 \times 4.75}{13} = \frac{28.5}{13} \approx 2.19$$

2. Side 2:

$$3k = \frac{3 \times 4.75}{13} = \frac{14.25}{13} \approx 1.10$$

3. Side 3:

$$4k = \frac{4 \times 4.75}{13} = \frac{19}{13} \approx 1.46$$

Therefore, the approximate lengths of the sides are:

- Side 1: 2.19 units
- Side 2: 1.10 units
- Side 3: 1.46 units

These lengths satisfy the perimeter condition and maintain the proportional relationships indicated by the ratio.

Verifying the Triangle's Validity

Triangle Inequality Theorem

For any set of three side lengths to form a valid triangle, they must satisfy the triangle inequality theorem:

- Sum of any two sides $>$ the third side

Check:

1. $(2.19 + 1.10 = 3.29 > 1.46)$ (True)

2. $(2.19 + 1.46 = 3.65 > 1.10)$ (True)

3. $(1.10 + 1.46 = 2.56 > 2.19)$ (True)

All inequalities hold; hence, the side lengths form a valid triangle.

Properties and Applications of These Triangles

Special Triangle Types

Given the side lengths, it's insightful to analyze the type of triangle:

- Scalene Triangle: All sides are of different lengths.
- Acute, Right, or Obtuse: To determine the nature of the angles, use the Law of Cosines or compare squares of the sides.

Calculate the squares:

- $(2.19^2 \approx 4.80)$

- $(1.10^2 \approx 1.21)$

- $(1.46^2 \approx 2.13)$

Identify the largest side (2.19). Check if the square of the largest side equals the sum of the squares of the other two:

$$\begin{aligned} & 4.80 \stackrel{?}{=} 1.21 + 2.13 = 3.34 \end{aligned}$$

Since $4.80 > 3.34$, the triangle is obtuse, with the obtuse angle opposite the longest side.

Real-World Applications of Triangle Side Ratios

Understanding ratios like these is critical in:

- Engineering design: Ensuring structural stability based on side proportions.
- Navigation and surveying: Calculating distances based on proportional relationships.
- Art and architecture: Creating aesthetically pleasing and balanced structures.

Summary and Key Takeaways

- Ratios of a triangle's sides can be expressed as whole numbers by finding a common denominator.
- Using the given perimeter, the scale factor (k) is calculated to find exact side lengths.
- The sides are approximately 2.19, 1.10, and 1.46 units, forming a valid, obtuse triangle.
- Recognizing the properties of the triangle aids in various scientific and practical applications.

Conclusion

In conclusion, analyzing the ratio of the sides of a triangle and its perimeter provides valuable insights into the triangle's dimensions and properties. The process involves converting ratios into a workable form, calculating the scale factor, and verifying the validity of the triangle through the triangle inequality theorem. This approach is fundamental in geometry and has numerous practical applications in fields like engineering, architecture, and navigation. Understanding such relationships enhances problem-solving skills and deepens comprehension of geometric principles.

Frequently Asked Questions

What are the ratios of the sides of the triangle given as $\frac{1}{4} : \frac{1}{8} : \frac{1}{6}$?

The ratios are given as fractions, so to compare them directly, we can find a common denominator or convert them to equivalent ratios. The ratios are $\frac{1}{4} : \frac{1}{8} : \frac{1}{6}$.

How do you find the actual lengths of the sides of the triangle with the given ratios and perimeter?

First, convert the ratios to common terms or find a common multiplier to express each side as a multiple of a variable. Then, use the perimeter to solve for that variable to find the actual side lengths.

What is the method to determine the scale factor for the sides based on the given ratios and perimeter?

Add the ratio parts after converting all to a common denominator to find the sum of parts. Then, divide the total perimeter by this sum to get the scale factor, which when multiplied by each ratio gives the side lengths.

Calculate the actual lengths of the sides of the triangle given the ratios $\frac{1}{4} : \frac{1}{8} : \frac{1}{6}$ and a perimeter of 4.75.

Convert ratios to common denominator: $\frac{1}{4} = \frac{6}{24}$, $\frac{1}{8} = \frac{3}{24}$, $\frac{1}{6} = \frac{4}{24}$. Sum: $6 + 3 + 4 = 13$ parts. Perimeter = 4.75, so scale factor = $4.75 / 13 \approx 0.365$. Side lengths: $6 \times 0.365 \approx 2.19$, $3 \times 0.365 \approx 1.095$, $4 \times 0.365 \approx 1.46$.

Are the given ratios consistent with forming a valid triangle?

Yes, since the sum of the lengths of any two sides exceeds the third, the ratios are consistent with forming a valid triangle.

What is the significance of understanding ratios and perimeter in solving triangle problems?

Understanding ratios and perimeter allows us to determine side lengths accurately, analyze the triangle's properties, and solve related geometric problems efficiently.

Additional Resources

The Ratio Of The Measures Of The Three Sides Of A Triangle Is $\frac{1}{4} : \frac{1}{8} : \frac{1}{6}$. Its Perimeter Is 4.75

In the realm of geometry, understanding the relationships between the sides of a triangle is fundamental. When the measures of the sides are expressed in a specific ratio, it opens avenues for deeper analysis—be it for solving geometric problems, exploring properties, or applying these concepts to real-world scenarios. One such intriguing case involves a triangle whose sides are in the ratio of $\frac{1}{4} : \frac{1}{8} : \frac{1}{6}$, with the added piece of information that its perimeter is 4.75. This article aims to dissect this problem comprehensively, providing clarity on the underlying mathematics, the process of deriving actual side lengths, and exploring the broader context of ratios in triangle geometry.

Understanding the Given Ratios: Decoding the Side Lengths

Interpreting the Ratios

The initial challenge in this problem is interpreting the given ratios: $\frac{1}{4} : \frac{1}{8} : \frac{1}{6}$. Ratios are a way of expressing the relative sizes of quantities—in this case, the sides of a triangle. To analyze and calculate actual lengths, it's easier to work with ratios expressed as whole numbers or common denominators rather than fractions.

The given ratios are:

- First side: $\frac{1}{4}$
- Second side: $\frac{1}{8}$
- Third side: $\frac{1}{6}$

These fractions indicate the proportional sizes of each side relative to some common baseline. To facilitate calculations, converting these fractions into a common denominator is advantageous.

Converting Fractions to a Common Denominator

Let's find the least common denominator (LCD) of 4, 8, and 6:

- Factors:

- $4 = 2^2$

- $8 = 2^3$

- $6 = 2 \times 3$

- LCD is the least common multiple (LCM):

- LCM of 4, 8, and 6:

- 8 (since it's the highest power of 2 among them)

- 3 (from 6)

- Therefore, $\text{LCM} = 8 \times 3 = 24$

Now, convert each fraction to an equivalent fraction with denominator 24:

- $\frac{1}{4} = \frac{(1 \times 6)}{(4 \times 6)} = \frac{6}{24}$

- $\frac{1}{8} = \frac{(1 \times 3)}{(8 \times 3)} = \frac{3}{24}$

- $\frac{1}{6} = \frac{(1 \times 4)}{(6 \times 4)} = \frac{4}{24}$

Thus, the ratios become:

$$\frac{6}{24} : \frac{3}{24} : \frac{4}{24}$$

which simplifies to:

$$6 : 3 : 4$$

This means that if the sides are represented by a common scale factor (k) , then:

- Side 1 = $6k$

- Side 2 = $3k$

- Side 3 = $4k$

Determining the Actual Side Lengths

Using the Perimeter to Find the Scale Factor

Given the total perimeter of the triangle is 4.75, and the sides are proportional to $6k$, $3k$, and $4k$ respectively, the perimeter (P) can be expressed as:

$$P = (6k) + (3k) + (4k) = (6 + 3 + 4)k = 13k$$

Substituting the known perimeter:

$$13k = 4.75$$

Solving for k :

$$k = \frac{4.75}{13} \approx 0.3653846$$

With k determined, we can now find the actual lengths of each side:

- Side 1: $6 \times 0.3653846 \approx 2.1923$
- Side 2: $3 \times 0.3653846 \approx 1.0962$
- Side 3: $4 \times 0.3653846 \approx 1.4615$

Summary of side lengths:

Side	Length (approximate)
1	2.1923
2	1.0962
3	1.4615

Verifying the Triangle Inequality

Before proceeding further, it's essential to verify that these side lengths indeed form a valid triangle. The triangle inequality states that the sum of any two sides must be greater than the third.

Let's verify:

- $2.1923 + 1.0962 = 3.2885 > 1.4615$ $\square$
- $2.1923 + 1.4615 = 3.6538 > 1.0962$ $\square$
- $1.0962 + 1.4615 = 2.5577 > 2.1923$ $\square$

All inequalities hold true, confirming the sides form a valid triangle.

Analytical Insights into the Triangle

Type of Triangle

Analyzing the side lengths, the triangle appears scalene (all sides different). To determine if it's acute, right, or obtuse, we can apply the Pythagorean theorem or compare squares of sides.

Calculate the squares:

- $(2.1923)^2 \approx 4.80$
- $(1.4615)^2 \approx 2.14$
- $(1.0962)^2 \approx 1.20$

Order the sides:

- Longest side: approximately 2.1923

Sum of squares of the other two sides:

$$2.14 + 1.20 = 3.34$$

Compare with the square of the longest side:

$$4.80$$

Since $4.80 > 3.34$, the triangle is obtuse—specifically, the angle opposite the longest side exceeds 90 degrees.

Implication: This insight is critical in understanding the triangle's properties, especially if these measures are used in real-world applications like construction or design.

Broader Context: Ratios and Their Significance in Geometry

Ratios as a Tool for Generalization

Ratios are a cornerstone in geometry because they allow for the study of shapes independent of

absolute size. By expressing sides in ratios, mathematicians can analyze the properties of similar triangles, infer angles, and solve for unknowns efficiently.

In the problem at hand, the initial ratios of $1/4 : 1/8 : 1/6$ serve as a blueprint for constructing specific triangles with desired proportional relationships. Such techniques are widely used in similar triangles, scale modeling, and even in fields like architecture and engineering, where proportional relationships ensure structural integrity and aesthetic harmony.

Real-World Applications

Understanding ratios and their applications extends beyond theoretical mathematics:

- Design and Architecture: Ensuring aesthetic proportions and structural stability relies heavily on ratios.
- Navigation and Mapping: Ratios help in scaling maps and models accurately.
- Biology and Nature: Many natural phenomena exhibit proportional relationships, such as the Fibonacci sequence in sunflower seeds or spiral shells.
- Physics and Engineering: Ratios define relationships between forces, velocities, and other physical quantities.

Mathematical Techniques and Educational Significance

Learning how to convert fractional ratios into actual measurements enhances problem-solving skills, critical thinking, and the ability to apply mathematical concepts to real situations. It also fosters an understanding of scale, proportion, and the importance of unit analysis.

Conclusion: The Intersection of Ratios, Geometry, and Practical Computations

The exploration of a triangle with side ratios of $1/4 : 1/8 : 1/6$ and a known perimeter of 4.75 demonstrates the elegance and utility of ratio analysis in geometry. Converting fractional ratios into whole numbers simplifies calculations, revealing the actual side lengths and enabling further geometric analysis, such as verifying the triangle's validity and classifying its type.

This process exemplifies a core principle in mathematics: the power of proportional reasoning. It underscores how initial fractional relationships can be transformed into tangible measurements, bridging the abstract and the concrete.

Moreover, the insights gained extend beyond this specific problem, illustrating the importance of ratios in various fields and their role in fostering a deeper understanding of the natural and constructed world. Whether in designing bridges, analyzing biological structures, or creating art, the principles of ratios and proportionality remain fundamental.

In essence, this problem encapsulates the beauty of mathematical reasoning—starting from simple ratios, applying systematic conversions, and arriving at meaningful, real-world insights. It highlights that behind every geometric figure lies a network of relationships waiting to be unraveled, understood, and appreciated.

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