The Region Bounded By The Graphs Of Y = In X, Y = 0, And X = E Is Revolved About The Y-axis. Find The

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Understanding how to compute the volume of solid objects generated by revolving a region around an axis is a fundamental concept in integral calculus. In particular, the problem involving the region bounded by the graphs of $y = \ln x$, $y = 0$, and $x = e$ offers a classic example of applying the disk or washer method to find the volume when this region is revolved about the y-axis. This article provides an in-depth explanation of the problem and guides you through the step-by-step solution process, including setting up the integral, choosing the method, and performing the calculations.

Understanding the Geometric Region

Before diving into the calculus, it’s essential to visualize and understand the region that is being revolved:

- The graph of $y = \ln x$ is a natural logarithm curve, which is increasing for $x > 0$, passing through the point $(1, 0)$ and tending towards $-\infty$ as $x \to 0^+$.

- The line $y = 0$ is the x-axis.

- The vertical line $x = e$ bounds the region on the right.

Key Points:

- The region lies between the x-axis ($y=0$) and the curve $y=\ln x$.

- It is bounded on the left by the vertical asymptote at $x=0$, but since the region is from $x=1$ (where $y=0$) to $x=e$, the domain is from 1 to e.

Visual Representation:

Imagine the area under the curve $y = \ln x$ from $x=1$ to $x=e$. This region is bounded below by $y=0$ and above by $y=\ln x$.

Setting Up the Problem: Revolving Around the Y-Axis

When this region is revolved about the y-axis, it generates a three-dimensional solid. To compute its volume, we use methods from integral calculus, notably the washer method, which accounts for the shape of the cross-sectional disks or washers.

Key insight:

- Since the axis of revolution is the y-axis, it’s often easier to set up the integral with respect to y.

- To do this, we need to express x as a function of y.

Expressing x in terms of y:

Given $y = \ln x$, then:

$$
x = e^{y}
$$

This inverse relationship allows us to consider slices perpendicular to the y-axis, which simplifies the integral setup.

Method Selection: Disk/Washer Method

Because the region is being revolved around the y-axis, and the boundary curves are functions of x and y, the washer method is suitable here.

Why the Washer Method?

- The cross-section at a particular y is a washer (a disk with a hole), since the region bounded by $y=\ln x$ and $y=0$ will generate a hollow or solid shape depending on the bounds.

- The method involves integrating the area of these washers along the axis of revolution.

Determining the Radii:

- Inner radius R_{inner}: Since the region starts at $x=1$, the inner radius at height y is $x_{inner} = 1$.

- Outer radius R_{outer}: Corresponds to the outer boundary of the region at $x = e^{y}$.

But because the region is from $x=1$ to $x=e$, and the revolution is around the y-axis, the radius of a typical washer at height y is given by the x-value at that y:

$$
r(y) = x = e^{y}
$$

The region is between $x=1$ and $x=e$, so the outer radius varies from 1 to e, but for the revolution about the y-axis, the cross-sectional radius at height y is simply e^{y}.

Setting Up the Integral for the Volume

To find the volume, we need to integrate the area of the washers along the y-axis over the relevant interval.

Step 1: Determine the bounds for y:

- At $x=1$:

$$
y = \ln 1 = 0
$$

- At $x=e$:

$$
y = \ln e = 1
$$

So, the region corresponds to y in the interval $[0, 1]$.

Step 2: Express the cross-sectional area:

Each washer at height y has a:

- Outer radius: $R_{outer} = e^{y}$

- Inner radius: Since the region is bounded below by $y=0$ and the inner boundary is at $x=1$, which corresponds to $y=0$, the inner radius at any y is:

$$
R_{inner} = 1
$$

(Note: The inner radius is constant because the region between $x=1$ and the curve $y=\ln x$ is being revolved, resulting in a hollow shape with an inner radius of 1 and outer radius e^{y}.)

Step 3: Write the formula for the volume:

Using the washer method, the volume V is:

$$
V = \pi \int_{y=0}^{1} \left[R_{outer}^2 - R_{inner}^2 \right] dy
$$

Substituting the radii:

$$
V = \pi \int_{0}^{1} \left[(e^{y})^{2} - 1^{2} \right] dy = \pi \int_{0}^{1} \left(e^{2y} - 1 \right) dy
$$

Calculating the Integral

Now, perform the integration:

$$
V = \pi \left[\int_{0}^{1} e^{2y} dy - \int_{0}^{1} 1 dy \right]
$$

Integral 1:

$$
\int e^{2y} dy = \frac{1}{2} e^{2y} + C
$$

Integral 2:

$$
\int 1 dy = y + C
$$

Putting it all together:

$$
V = \pi \left[\left. \frac{1}{2} e^{2y} \right|_{0}^{1} - \left. y \right|_{0}^{1} \right]
$$

Calculate each term:

$$
V = \pi \left(\frac{1}{2} e^{2 \times 1} - \frac{1}{2} e^{0} - (1 - 0) \right)
$$

$$
V = \pi \left(\frac{1}{2} e^{2} - \frac{1}{2} \times 1 - 1 \right)
$$

Simplify:

$$
V = \pi \left(\frac{e^{2}}{2} - \frac{1}{2} - 1 \right)
$$

$$
V = \pi \left(\frac{e^{2} - 1}{2} - 1 \right)
$$

Express as a single fraction:

$$
V = \pi \left(\frac{e^{2} - 1 - 2}{2} \right) = \pi \left(\frac{e^{2} - 3}{2} \right)
$$

Final Result

The volume of the solid generated by revolving the region bounded by $y = \ln x$, $y=0$, and $x=e$ about the y-axis is:

$$
\boxed{
V = \frac{\pi}{2} (e^{2} - 3)
}
$$

This formula provides a precise measure of the volume, combining exponential and geometric components.

Summary and Key Takeaways

- Understanding the region: Visualize the bounded area and its limits to set up the integral correctly.

- Choosing the axis of revolution: Revolving around the y-axis benefits from expressing x as a function of y.

- Using the washer method: When the region creates a hollow shape upon revolution, the washer method accounts for inner and outer radii.

- Setting bounds: Use the intersection points of the boundary curves to determine the limits of integration in the y-direction.

- Performing the integral: Apply standard integral formulas for exponential functions and polynomial terms.

- Final expression: The volume involves exponential functions and constants, emphasizing the importance of careful algebraic manipulation.

Additional Resources for Further Study

- Calculus Textbooks: For foundational

Frequently Asked Questions

What is the method used to find the volume of the region bounded by y = ln x, y = 0, and x = e when revolved about the y-axis?

The method used is the disk or washer method, where the volume is computed by integrating the cross-sectional areas perpendicular to the y-axis, typically converting x in terms of y and integrating with respect to y.

How do you express x in terms of y for the region bounded by y = ln x when revolving around the y-

axis?

Since $y = \ln x$, solving for x gives $x = e^y$, which serves as the radius of the disks when integrating with respect to y.

What are the bounds of integration for finding the volume of the given region when revolved about the y-axis?

The bounds in y are from $y = 0$ (corresponding to $x = 1$) up to $y = 1$ (corresponding to $x = e$), since at $x = e$, $y = 1$, and at $x = 1$, $y = 0$.

How is the volume of the solid computed when revolving the region around the y-axis?

The volume V is calculated using the integral $V = \pi \int$ from y=0 to y=1 of $[x(y)]^2\, dy$, where $x(y) = e^y$, resulting in $V = \pi \int_0^1 (e^y)^2\, dy$.

What is the final integral expression to find the volume of the solid formed by revolving the region about the y-axis?

The volume is given by $V = \pi \int_0^1 e^{2y}\, dy$, which can be evaluated as $V = \pi [e^{2y} / 2]$ evaluated from 0 to 1.

What is the calculated volume for the solid formed by revolving the region bounded by y = ln x, y = 0, and x = e about the y-axis?

Evaluating the integral gives $V = \pi [e^{2} / 2 - 1 / 2] = (\pi / 2)(e^{2} - 1)$.

Additional Resources

The Region Bounded By The Graphs Of Y = ln(x), Y = 0, And X = e Is Revolved About The Y-axis is a classic problem in calculus that combines the concepts of areas, volumes, and the application of the disk or washer method. This problem provides an excellent opportunity to explore the process of setting up and evaluating integrals that describe the volume generated by revolving a region around an axis. Such problems are foundational in understanding how to translate geometric boundaries into mathematical expressions that facilitate volume calculations.

In this guide, we'll walk through the step-by-step process of solving this problem, from understanding the region to applying the integral calculus techniques necessary for finding the volume. Whether you're a student studying calculus or a professional brushing up on integral methods, this detailed breakdown aims to clarify each stage of the process.

Understanding the Region

Before diving into the calculations, it's crucial to visualize and comprehend the region described by the boundaries:

- Y = ln(x): The natural logarithm function, which is defined for x > 0, is increasing and passes through (1, 0). As x approaches 0 from the right, ln(x) approaches -∞.
- Y = 0: The x-axis, serving as the lower boundary for the region.
- X = e: A vertical line at x = e, which will serve as one of the boundary limits.

Visualizing the Region

- The region is bounded below by the x-axis (Y=0).
- It is above the x-axis, bounded on the right by the vertical line x = e.
- It is below the curve y = ln(x) within the interval from x = 1 to x = e.

Graphically, the region looks like a "slice" under the curve y = ln(x), starting from x=1 where ln(1)=0, up to x=e where ln(e)=1.

Setting Up the Problem

The goal: Find the volume of the solid obtained by revolving this region about the y-axis.

Key considerations:

- The axis of rotation is the y-axis (x=0).
- The region is in the x-y plane, bounded as previously described.
- To find the volume, we need to select an appropriate method. Since the region is described with respect to x, and the axis of revolution is vertical (the y-axis), the washer method or disk method is suitable if we express everything as a function of x.

Choosing the Method

Because the region is described in terms of x, and the rotation is about the y-axis, it is often easiest to integrate with respect to x.

The general volume element when revolving around the y-axis (using the shell method) involves cylindrical shells:

- Shell method volume element: \(dV = 2\pi \times (\text{radius}) \times

$(\text{height}) \times dx$

Where:

- radius: distance from the y-axis to the shell (which is x itself).
- height: the vertical extent of the shell, which is from y=0 up to y=ln(x).

Why Shells?

- Since the region is bounded by x-values, and the axis of rotation is vertical, the shell method simplifies the calculation.
- The shell method involves integrating around the x-values, which aligns with the region's description.

Formal Setup of the Integral

1. Determine the limits of integration:

- The x-values go from 1 to e, because:

- at x=1, y=ln(1)=0
- at x=e, y=ln(e)=1

2. Express radius and height:

- Radius of each shell: x
- Height of each shell: y = ln(x)

3. Write the volume integral:

Using the shell method, the volume V is:

$$V = \int_{x=1}^{x=e} 2\pi \times \text{radius} \times \text{height} \, dx$$

Substituting the expressions:

$$V = \int_{1}^{e} 2\pi x \times \ln(x) \, dx$$

This integral captures the volume of the solid of revolution.

Computing the Integral

The integral:

$$V = 2\pi \int_{1}^{e} x \ln(x) \, dx$$

This integral combines the polynomial term x and the logarithmic function, suggesting the use of integration by parts.

1. Applying Integration by Parts

Recall the integration by parts formula:

$$ \int u \, dv = uv - \int v \, du $$

Choose:

- $u = \ln(x)$ (since its derivative simplifies)
- $dv = x \, dx$

Then:

- $du = \frac{1}{x} dx$
- $v = \frac{x^2}{2}$

2. Computing the integral:

$$ \int x \ln(x) \, dx = \frac{x^2}{2} \ln(x) - \int \frac{x^2}{2} \times \frac{1}{x} dx $$

Simplify the second integral:

$$ \int \frac{x^2}{2} \times \frac{1}{x} dx = \frac{1}{2} \int x \, dx = \frac{1}{2} \times \frac{x^2}{2} = \frac{x^2}{4} $$

Putting it all together:

$$ \int x \ln(x) \, dx = \frac{x^2}{2} \ln(x) - \frac{x^2}{4} + C $$

3. Evaluate from 1 to e:

$$ V = 2\pi \left[\frac{x^2}{2} \ln(x) - \frac{x^2}{4} \right]_{1}^{e} $$

Compute at the bounds:

- At $x = e$:

$$ \frac{e^2}{2} \times 1 - \frac{e^2}{4} = \frac{e^2}{2} - \frac{e^2}{4} = \frac{2e^2}{4} - \frac{e^2}{4} = \frac{e^2}{4} $$

- At $x = 1$:

$$ \frac{1^2}{2} \times 0 - \frac{1^2}{4} = 0 - \frac{1}{4} = -\frac{1}{4} $$

Subtracting:

$$ \left(\frac{e^2}{4} \right) - \left(-\frac{1}{4} \right) = \frac{e^2}{4} $$

+ \frac{1}{4} = \frac{e^2 + 1}{4} \]

4. Final volume:

\[V = 2\pi \times \frac{e^2 + 1}{4} = \frac{\pi}{2} (e^2 + 1) \]

Final Answer

The volume of the solid obtained by revolving the region bounded by \(y = \ln(x) \), \(y=0 \), and \(x=e \) about the y-axis is:

\[\boxed{V = \frac{\pi}{2} (e^2 + 1)} \]

Additional Insights and Variations

Changing the Axis of Rotation

- If, instead of revolving around the y-axis, the region were revolved around the x-axis, the setup would change, possibly requiring the disk method with respect to y.

Considerations in Other Scenarios

- For more complex regions, multiple integrals or numerical methods may be necessary.
- When the region is bounded differently, or the functions involve other transformations, the same principles apply but with altered integrals.

Summary of Key Steps

1. Identify the region and its boundaries.
2. Visualize the region and determine the limits of integration.
3. Choose an appropriate method (shell or disk).
4. Express the volume integral based on the selected method.
5. Simplify and evaluate the integral, often requiring advanced techniques (substitution, parts).
6. Interpret the result in the context of the problem.

Conclusion

The problem of calculating the volume of a region bounded by specific curves and revolved around an axis is a fundamental calculus exercise, blending geometric intuition with integral calculus techniques. By carefully analyzing the region, selecting the appropriate method, and executing systematic integration, we can derive precise formulas for such volumes. The example of revolving the region bounded by \(y = \ln(x) \), \(y=0 \), and \(x=e \) about the y-axis illustrates the power of the shell method and integration by parts, culminating in a neat, exact result: \(\frac{\pi}{2} (e^2 + 1) \). Mastery of these methods equips you with versatile tools for tackling a wide range of volume problems in calculus.

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