The Sum Of The Number Orange In Two Different Basket Is 22 Their Products Is 120 What Is Their Number

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When faced with a problem involving two numbers, such as the sum and product, it often resembles classic algebraic puzzles. These puzzles challenge us to find the original numbers based on given conditions. In this case, we are asked to determine two numbers placed in separate baskets, with the sum of their numbers being 22 and their product being 120. This type of problem is not only common in math exercises but also an excellent way to develop critical thinking and algebraic skills.

In this comprehensive guide, we will explore how to solve this problem step-by-step, introduce relevant mathematical concepts, and provide tips for solving similar problems efficiently. Whether you're a student brushing up on algebra or a curious learner, this article aims to clarify the process and deepen your understanding of solving systems of equations.

Understanding the Problem

Before diving into solutions, it's essential to interpret the problem accurately. The problem states:

- The sum of two numbers (each in a different basket) is 22.
- The product of these two numbers is 120.
- The goal is to find what these two numbers are.

This is a typical algebraic problem that can be approached through various methods, including substitution, factoring, or the quadratic formula.

Setting Up the Equations

To solve the problem systematically, we need to translate the word problem into algebraic equations.

Let:

- \(x \) = the number in the first basket.
- \(y \) = the number in the second basket.

Based on the problem statement, the equations are:

1. Sum Equation:
\[
x + y = 22
\]
2. Product Equation:
\[
xy = 120
\]

Our goal is to find the values of \(x \) and \(y \) that satisfy both equations.

Solving the System of Equations

There are multiple methods to solve these equations, but the most straightforward in this context is substitution or using the quadratic approach.

Using Substitution Method

1. Express \(y \) in terms of \(x \):

\[
y = 22 - x
\]

2. Substitute into the product equation:

\[
x(22 - x) = 120
\]

3. Expand and rearrange:

\[
22x - x^2 = 120
\]

4. Rearranged into standard quadratic form:

\[
-x^2 + 22x - 120 = 0

\]

Multiply through by -1 to make calculations easier:

$$
x^2 - 22x + 120 = 0
$$

5. Solve using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1$, $b=-22$, $c=120$.

Applying the Quadratic Formula

Calculating the discriminant:

$$
\Delta = b^2 - 4ac = (-22)^2 - 4 \times 1 \times 120 = 484 - 480 = 4
$$

Since the discriminant is positive, there are two real solutions:

$$
x = \frac{22 \pm \sqrt{4}}{2} = \frac{22 \pm 2}{2}
$$

Calculate the two possibilities:

- When using the plus sign:

$$
x = \frac{22 + 2}{2} = \frac{24}{2} = 12
$$

- When using the minus sign:

$$
x = \frac{22 - 2}{2} = \frac{20}{2} = 10
$$

Now, find the corresponding y for each x:

- For $x=12$:

$$
y = 22 - 12 = 10
$$

- For $x=10$:

$$
y = 22 - 10 = 12
$$

Thus, the two numbers are 10 and 12.

Summary of the Solution

- The numbers in the two baskets are 10 and 12.
- These satisfy both the sum and the product conditions:

$$
10 + 12 = 22
$$
$$
10 \times 12 = 120
$$

This confirms our solution is correct.

Understanding the Mathematical Concepts

This problem illustrates core algebraic concepts, including:

Systems of Equations
- Simultaneously solving equations involving multiple variables.
- Using substitution to reduce the system to a single-variable quadratic.

Quadratic Equations
- Understanding how to derive and solve quadratic equations.
- Using the quadratic formula to find roots when factoring is not straightforward.

Discriminant
- The value under the square root in the quadratic formula ($b^2 - 4ac$) determines the nature of roots:

- Positive: two real solutions.
- Zero: one real solution.
- Negative: complex solutions.

Practical Applications of Such Problems

While solving for two unknowns with sum and product conditions is a classic algebra problem, similar logic applies in various real-world scenarios:

- Financial calculations involving investments with known total amounts and returns.
- Inventory management determining quantities based on total weight and combined value.
- Physics problems involving combined velocities or forces with known sums and products.

Tips for Solving Similar Problems

- Always translate the word problem into clear algebraic equations.
- Check the nature of the problem: is it linear, quadratic, or a system?
- Use substitution or elimination methods for systems.
- When quadratic equations are involved, consider the discriminant first.
- Verify solutions by substituting back into original equations.
- Be mindful of extraneous solutions, especially in real-world contexts.

Additional Practice Problems

To reinforce your understanding, try solving these similar problems:

1. The sum of two numbers is 30, and their product is 221. What are the numbers?
2. Two numbers in separate baskets have a sum of 50 and a product of 649. Find the numbers.
3. A problem states that the sum of two numbers is 15, and their product is 54. What are these numbers?

Conclusion

In solving the problem where the sum of two numbers is 22 and their product is 120, we utilized fundamental algebraic techniques to arrive at the solution. The key steps involved translating the problem into equations, applying the quadratic formula, and verifying the results. The numbers in the two baskets are 10 and 12.

Understanding these methods not only helps solve this specific problem but also builds a strong foundation for tackling various algebraic challenges. Remember, approaching such problems systematically and verifying your solutions ensures accuracy and strengthens your mathematical reasoning skills.

Whether you're preparing for exams or simply enjoy solving puzzles, mastering these techniques will serve you well across many areas of mathematics and real-life problem-solving scenarios.

Frequently Asked Questions

What are the two numbers in the baskets if their sum is 22 and their product is 120?

The two numbers are 10 and 12.

How can I find the numbers if I know their sum and product?

Use the system of equations: $x + y = 22$ and $xy = 120$, then solve for x and y using substitution or quadratic equations.

What quadratic equation represents these two numbers?

The quadratic equation is $t^2 - 22t + 120 = 0$.

How do I solve the quadratic equation to find the numbers?

Apply the quadratic formula $t = [22 \pm \text{sqrt}(22^2 - 41120)] / 2$, which simplifies to $t = [22 \pm \text{sqrt}(484 - 480)] / 2$, resulting in $t = [22 \pm 2] / 2$, leading to $t = 12$ or $t = 10$.

Are the two numbers 10 and 12 the only solutions?

Yes, the two numbers are 10 and 12 since they satisfy both the sum and product conditions.

Can the numbers be negative?

No, because their sum is positive (22) and their product is positive (120), so both are positive numbers.

What is the significance of the color orange in this problem?

The mention of 'orange' appears to be a thematic or contextual detail, but it does not affect the mathematical solution.

How would the problem change if the sum were different, say 30?

You would set up the equations x + y = 30 and xy = 120, then solve similarly for the new sum.

Is this problem an example of quadratic equations in real-world scenarios?

Yes, it illustrates how quadratic equations can be used to solve problems involving sums and products of numbers in real-world contexts.

What mathematical concepts are involved in solving this problem?

The problem involves algebra, systems of equations, quadratic equations, and the quadratic formula.

Additional Resources

Mathematical Puzzle Analysis: Deciphering the Numbers Orange in Two Baskets

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In the world of mathematical riddles and algebraic problem-solving, certain puzzles stand out due to their intriguing blend of simplicity and challenge. One such problem involves identifying two specific numbers, referred to here as "orange", that are placed in separate baskets. The problem states:

> The sum of the number orange in two different baskets is 22. Their product

is 120. What are their numbers?

This puzzle is a classic example of applying fundamental algebraic principles to find unknown values based on given conditions. In this article, we will explore the problem thoroughly, breaking down each component, analyzing possible solutions, and offering insights into the methods used to reach the answer.

Understanding the Problem: Breaking Down the Conditions

Before delving into calculations, it's essential to understand what is being asked and the constraints imposed by the problem.

The Given Data

- Sum of the two numbers: 22
Mathematically, if the two numbers are x and y, then:
$$
x + y = 22
$$
- Product of the two numbers: 120
Which translates to:
$$
xy = 120
$$

The Objective

Determine the two numbers (referred to as "orange") that satisfy these conditions.

Mathematical Approach: Formulating Equations

The problem is straightforward but requires a structured approach to solve.

Step 1: Assign Variables

Let the numbers in the two baskets be:

$$
x = \text{Number in Basket 1}

\]
\[
y = \text{Number in Basket 2}
\]

Step 2: Write the System of Equations

Based on the problem:

\[
\begin{cases}
x + y = 22 \quad \text{(Sum condition)} \\
xy = 120 \quad \text{(Product condition)}
\end{cases}
\]

Solving the Equations: Algebraic Techniques

The primary strategy involves leveraging the relationships between sums and products. The key methods include substitution, factoring, and using quadratic equations.

Step 3: Express One Variable in Terms of the Other

From the first equation:

\[
y = 22 - x
\]

Substitute into the second:

\[
x(22 - x) = 120
\]

Expanding:

\[
22x - x^2 = 120
\]

Rearranged into standard quadratic form:

\[
x^2 - 22x + 120 = 0
\]

Quadratic Equation Solution

Now, solve for x using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a = 1$, $b = -22$, and $c = 120$.

Step 4: Calculate the Discriminant

$$
\Delta = b^2 - 4ac = (-22)^2 - 4 \times 1 \times 120 = 484 - 480 = 4
$$

Since the discriminant is positive, there are two real solutions.

Step 5: Find the Roots

$$
x = \frac{22 \pm \sqrt{4}}{2} = \frac{22 \pm 2}{2}
$$

Calculating each:

- When using the positive square root:

$$
x = \frac{22 + 2}{2} = \frac{24}{2} = 12
$$

- When using the negative square root:

$$
x = \frac{22 - 2}{2} = \frac{20}{2} = 10
$$

Step 6: Determine Corresponding Values of y

Recall $y = 22 - x$:

- If $x = 12$:

$$
y = 22 - 12 = 10
$$

- If $x = 10$:

$$
y = 22 - 10 = 12
$$

Solutions in Context

The two solutions are:

1. Number 1: 12
2. Number 2: 10

or vice versa. Because the problem involves two distinct baskets, and the numbers are interchangeable, the pairings are:

- Basket 1: 12, Basket 2: 10
- Basket 1: 10, Basket 2: 12

Both satisfy the sum and product conditions.

Interpreting the Results: Real-World Implications and Insights

This mathematical exercise illustrates several key points:

Symmetry of Solutions

The problem's symmetry emphasizes that the order of the numbers doesn't affect the conditions—they are interchangeable in the context of sum and product.

Significance of the Discriminant

The discriminant's value (4) confirms the existence of two real solutions, emphasizing the importance of quadratic discriminants in solving such problems.

Practical Applications

While this problem is theoretical, similar logic applies in various real-world scenarios:

- Financial calculations: Distributing resources evenly while maintaining specific product-based constraints.
- Inventory management: Balancing quantities in different baskets or containers based on sum and product considerations.
- Engineering: Designing systems where parameters must satisfy multiple multiplicative and additive conditions.

Expanding the Problem: What If Conditions Change?

To deepen understanding, consider how altering conditions affects solutions.

Scenario 1: Different Sum or Product

Suppose the sum is 24, and the product remains 120:

$$
x + y = 24
$$
$$
xy = 120
$$

The quadratic becomes:

$$
x^2 - 24x + 120 = 0
$$

Discriminant:

$$
\Delta = 24^2 - 4 \times 120 = 576 - 480 = 96
$$

Square root of discriminant:

$$
\sqrt{96} \approx 9.8
$$

Solutions:

$$
x = \frac{24 \pm 9.8}{2}
$$

- $x \approx \frac{24 + 9.8}{2} = \frac{33.8}{2} = 16.9$
- $x \approx \frac{24 - 9.8}{2} = \frac{14.2}{2} = 7.1$

Corresponding y:

- $y \approx 24 - 16.9 = 7.1$
- $y \approx 24 - 7.1 = 16.9$

This demonstrates how changing sums affects the solutions' values but still yields real solutions.

Scenario 2: Introducing Constraints

If the problem states the numbers must be integers, then solutions must be integer pairs:

- For the original problem, integer solutions are $(12, 10)$ and $(10, 12)$.
- For other sums or products, solutions might not be integers, indicating the need for rational or real solutions.

Conclusion: The Power of Algebra in Problem Solving

The problem of identifying two numbers with a specified sum and product exemplifies fundamental algebraic techniques, notably quadratic equations. By translating the problem into an algebraic system, applying the quadratic formula, and interpreting the discriminant, we arrive at the solutions:

$$
\boxed{
\text{The two numbers are } 10 \text{ and } 12
}
$$

This exercise not only reinforces key mathematical concepts but also demonstrates how algebra can be used to solve practical and theoretical problems efficiently. Whether in academic settings, real-world applications, or brain-teasing puzzles, mastering these techniques enhances analytical skills and problem-solving confidence.

In essence, the "orange" numbers in the two baskets are 10 and 12—an elegant example of algebra in action, illustrating how simple conditions can lead to precise solutions when approached methodically and thoughtfully.

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