

The Sum Of Two Numbers Is 23. Twice One Number Increased By 6 Is The Same As 4 Times The Other Number

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When dealing with algebraic problems, especially those involving multiple variables, it's essential to understand how to translate word problems into mathematical equations. The statement "The sum of two numbers is 23. Twice one number increased by 6 is the same as 4 times the other number" provides a perfect example of how to approach such problems systematically. In this article, we will explore how to solve this problem step-by-step, understand the underlying principles, and discuss related concepts that can help improve your algebraic problem-solving skills.

Understanding the Problem Statement

Before diving into the solution, it's crucial to interpret the problem correctly. The statement contains two parts:

1. The sum of two numbers is 23.
2. Twice one number increased by 6 is the same as 4 times the other number.

Let's define the variables:

- Let x be the first number.
- Let y be the second number.

Using these variables, we can translate the statements into equations.

Translating Words into Mathematical Equations

Based on the problem, the following equations can be formulated:

1. Sum of two numbers:

$$\begin{aligned} &\backslash \\ &x + y = 23 \end{aligned}$$

$\backslash$

2. Twice one number increased by 6 equals four times the other number:

$\backslash$

$$2x + 6 = 4y$$

\]

Now, the problem reduces to solving this system of two equations with two variables.

Solving the System of Equations

To find the values of x and y , we can employ various methods such as substitution, elimination, or graphical method. For this problem, substitution is straightforward.

Step 1: Express one variable in terms of the other

From the first equation:

\[

$$x + y = 23 \implies y = 23 - x$$

\]

Step 2: Substitute into the second equation

Replace y in the second equation:

\[

$$2x + 6 = 4(23 - x)$$

\]

Simplify:

\[

$$2x + 6 = 92 - 4x$$

\]

Step 3: Solve for x

Bring all x terms to one side:

\[

$$2x + 4x = 92 - 6$$

\]

\[

$$6x = 86$$

\]

Divide both sides by 6:

\[

$$x = \frac{86}{6} = \frac{43}{3} \approx 14.33$$

\]

Step 4: Find y

Recall:

\[

$$y = 23 - x$$

\]

Substitute x:

\[

$$y = 23 - \frac{43}{3} = \frac{69}{3} - \frac{43}{3} = \frac{26}{3} \approx 8.67$$

\]

Interpreting the Solution

The solution:

\[

$$x = \frac{43}{3} \approx 14.33, \text{quad } y = \frac{26}{3} \approx 8.67$$

\]

indicates that the two numbers are fractional, approximately 14.33 and 8.67, which satisfy the given conditions.

Verification:

- Sum check:

\[

$$14.33 + 8.67 \approx 23$$

\]

- Twice x plus 6:

\[

$$2 \times 14.33 + 6 \approx 28.66 + 6 = 34.66$$

\]

- Four times y:

\[

$$4 \times 8.67 \approx 34.68$$

\]

The slight differences are due to rounding, confirming the solution's correctness.

Alternative Approaches and General Tips

While substitution works well here, other methods like elimination or graphing can also be employed.

Elimination Method

Multiply the first equation if necessary to align coefficients and then subtract to eliminate one variable.

Graphical Method

Plot both equations on a coordinate plane; their intersection point gives the solution.

Understanding the Broader Context

This problem illustrates fundamental algebraic techniques applicable in various real-world situations, such as budgeting, measurement conversions, and physics problems involving relationships between quantities.

Key points to remember:

- Always define variables clearly.
- Translate word problems into equations carefully.
- Use substitution or elimination based on which simplifies the problem.
- Check solutions by substituting back into original equations.
- Be prepared to work with fractions or decimals.

Related Concepts and Extensions

Once you master solving two-variable systems, you can explore more advanced topics:

- Systems with more variables
- Nonlinear systems
- Word problems involving percentages, rates, and proportions
- Application of matrices and determinants for larger systems

Practical Applications of Such Problems

Understanding how to solve systems of equations has numerous practical applications:

- Finance: Calculating investments and interest rates
- Engineering: Balancing forces and electrical circuits
- Science: Analyzing chemical reactions or population models
- Economics: Supply and demand analysis

Conclusion

Solving the problem "The sum of two numbers is 23. Twice one number increased by 6 is the same as 4 times the other number" demonstrates essential algebraic skills, including setting up equations, solving systems, and verifying solutions. Mastery of these techniques not only enhances mathematical proficiency but also equips you with tools to analyze and solve complex real-world problems effectively.

Remember, practice is key. Try creating similar problems with different numbers or conditions to strengthen your understanding. With consistent effort, solving such algebraic problems will become intuitive and straightforward.

Keywords for SEO Optimization:

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- Solving systems of equations
- Two-variable equations
- Word problem solutions
- Algebra techniques
- Math problem solving
- Systems of equations examples

- How to solve algebraic problems
- Real-world math applications
- Basic algebra tips

Frequently Asked Questions

What are the two numbers if their sum is 23 and twice one increased by 6 equals four times the other?

Let the two numbers be x and y . From the conditions: $x + y = 23$ and $2x + 6 = 4y$. Solving these equations, we find $x = 7$ and $y = 16$.

How do you set up equations for the problem where the sum of two numbers is 23 and twice one plus 6 equals four times the other?

Assign variables to the numbers, say x and y . Write the equations: $x + y = 23$ and $2x + 6 = 4y$. These form the system to solve.

What is the solution to the system of equations: $x + y = 23$ and $2x + 6 = 4y$?

Solving these equations, first express y from the first: $y = 23 - x$. Substitute into the second: $2x + 6 = 4(23 - x)$. Simplify to find $x = 7$, then $y = 16$.

Can this problem be solved using substitution or elimination methods?

Yes, both substitution and elimination methods can be used. Substitution involves expressing one variable in terms of the other; elimination involves adding or subtracting equations to eliminate a variable.

What are the practical applications of solving such algebraic word problems?

Solving these problems helps develop critical thinking and problem-solving skills, useful in fields like engineering, finance, and computer science where modeling real-world situations mathematically is essential.

What are common mistakes to avoid when solving these types of algebraic problems?

Common mistakes include mislabeling variables, incorrect algebraic manipulations, forgetting to check solutions in the original equations, and arithmetic errors during calculations.

How can understanding this problem help in learning more advanced algebra topics?

This problem introduces systems of equations, which are foundational for understanding linear algebra, solving real-world problems involving multiple variables, and progressing to more complex topics like quadratic equations and inequalities.

Additional Resources

The Sum Of Two Numbers Is 23. Twice One Number Increased By 6 Is The Same As 4 Times The Other Number

Introduction

Mathematics often presents us with intriguing problems that test our logical reasoning and algebraic skills. Among these, systems of equations stand out as fundamental tools, enabling us to analyze relationships between multiple variables. One such problem, deceptively simple at first glance, involves two numbers whose sum is 23, and a further relationship linking their doubled values. This problem not only exemplifies the core principles of algebra but also serves as a gateway into more complex problem-solving paradigms.

In this article, we delve into the problem statement:

> "The sum of two numbers is 23. Twice one number increased by 6 is the same as 4 times the other number."

Our goal is to explore this problem thoroughly—unraveling its underlying structure, solving the equations involved, and examining the implications of the solutions. We will also discuss potential strategies for approaching similar problems and the relevance of such problems in educational and practical contexts.

Restating the Problem: Clarifying the Conditions

Before diving into algebraic manipulations, it's essential to clearly understand what the problem entails. Restated, the conditions are:

- Sum condition: The sum of two numbers, say x and y , equals 23:

$$\begin{aligned} &[\\ &x + y = 23 \\ &] \end{aligned}$$

- Relationship condition: Twice one number, increased by 6, equals four times the other number:

[

$$2x + 6 = 4y$$

\]

Alternatively, the problem could be interpreted with the roles of x and y reversed, but for now, we assume x and y are variables representing the two numbers, with no initial constraints on their order.

Formulating the Equations

The problem naturally translates into a system of two equations:

1. Equation 1: $x + y = 23$
2. Equation 2: $2x + 6 = 4y$

Our task is to find the values of x and y that satisfy both equations simultaneously.

Solving the System of Equations

Step 1: Express one variable in terms of the other

From Equation 1:

$$\begin{aligned} & \text{[} \\ x + y &= 23 \Rightarrow y = 23 - x \\ & \text{]} \end{aligned}$$

Substitute this into Equation 2:

$$\begin{aligned} & \text{[} \\ 2x + 6 &= 4(23 - x) \\ & \text{]} \end{aligned}$$

Step 2: Simplify and solve for x

Expanding the right side:

$$\begin{aligned} & \text{[} \\ 2x + 6 &= 92 - 4x \\ & \text{]} \end{aligned}$$

Bring all terms to one side:

$$\begin{aligned} & \text{[} \\ 2x + 4x &= 92 - 6 \\ & \text{]} \end{aligned}$$

\]

$$6x = 86$$

\]

\[

$$x = \frac{86}{6} = \frac{43}{3} \approx 14.33$$

\]

Step 3: Find y

Using $y = 23 - x$:

\[

$$y = 23 - \frac{43}{3} = \frac{69}{3} - \frac{43}{3} = \frac{26}{3} \approx 8.67$$

\]

Interpretation of the Solutions

The solutions are:

\[

$$x = \frac{43}{3} \quad \text{and} \quad y = \frac{26}{3}$$

\]

which approximate to:

\[

$$x \approx 14.33, \quad y \approx 8.67$$

\]

It's noteworthy that both solutions are rational numbers, not integers, indicating that the problem admits fractional solutions. This is common in algebraic systems where the constraints do not restrict variables to integers.

Verifying the Solutions

Verification of the sum:

\[

$$x + y = \frac{43}{3} + \frac{26}{3} = \frac{69}{3} = 23$$

\]

which confirms the first condition.

Verification of the second condition:

Calculate $2x + 6$:

$$2 \times \frac{43}{3} + 6 = \frac{86}{3} + 6 = \frac{86}{3} + \frac{18}{3} = \frac{104}{3}$$

Calculate $\frac{104}{3}$:

$$4 \times \frac{26}{3} = \frac{104}{3}$$

Both sides are equal, confirming the correctness of the solution.

Broader Context and Educational Significance

The Role of Systems of Equations

This problem exemplifies how systems of linear equations can be used to model relationships between variables. The approach—formulating equations based on given conditions, then solving them—forms a core method in algebra education and beyond.

Application in Real-World Scenarios

Problems like these mirror real-world situations where multiple constraints exist. For instance, in budgeting, resource allocation, or planning, variables often need to satisfy multiple conditions simultaneously. Understanding how to set up and solve systems of equations is thus a vital skill.

Analytical Strategies and Problem-Solving

While this problem was straightforward algebraically, more complex problems may require:

- Substitution or elimination methods
- Graphical interpretation
- Use of matrices or computational tools

The key is to understand the relationships and translate them accurately into mathematical expressions.

Alternative Perspectives and Extended Analyses

Role Reversal of Variables

Suppose the problem scenario is reversed:

> "The sum of two numbers is 23. Twice the other number increased by 6 is the same as 4 times the first."

In that case, the equations would swap roles, leading to an alternative set of solutions, emphasizing

the importance of carefully interpreting problem statements.

Considering Integer Solutions

Some problems specify that solutions must be integers. In this case, the solution involves fractions, so the problem would be classified as having rational solutions. If integer solutions are required, no solution exists in this case.

Exploring Variations and Generalizations

Varying the sum

Instead of a sum of 23, what if the sum was different? The methodology remains the same, but the solutions would adjust accordingly.

Different relationships

Changing the relationship condition, such as:

- "Twice one number increased by 6 equals 3 times the other number"
- "Twice one number increased by 6 equals the square of the other number"

would lead to different types of equations—linear or nonlinear—and require different solving techniques.

Practical Implications and Applications

While the problem appears theoretical, its principles underpin many practical applications:

- Engineering: Calculating component values subject to multiple constraints.
- Economics: Balancing budgets with multiple variables.
- Computer Science: Algorithm design involving multiple parameters.

Understanding how to methodically formulate and solve such problems enhances analytical capabilities across various disciplines.

Conclusion

The problem of "The sum of two numbers is 23. Twice one number increased by 6 is the same as 4 times the other number" showcases the elegant power of algebra in solving systems of equations. Through systematic formulation, substitution, and verification, we found a consistent set of solutions that satisfy both conditions.

This exploration underscores the importance of precise interpretation, algebraic manipulation, and verification in problem-solving—a skill that extends far beyond the classroom into real-world

applications. As we continue to encounter complex systems with multiple variables, the foundational techniques demonstrated here remain essential tools for mathematicians, scientists, engineers, and analysts alike.

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