

The Test Statistic Of $Z=2.46$ Is Obtained When Testing The Claim That $p < 0.25$.a. Using A Significance

The Test Statistic Of $Z=2.46$ Is Obtained When Testing The Claim That $p < 0.25$. Using A Significance

When conducting hypothesis tests about population proportions, the test statistic is a critical component that helps determine whether to accept or reject a null hypothesis. In the scenario where the test statistic $Z=2.46$ is obtained while testing the claim that $p < 0.25$, understanding the context, calculation, and interpretation of this Z-value is essential. This article explores the process of hypothesis testing for proportions, how to interpret a Z-score of 2.46, and the significance of this result in statistical decision-making.

Understanding Hypothesis Testing for Population Proportions

Hypothesis testing is a fundamental statistical method used to make inferences about a population parameter based on sample data. When dealing with proportions, the goal is often to test claims or assumptions about the true proportion p in the population.

The Null and Alternative Hypotheses

In testing the claim $p < 0.25$, the hypotheses are typically formulated as:

- Null hypothesis (H_0): $p \geq 0.25$
- Alternative hypothesis (H_1): $p < 0.25$

This is a one-tailed test, focusing on whether the proportion in the population is less than 0.25.

Significance Level (α)

The significance level, denoted as α , represents the probability of rejecting the null hypothesis when it is actually true (Type I error). Common choices for α include 0.05, 0.01, and 0.10. The significance level determines the critical value against which the test statistic is compared.

Calculating the Z-Score for Proportion Tests

The Z-score for testing a population proportion is calculated using the formula:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Where:

- $\hat{p}$ = sample proportion
- p_0 = hypothesized population proportion (in this case, 0.25)
- n = sample size

This Z-score indicates how many standard errors the sample proportion is away from the hypothesized proportion under the null hypothesis.

Interpreting a Z-Score of 2.46

In the provided scenario, a test statistic $Z=2.46$ is obtained. Since the alternative hypothesis is $p < 0.25$, the test is left-tailed. However, a Z-value of 2.46 is positive, which suggests that the sample proportion is higher than the hypothesized proportion, not lower.

Important note: In the context of testing $p < 0.25$, a Z-value of 2.46 typically would lead to not rejecting the null hypothesis because it indicates the sample proportion is significantly greater than 0.25. This implies that the sample data does not support the claim that p is less than 0.25.

Possible reasons for this Z-value include:

- The sample proportion $\hat{p}$ is greater than 0.25
- The sample size n is sufficiently large to produce a Z of 2.46 if the sample proportion exceeds 0.25

Critical Values and P-Values

The critical value corresponding to a Z of 2.46 in a left-tailed test at common significance levels can be identified from Z-tables:

Significance Level (α)	Critical Z-Value	Interpretation
0.05	-1.645	Since $Z=2.46 > -1.645$, we do not reject H_0 at $\alpha=0.05$
0.01	-2.33	$Z=2.46 > -2.33$, so no rejection at $\alpha=0.01$

Because the Z-value is positive and large, it indicates strong evidence against the claim $p < 0.25$ in favor of the null hypothesis $p \geq 0.25$, unless the test is set up differently.

P-Value Calculation

The p-value represents the probability of observing a test statistic as extreme as or more extreme than the one calculated, assuming the null hypothesis is true.

For $Z=2.46$ in a left-tailed test:

- Since the Z-value is positive, the p-value is the probability of Z being less than 2.46, which is approximately 0.9932.
- However, because the alternative hypothesis is $p < 0.25$, we are concerned with the lower tail ($Z < 0$), so the p-value would be $1 - 0.9932 = 0.0068$ if considering the upper tail.

But in a left-tailed test, the p-value is:

$$p = \Phi(Z)$$

where $\Phi(Z)$ is the cumulative distribution function (CDF) of the standard normal distribution at $Z=2.46$, which is approximately 0.9932. Since this is a high probability, it indicates that the observed data is not unusual under the null hypothesis.

Conclusion: The p-value is about 0.0068, which is less than common significance levels like 0.05. This suggests that if the test were set up as $p > 0.25$, the evidence would be against the null hypothesis. But since the hypotheses are $p < 0.25$, the positive Z-value indicates that the sample proportion is significantly greater than 0.25, leading us not to reject H_0 in that case.

Deciding Whether to Reject the Null Hypothesis

The decision to reject or fail to reject H_0 depends on the comparison between the p-value and the significance level α :

- If $p\text{-value} \leq \alpha$: reject H_0
- If $p\text{-value} > \alpha$: fail to reject H_0

In the case of $Z=2.46$, with a p-value of approximately 0.0068:

- At $\alpha=0.05$: $p\text{-value} < 0.05$, so reject H_0 if the test is set to detect $p > 0.25$ (which aligns with a right-tailed test).
- But for the original claim $p < 0.25$, the test is left-tailed, and the positive Z indicates the sample proportion is not less than 0.25, thus the evidence does not support the claim.

Implications of the Test Results

Based on the Z-value and p-value, the following conclusions can be drawn:

- If testing $p < 0.25$ (left-tailed): A Z of 2.46 indicates the sample proportion is significantly

greater than 0.25, so there is insufficient evidence to support the claim $p < 0.25$.

- If testing $p > 0.25$: The Z-score suggests strong evidence that p is greater than 0.25.

Key Takeaways:

- The sign and magnitude of the Z-score are crucial in hypothesis testing.
- A large positive Z-value in a test for $p < 0.25$ indicates the sample proportion is likely greater than 0.25.
- Proper formulation of hypotheses is essential to interpret the test statistic correctly.

Practical Applications of Z-Tests for Proportions

Z-tests for proportions are widely used across various fields, including:

- Medicine: To determine if a new treatment has a higher success rate.
- Marketing: To assess whether a new advertisement increases the proportion of customers making a purchase.
- Quality Control: To evaluate if the proportion of defective items exceeds acceptable limits.
- Polls and Surveys: To infer population opinions based on sample data.

Steps to Conduct a Z-Test for Population Proportions

1. State hypotheses: Null and alternative.
2. Choose significance level (α).
3. Calculate the sample proportion ($\hat{p}$).
4. Compute the test statistic (Z).
5. Determine the critical value or p-value.
6. Make a decision: Reject or fail to reject H_0 .
7. Interpret results in context.

Conclusion

The test statistic $Z=2.46$ plays a pivotal role in hypothesis testing regarding population proportions. Understanding how to compute, interpret, and apply this Z-value ensures accurate decision-making. In the context of testing whether $p < 0.25$, a $Z=2.46$ suggests that the sample data provides strong evidence against the claim if the test is set as a left-tailed hypothesis. Properly aligning hypotheses, significance levels, and test statistics is fundamental to deriving valid conclusions in statistical analysis.

Whether you're conducting a business survey, clinical trial, or quality check, mastering the interpretation of Z-scores like 2.46 enables you to make informed, data-driven decisions with confidence.

Frequently Asked Questions

What does a Z-test statistic of 2.46 indicate when testing the claim that $p < 0.25$?

A Z-value of 2.46 suggests that the sample proportion is 2.46 standard deviations above the hypothesized proportion of 0.25, indicating evidence against the null hypothesis if testing $p < 0.25$.

How do you interpret the significance of a $Z=2.46$ in hypothesis testing at a typical alpha level?

At an alpha level of 0.05, a $Z=2.46$ corresponds to a p-value less than 0.05, leading to rejection of the null hypothesis that $p \geq 0.25$, supporting the claim that $p < 0.25$.

What is the critical value for a one-tailed test at $\alpha = 0.05$, and how does $Z=2.46$ compare?

The critical value for a one-tailed test at $\alpha=0.05$ is approximately 1.645. Since $2.46 > 1.645$, the result is statistically significant, and the null hypothesis can be rejected.

How is the Z statistic of 2.46 calculated in hypothesis testing for proportions?

The Z statistic is calculated as (sample proportion - hypothesized proportion) divided by the standard error of the proportion, i.e., $Z = (\hat{p} - p_0) / \sqrt{(p_0(1 - p_0)/n)}$.

What assumptions are made when using the Z-test with $Z=2.46$ for proportion testing?

Assumptions include a sufficiently large sample size for the normal approximation, random sampling, and that the data follows a binomial distribution for the proportion.

If the Z-value of 2.46 is obtained, what does this mean about the null hypothesis $p \geq 0.25$?

It suggests strong evidence against the null hypothesis, supporting the alternative hypothesis that $p < 0.25$, assuming the test is one-tailed and significance level is appropriate.

What is the approximate p-value associated with $Z=2.46$ in a one-tailed test?

The p-value is approximately 0.0071, indicating a low probability of observing such a Z-value if the null hypothesis is true, supporting its rejection at common significance levels.

How does the obtained $Z=2.46$ influence decision-making in hypothesis testing for $p < 0.25$?

Since $Z=2.46$ exceeds the critical value for typical significance levels, it leads to rejecting the null hypothesis and concluding there is evidence that p is less than 0.25.

Can a Z-value of 2.46 be considered statistically significant at the 1% significance level?

No, because the critical Z-value at 1% significance for a one-tailed test is approximately 2.33, and since $2.46 > 2.33$, it is statistically significant at the 1% level, leading to rejection of the null hypothesis.

Additional Resources

Test Statistic $Z=2.46$: An In-Depth Analysis of Its Role in Hypothesis Testing for $p < 0.25$

Introduction

In the realm of statistical inference, hypothesis testing serves as a fundamental tool for making data-driven decisions. Among the various types of tests, those involving proportions—particularly when examining claims about population proportions—are prevalent in fields ranging from medicine and social sciences to quality control and market research. A pivotal component in these tests is the test statistic, which quantifies how much the observed data deviate from the null hypothesis.

One such test statistic, $Z=2.46$, often arises in the context of testing a claim that a population proportion (p) is less than 0.25. This article explores the significance of this Z value, delves into the intricacies of hypothesis testing, and provides a comprehensive understanding of how such a statistic informs decision-making, especially when evaluated against a specified significance level.

Understanding the Context: Testing the Claim $(p < 0.25)$

When researchers or analysts test a hypothesis like "the population proportion (p) is less than 0.25", the goal is to determine whether the observed data provide sufficient evidence to support this claim or if the data are consistent with the null hypothesis.

Null Hypothesis (H_0) : $p = 0.25$

Alternative Hypothesis (H_a) : $p < 0.25$

This setup is a left-tailed test because we are examining whether the true proportion is less than a specific value.

The Role of the Test Statistic in Hypothesis Testing

A test statistic is a standardized value that measures how far the sample data deviate from the null hypothesis, scaled by the standard error. Specifically, in tests involving proportions, the Z-statistic is employed because of the normal approximation to the binomial distribution, provided certain conditions are met.

Why Z?

The Z-test for proportions relies on the Central Limit Theorem, which states that for sufficiently large samples, the sampling distribution of the proportion approximates a normal distribution.

Calculating the Z-statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Where:

- $\hat{p}$ = observed sample proportion
- p_0 = hypothesized population proportion (here, 0.25)
- n = sample size

This formula standardizes the difference between the observed proportion and the hypothesized proportion, allowing for comparison with the standard normal distribution.

Interpreting $Z=2.46$ in the Context of Testing $(p < 0.25)$

At first glance, a Z value of 2.46 indicates that the sample proportion is 2.46 standard errors above the hypothesized value (0.25). However, this interpretation must be contextualized within the test's directionality and significance level.

Important Note:

In the scenario described, the obtained $Z=2.46$ suggests the sample proportion is greater than 0.25 (assuming the calculation is as per the standard formula). But since the claim is $(p < 0.25)$, a Z-value greater than zero contradicts the alternative hypothesis, hinting at the sample data being inconsistent with the claim.

Is there a discrepancy?

Yes. Typically, for testing $(p < 0.25)$, a small Z-value (negative and large in magnitude) indicates evidence supporting the claim. A $Z=2.46$ suggests the sample proportion exceeds 0.25 significantly, leading to a potential rejection of the null in favor of an alternative like $(p > 0.25)$.

Therefore, in an actual analysis, the sign and the context of the Z value are crucial.

For the purpose of this article, we'll assume the $Z=2.46$ was computed from data where the observed proportion was indeed greater than 0.25, and we are exploring the implications of

this Z in hypothesis testing.

Significance Level and Critical Regions

In hypothesis testing, the significance level (α) represents the threshold for deciding whether to reject H_0 . Common α values include 0.05, 0.01, and 0.10.

How does the significance level relate to the Z-value?

- For a left-tailed test at $\alpha=0.05$, the critical Z-value is approximately -1.645.
- For a right-tailed test at $\alpha=0.05$, critical Z is approximately +1.645.
- For a two-tailed test at $\alpha=0.05$, critical Z-values are approximately ± 1.96 .

Since the $Z=2.46$ is positive, it falls into the right tail of the normal distribution, which is relevant if the alternative hypothesis is $p > 0.25$.

Implication:

- If testing $p < 0.25$, a large positive Z-value (like 2.46) suggests evidence against the null hypothesis in favor of $p > 0.25$.
- Conversely, if the Z-value is negative and less than the critical threshold, it supports $p < 0.25$.

Decision-Making: Rejecting or Failing to Reject H_0

Given the $Z=2.46$, and assuming a significance level $\alpha=0.05$:

- Critical value for right tail: +1.645
- Observed Z: 2.46

Since $2.46 > 1.645$, we reject the null hypothesis at the 5% significance level, indicating the data provide sufficient evidence to conclude $p > 0.25$.

But what about the claim $p < 0.25$?

In this case, the Z-value contradicts the claim, leading us to fail to support the hypothesis that $p < 0.25$. Instead, the data suggest the proportion might be greater than 0.25.

Practical Example: Applying the Test

Suppose a researcher is testing whether a new manufacturing process results in a defect rate less than 25%. They collect a sample of 200 units and find 60 defective units.

Step 1: Calculate $\hat{p}$:

$$\hat{p} = \frac{60}{200} = 0.30$$

\]

Step 2: State hypotheses:

\[

$H_0: p = 0.25$

\]

\[

$H_a: p < 0.25$

\]

Step 3: Compute the Z-statistic:

\[

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = \frac{0.30 - 0.25}{\sqrt{\frac{0.25 \times 0.75}{200}}}$$

\]

Calculate denominator:

\[

$$\sqrt{\frac{0.1875}{200}} = \sqrt{0.0009375} \approx 0.0306$$

\]

Calculate numerator:

\[

0.05

\]

Thus,

\[

$$Z \approx \frac{0.05}{0.0306} \approx 1.63$$

\]

Step 4: Interpret the Z-value:

- For $(\alpha=0.05)$, critical Z-value (left tail): -1.645

- Since $1.63 > -1.645$, and in fact, it's positive, we do not reject (H_0) .

- Because the Z-value is positive and not in the left tail, the data do not support the claim that $(p < 0.25)$. Instead, the sample suggests $(p \geq 0.25)$.

Broader Implications and Expert Insights

Understanding the $Z=2.46$ value and its significance is crucial in rigorous hypothesis testing.

It emphasizes the importance of:

- Correctly specifying hypotheses
- Calculating the test statistic accurately
- Comparing it against the appropriate critical value based on the significance level
- Interpreting the results within the context of the research question

In practice, such statistical findings influence critical decisions, such as approving quality control processes, validating clinical trial claims, or assessing market strategies.

Limitations and Considerations

While the Z-test is powerful, it relies on assumptions that should be validated:

- Sample Size: The approximation to the normal distribution is reliable when both (np_0) and $(n(1-p_0))$ are greater than 5 or 10.
- Random Sampling: The data should be collected randomly to avoid bias.
- Independence: Observations must be independent.

Violating these assumptions can lead to misleading conclusions, underscoring the importance of proper study design and analysis.

Final Thoughts

The test statistic $Z=2.46$ serves as a pivotal metric in hypothesis testing concerning proportions. Its magnitude and sign directly influence whether we accept or reject the null hypothesis, especially when evaluated against a pre-determined significance level.

In the specific context

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