

This Circle Is Centered At The Origin, And The Length Of Its Radius Is 5. What Is The Equation Of The

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Understanding the equation of a circle is fundamental in coordinate geometry, especially when dealing with circles centered at the origin. When a circle is centered at the origin (0,0), and its radius is known, deriving its equation becomes straightforward. In this article, we will explore the specific case where the circle is centered at the origin, with a radius of 5, to determine its exact equation. Whether you're a student preparing for exams or someone interested in geometry, this comprehensive guide will clarify the concepts and provide practical insights.

Fundamentals of the Equation of a Circle

Before delving into the specific case, it's essential to understand the general form of a circle's equation and the key components involved.

Standard Equation of a Circle

The most common form of a circle's equation in coordinate geometry is:

- $(x - h)^2 + (y - k)^2 = r^2$

Where:

- (h, k) are the coordinates of the circle's center.
- r is the radius of the circle.

Special Case: Circle Centered at the Origin

When the circle's center is at the origin (0,0), the equation simplifies to:

- $x^2 + y^2 = r^2$

This is because $h = 0$ and $k = 0$, removing the linear terms involving $(x - h)$ and $(y - k)$.

Determining the Equation for a Circle Centered at the Origin with Radius 5

Given the specific details:

- Center: (0, 0)
- Radius: 5

we can directly substitute these into the standard form.

Step-by-Step Derivation

1. Identify the center and radius:

- Center (h, k): (0, 0)
- Radius (r): 5

2. Substitute into the standard form:

- $x^2 + y^2 = r^2$

3. Calculate r^2 :

- $5^2 = 25$

4. Write the final equation:

- $x^2 + y^2 = 25$

Therefore, the equation of the circle centered at the origin with a radius of 5 is:

- $x^2 + y^2 = 25$

Understanding the Graph of the Circle

Visualizing the circle helps in grasping its geometric properties.

Graphical Representation

- The circle is centered at the point (0, 0).
- All points on the circle are exactly 5 units away from the origin.
- The circle intersects the axes at $(\pm 5, 0)$ and $(0, \pm 5)$.

Key Coordinates on the Circle

- Points on the x-axis: (5, 0) and (-5, 0)
- Points on the y-axis: (0, 5) and (0, -5)

These points satisfy the equation $x^2 + y^2 = 25$.

Applications and Significance of the Equation

Understanding this specific equation has practical applications in various fields.

Coordinate Geometry and Trigonometry

- The equation helps in analyzing distances and angles relative to the origin.
- Useful in trigonometric problems involving circles.

Physics and Engineering

- Modeling circular motion where the center is at the origin.
- Designing components that require precise circular dimensions.

Computer Graphics and Visualization

- Rendering circles centered at the origin in digital environments.
- Calculating pixel positions along the circle's perimeter.

Additional Insights and Variations

While the specific case discussed is straightforward, understanding variations enhances your overall grasp.

Circle with Different Centers or Radii

- If the center shifts away from the origin, the equation becomes $(x - h)^2 + (y - k)^2 = r^2$.
- The radius can be any positive value, affecting the size of the circle.

Equations in Different Forms

- General form: $Ax^2 + By^2 + Cx + Dy + E = 0$
- Parametric equations: $x = h + r \cos \theta$, $y = k + r \sin \theta$

Practice Problems to Reinforce Learning

To solidify your understanding, consider solving these problems:

1. Write the equation of a circle with center at (0, 0) and radius 7.
2. Find the equation of a circle centered at the origin with radius 3.
3. Determine whether the point (4, 3) lies on the circle $x^2 + y^2 = 25$.
4. Graph the circle described by $x^2 + y^2 = 25$ and identify its intercepts.
5. How does the equation change if the circle's center is at (2, -3) with the same radius?

Answers:

1. $x^2 + y^2 = 49$
2. $x^2 + y^2 = 9$
3. Yes, because $4^2 + 3^2 = 16 + 9 = 25$, which satisfies the equation.
4. Intercepts at $(\pm 5, 0)$ and $(0, \pm 5)$.
5. $(x - 2)^2 + (y + 3)^2 = 25$

Conclusion

The equation of a circle centered at the origin with a radius of 5 is succinctly expressed as $x^2 + y^2 = 25$. This fundamental form is essential for solving a wide range of problems in geometry, physics, engineering, and computer graphics. Understanding how to derive and interpret this equation enhances your mathematical fluency and provides a foundation for tackling more complex geometric figures. Remember, whenever the circle is centered at the origin, simply square the radius and set the sum of the squares of x and y equal to this value — a straightforward yet powerful principle in coordinate geometry.

Frequently Asked Questions

What is the standard form of the equation for a circle centered at the origin with radius 5?

The equation is $x^2 + y^2 = 25$.

How do you determine the equation of a circle centered at the origin with a radius of 5?

Use the formula $x^2 + y^2 = r^2$, so with $r = 5$, it becomes $x^2 + y^2 = 25$.

What is the significance of the radius in the equation of a circle centered at the origin?

The radius determines the distance from the origin to any point on the circle, and in the equation, r^2

appears as the constant term.

Can the equation $x^2 + y^2 = 25$ represent any other circle besides the one centered at the origin with radius 5?

No, $x^2 + y^2 = 25$ specifically represents a circle centered at the origin with radius 5; shifting the center would change the equation.

If a point (3, 4) lies on the circle, does it satisfy the equation $x^2 + y^2 = 25$?

Yes, because $3^2 + 4^2 = 9 + 16 = 25$, which matches the equation of the circle.

How does the equation change if the circle is centered at a point other than the origin?

The equation becomes $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the new center.

What is the geometric interpretation of the equation $x^2 + y^2 = 25$?

It represents all points in the plane that are exactly 5 units away from the origin.

Is the circle with the equation $x^2 + y^2 = 25$ a perfect circle, and why?

Yes, because the equation satisfies the standard form of a circle with a constant radius, indicating a perfect circle.

How can I graph the circle given by $x^2 + y^2 = 25$?

Plot the center at the origin $(0,0)$ and draw a circle with radius 5 units in all directions from the origin.

Additional Resources

This Circle Is Centered At The Origin, And The Length Of Its Radius Is 5. What Is The Equation Of The

Understanding the fundamental aspects of circles in coordinate geometry is essential for students and enthusiasts alike. The statement, "This circle is centered at the origin, and the length of its radius is 5," provides a straightforward yet vital scenario that encapsulates core concepts of circle equations. In this article, we will explore the underlying principles, derive the equation, and examine related concepts to deepen your understanding of circle geometry.

Understanding the Basic Equation of a Circle

Standard Form of a Circle's Equation

The general equation of a circle in the coordinate plane is given by:

$$(x - h)^2 + (y - k)^2 = r^2$$

where:

- (h, k) is the center of the circle,
- r is the radius of the circle.

This form is known as the standard form of the circle's equation. It clearly indicates the position of the circle and its size.

Key features:

- The center is located at (h, k) .
- The radius is the distance from the center to any point on the circle.

Deriving the Equation for the Given Circle

Identifying the Parameters

From the given statement:

- The circle is centered at the origin, meaning $h = 0$ and $k = 0$.
- The radius is 5, meaning $r = 5$.

Plugging these values into the standard form:

$$(x - 0)^2 + (y - 0)^2 = 5^2$$

which simplifies to:

$$x^2 + y^2 = 25$$

Therefore, the equation of the circle is:

$$\boxed{x^2 + y^2 = 25}$$

This is the most straightforward form of the circle's equation with the specified properties.

Analyzing the Circle's Equation

Graphical Interpretation

- The circle is centered at the origin $((0, 0))$.
- The radius extends 5 units in all directions from the origin, covering points like $((5, 0))$, $((0, 5))$, $((-5, 0))$, and $((0, -5))$.

Visual features:

- Symmetrical around both axes.
- All points satisfying $(x^2 + y^2 = 25)$ lie on the circle.

Properties of the Circle

- Diameter: $(2r = 10)$
- Circumference: $(2\pi r = 10\pi \approx 31.416)$
- Area: $(\pi r^2 = 25\pi \approx 78.54)$

Applications and Contexts

Coordinate Geometry Problems

Knowing the equation allows for solving various problems such as:

- Finding intersection points with other shapes or lines.
- Determining if a given point lies on the circle.
- Computing distances between points and the circle.

Real-World Examples

- Designing circular tracks or arenas with known centers and radii.
- Analyzing orbit paths in physics, where the origin may represent a central point like a star or

planet.

- Creating graphics in computer-aided design (CAD) software.

Extending the Concept: Variations and Related Equations

Circle with Different Center

If the circle's center is moved away from the origin, say to $((h, k))$, the equation becomes:

$$\boxed{(x - h)^2 + (y - k)^2 = r^2}$$

For example, a circle centered at $((3, -4))$ with radius 5:

$$\boxed{(x - 3)^2 + (y + 4)^2 = 25}$$

Features:

- The shift affects the position but not the size.
- The graph shifts accordingly in the coordinate plane.

Circle with Variable Radius

If the radius varies, the equation becomes parametric or part of a family of circles:

$$\boxed{(x - h)^2 + (y - k)^2 = r(t)^2}$$

where $r(t)$ is a function of some parameter t .

Common Mistakes and Clarifications

- Confusing the form: Remember that the standard form explicitly shows the center and radius; the general form $(Ax^2 + By^2 + Cx + Dy + E = 0)$ can be converted to standard form.
- Incorrect signs: The signs in the equation are crucial. For a circle centered at $((h, k))$, the terms are $((x - h)^2)$ and $((y - k)^2)$.
- Radius squared: Always remember to square the radius in the equation.

Conclusion

The statement “This circle is centered at the origin, and the length of its radius is 5” succinctly captures a fundamental geometric shape. By applying the standard form of the circle’s equation, we derive:

$$[x^2 + y^2 = 25]$$

This simple yet powerful equation allows for a multitude of applications across mathematics, physics, engineering, and computer graphics. Understanding how to interpret and manipulate circle equations lays the foundation for more complex geometric and analytical problems.

In summary, the key takeaways are:

- The standard form is essential for understanding the properties and position of a circle.
- The parameters directly influence the shape and location.
- Extending these concepts enables solving more advanced problems involving circles.

Whether for academic purposes or practical applications, mastering the derivation and interpretation of circle equations is an indispensable skill in the toolkit of anyone working with coordinate geometry.

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