This System Of Equations Is Shown On The Graph: 2y 4x = 6 Y = 2x + 3 Which Statement About The System

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Understanding systems of equations is fundamental in algebra and many real-world applications. When two equations are graphed on the coordinate plane, their points of intersection reveal critical information about their solutions. In this article, we will analyze the system consisting of the equations $2y + 4x = 6$ and $y = 2x + 3$, explore how they are represented on a graph, and determine what statements can be made about their relationship. By delving into the graphing process, solving methods, and interpretations, you'll gain a comprehensive understanding of how to interpret systems of equations visually and algebraically.

Introduction to Systems of Equations

A system of equations involves two or more equations with the same variables. The goal is often to find the point(s) where these equations intersect, which represents the solution(s) to the system. There are three possible scenarios:

- **One solution:** The lines intersect at exactly one point.
- **Infinite solutions:** The lines coincide, meaning they are the same line.
- **No solution:** The lines are parallel and never intersect.

Identifying which scenario applies depends on the equations involved and their graphical representation.

Analyzing the Given Equations

Our focus is on the system:

```
2y + 4x = 6
y = 2x + 3
```

Let's examine each equation individually and then analyze their relationship.

Equation 1: $2y + 4x = 6$

This is a linear equation in standard form. To graph it, it's often easiest to convert it to slope-intercept form ($y = mx + b$).

Converting to slope-intercept form:

1. Start with $2y + 4x = 6$.
2. Subtract $4x$ from both sides:

$2y = -4x + 6$

3. Divide both sides by 2:

$y = -2x + 3$

Interpretation:

- Slope (m): -2
- Y-intercept (b): 3

This line crosses the y-axis at (0,3) and descends with a slope of -2.

Equation 2: $y = 2x + 3$

This is already in slope-intercept form, with:

- Slope (m): 2
- Y-intercept (b): 3

This line crosses the y-axis at (0,3) and ascends with a slope of 2.

Graphical Representation of the System

Plotting both lines will reveal their points of intersection and help determine the nature of their solutions.

Steps to Graph Each Line:

1. Identify the y-intercept (b): For both lines, it's 3, so plot the point (0,3) on the graph.

2. Use the slope (m) to find another point:

    - For $y = -2x + 3$, from (0,3), move down 2 units and right 1 unit to (1,1).
    - For $y = 2x + 3$, from (0,3), move up 2 units and right 1 unit to (1,5).

3. Draw straight lines through these points, extending across the graph.

Expected Graphical Outcome

- The line $y = -2x + 3$ (red or solid line, for example) slopes downward.
- The line $y = 2x + 3$ (blue or dashed line) slopes upward.
- Both lines cross at a single point, which indicates a unique solution.

Finding the Solution Algebraically

While the graphical approach provides visual insight, solving algebraically confirms the exact point of intersection.

Method: Substitution

Since $y = 2x + 3$ from the second equation, substitute into the first:

$2(2x + 3) + 4x = 6$

Simplify:

$4x + 6 + 4x = 6$

Combine like terms:

$8x + 6 = 6$

Subtract 6 from both sides:

$8x = 0$

Divide both sides by 8:

$x = 0$

Now, substitute $x = 0$ back into $y = 2x + 3$:

y = 2(0) + 3 = 3

Solution:

- The lines intersect at the point (0,3).

Interpreting the Solution and the System's Nature

Based on both the graphical and algebraic analysis, we observe:

- The lines intersect at exactly one point.
- The point of intersection is (0,3).

Thus, the system has a unique solution.

Implications of the Solution

- The unique solution indicates that the lines are neither parallel nor coincident.
- The different slopes (-2 and 2) confirm that the lines are not parallel.
- They are not the same line, as their equations differ beyond the slope and intercept.

What Statements Can Be Made About the System?

Given the analysis, the following statements are valid:

- The system has exactly one solution.
- The solution is the point (0,3).
- The lines intersect at a single point on the graph.
- The lines are neither parallel nor coincident.
- The system is consistent and independent.

Additional Notes:

- If the slopes were equal and intercepts different, the lines would be parallel with no solutions.
- If the equations were multiples of each other, they would be coincident, leading to infinitely many solutions.

Conclusion

Understanding the graphical representation of systems of equations, such as the one with 2y + 4x = 6 and y = 2x + 3, is essential for visualizing solutions and their relationships. In this case, the lines intersect at exactly one point, (0,3), confirming that the system has a unique solution. Recognizing the slopes and intercepts helps predict the nature of the solutions before solving algebraically, reinforcing the importance of both methods in algebra.

By mastering these concepts, students and practitioners can confidently analyze and interpret systems of equations in various mathematical and real-world contexts, from engineering to economics.

Frequently Asked Questions

What do the equations 2y + 4x = 6 and y = 2x + 3 represent on the graph?

They represent two lines plotted on the same coordinate plane, and their intersection point shows the solution to the system.

How can we determine if the system has one solution, no solution, or infinitely many solutions from the graph?

By observing whether the lines intersect at a single point (one solution), are parallel and do not intersect (no solution), or are coincident (infinite solutions).

Are the two equations in the system consistent or inconsistent?

They are consistent if they intersect at a point, which appears to be the case here, indicating the system has at least one solution.

What is the approximate solution to the system based on the graph?

The approximate point of intersection can be found by solving the equations algebraically or estimating from the graph; it appears near (0.75, 3.5).

Can the equations be rewritten in slope-intercept form? If so, what are their slopes?

Yes. The second equation is already in slope-intercept form y = 2x + 3 with slope 2. The first equation can be rewritten as y = -2x + 1. So, the slopes are 2 and -2.

Are the lines represented by these equations perpendicular, parallel, or intersecting at an angle?

Since their slopes are 2 and -2, which are negative reciprocals, the lines are perpendicular and intersect at a right angle.

What does the intersection point tell us about the solution to the system?

The intersection point gives the unique solution (x, y) that satisfies both equations simultaneously.

Additional Resources

Understanding the System of Equations: An In-Depth Analysis

When exploring the world of algebra, systems of equations are fundamental concepts that help us understand relationships between multiple variables simultaneously. The system in question, comprising the equations $2y + 4x = 6$ and $y = 2x + 3$, offers a rich opportunity to examine how algebraic methods translate into graphical representations and what these representations reveal about the nature of the solutions. This review delves into each component of the system, the graphical implications, and the key insights that can be gained from analyzing these equations together.

Breaking Down the System of Equations

Before interpreting the graph, it’s essential to understand each equation individually and how they relate to each other.

The First Equation: $2y + 4x = 6$

- Type of Equation: Linear
- Standard Form: Ax + By = C
- Rearranged for Slope-Intercept Form:

$$
2y + 4x = 6 \Rightarrow 2y = -4x + 6 \Rightarrow y = -2x + 3
$$

- Characteristics:
- Slope (m): -2
- Y-intercept (b): 3

The Second Equation: $y = 2x + 3$

- Type of Equation: Linear
- Slope-Intercept Form: Already provided
- Slope (m): 2
- Y-intercept (b): 3

Graphical Interpretation of the Equations

Understanding the graph of these equations involves plotting their lines based on slope and intercepts.

Graphing $y = -2x + 3$

- Slope: -2, indicates the line descends 2 units vertically for every 1 unit horizontally.
- Y-intercept: 3, meaning the line crosses the y-axis at $(0, 3)$.
- X-intercept: To find where the line crosses the x-axis:

$$
0 = -2x + 3 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2} = 1.5
$$

So, the line crosses the x-axis at $(1.5, 0)$.

Graphing $y = 2x + 3$

- Slope: 2, indicating the line ascends 2 units vertically for every 1 unit horizontally.
- Y-intercept: 3, crossing the y-axis at $(0, 3)$.
- X-intercept: To find the x-intercept:

$$
0 = 2x + 3 \Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2} = -1.5
$$

The line crosses the x-axis at $(-1.5, 0)$.

Analyzing the Graphs: Do the Lines Intersect?

The core of understanding the system lies in determining whether the lines intersect, are parallel, or coincide.

Comparison of Slopes

- The slope of $y = -2x + 3$ is -2.

- The slope of $y = 2x + 3$ is 2.

Since these slopes are not equal, the lines are not parallel. Therefore, they must intersect at exactly one point.

Point of Intersection

- The intersection point can be found algebraically by solving the system:
$$
y = -2x + 3
$$
$$
y = 2x + 3
$$
- Equate the two expressions:
$$
-2x + 3 = 2x + 3
$$
- Simplify:
$$
-2x + 3 = 2x + 3 \Rightarrow -2x = 2x \Rightarrow 4x = 0 \Rightarrow x = 0
$$
- Substitute $x = 0$ into either equation to find y:
$$
y = 2(0) + 3 = 3
$$
- Intersection Point: $(0, 3)$

Graphical Significance of the Intersection

- The point $(0, 3)$ lies on both lines.
- Graphically, this confirms the lines cross at this point, which is the unique solution to the system.

Classification of the System’s Nature

Based on the graphical and algebraic analysis, we can classify the system:

Consistent and Independent System

- Consistent: Because it has at least one solution (the intersection point).
- Independent: Because the lines are neither parallel nor coincident (they intersect at exactly one point).

Implications of the Solution

- The solution $(0, 3)$ satisfies both equations.
- The system describes two lines crossing at a single point, embodying the typical scenario of a unique solution in two-variable linear systems.

What Statements About the System Are Correct?

Given this analysis, certain statements about the system can be confidently evaluated:

Possible Correct Statements

- The system has exactly one solution.
This is confirmed by the intersection at $(0, 3)$.

- The lines are not parallel.
Since their slopes differ (-2 and 2), they intersect at one point.

- The solution point lies on both lines.
As shown, $(0, 3)$ is on both graphs.

- The system is consistent but not dependent.
It has a single unique solution, so it’s consistent and independent.

Incorrect Statements

- The lines are parallel.
Because their slopes are different, they are not parallel.

- The lines are coincident.
They are not coincident, as their slopes differ.

- The system has infinitely many solutions.
Only true if the lines coincided, which they do not.

Deep Dive: Algebraic vs. Graphical Representation

Understanding the relationship between algebraic equations and their graphical counterparts provides clarity on solving systems.

Algebraic Methods for Solving

- Substitution Method: Using the explicit form $y = 2x + 3$ to substitute into the other equation.
- Elimination Method: Combining equations to eliminate variables, as demonstrated above.

Graphical Method

- Plotting both lines on a coordinate plane.
- Observing the intersection point visually.
- Estimating solutions when exact algebraic calculations are complex or impossible.

Advantages and Limitations

- Algebraic solutions are precise but can be computationally intensive.
- Graphical solutions provide visual insight but may lack exactness unless plotted carefully.

Extending the Analysis: Real-World Applications

Systems of equations like these aren’t just mathematical exercises—they model real-world scenarios.

Applications in Business and Economics

- Cost and Revenue Analysis: Equations can represent cost functions and revenue functions, with the intersection indicating break-even points.

- Supply and Demand: Equilibrium in markets can be represented by systems similar to this, where the intersection point indicates market equilibrium.

Engineering and Physics

- Motion Analysis: Equations may describe objects' paths, with intersection points indicating collision points or interactions.
- Electrical Engineering: Circuit analysis often involves solving systems of equations to find currents or voltages.

In Education

- Systems of equations serve as foundational concepts to develop problem-solving skills and understanding of linear relationships.

Conclusion: Summary and Final Remarks

The system $2y + 4x = 6$ and $y = 2x + 3$ exemplifies many fundamental principles of algebra and graphing. The key takeaways include:

- The two lines are not parallel and intersect at a single point.
- The point of intersection is $(0, 3)$, which satisfies both equations.

- The system is consistent and independent, leading to a unique solution.
- Graphically, the lines cross at the point \((0, 3)\), illustrating how algebraic solutions translate into visual interpretations.

Understanding this system deepens comprehension of linear equations, their graphical behaviors, and their applications across various disciplines. It highlights the importance of analyzing both the algebraic form and the graphical representation to fully grasp the nature of solutions within systems of equations. Whether approached through algebraic manipulation or visual plotting,

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