Three Moles Of An Ideal Monatomic Gas Are At A Temperature Of 345 K. Then, 2438 J Of Heat Is Added To

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In thermodynamics, understanding the behavior of gases under various conditions is fundamental. When dealing with ideal monatomic gases, the relationships between heat, temperature, and internal energy are well-defined and provide valuable insights into energy transfer processes. This article explores a specific scenario involving three moles of an ideal monatomic gas initially at 345 K, to analyze the effects of adding 2438 J of heat. We will examine the concepts of internal energy, heat capacity, work done, and temperature change, providing a comprehensive understanding suitable for students, educators, and enthusiasts interested in thermodynamics.

Understanding the Initial Conditions of the Gas

Number of Moles and Nature of the Gas

- Number of Moles (n): 3 mol
- Type of Gas: Ideal monatomic gas (e.g., Helium, Argon)
- Initial Temperature (T_1): 345 K

An ideal monatomic gas consists of particles with negligible volume and no intermolecular forces. The kinetic theory provides that the internal energy of such a gas depends solely on temperature, simplifying calculations related to heat transfer and energy changes.

Initial Internal Energy of the Gas

The internal energy (U) of an ideal monatomic gas is given by:

$$ U = \frac{3}{2} n R T $$

Where:
- n = number of moles
- R = universal gas constant (8.314 J/mol·K)
- T = temperature in Kelvin

Calculating the initial internal energy:

$$
U_1 = \frac{3}{2} \times 3 \times 8.314 \times 345 \approx \frac{3}{2} \times 3 \times 8.314 \times 345
$$

$$
U_1 \approx 1.5 \times 3 \times 8.314 \times 345
$$

$$
U_1 \approx 4.5 \times 8.314 \times 345
$$

$$
U_1 \approx 4.5 \times 2867.13 \approx 12901.09\, \text{J}
$$

Thus, the initial internal energy of the gas is approximately 12,901.09 J.

Adding Heat to the Gas: Key Concepts

When heat is added to a gas, it can result in:

- An increase in the internal energy (leading to temperature rise)
- Work done by the gas during expansion or compression
- Both effects depending on the process conditions (constant volume, constant pressure, or others)

Understanding these effects requires analyzing the type of process involved, which influences how heat, work, and energy interplay.

Types of Thermodynamic Processes

- Isochoric process: Volume remains constant; heat added increases internal energy and temperature.
- Isobaric process: Pressure remains constant; heat added causes temperature increase and work done by the gas.
- Adiabatic process: No heat exchange; energy change is due to work done.

In this scenario, since the problem states only that heat is added, without specifying the process, the most straightforward assumption is an isochoric process (constant volume), unless otherwise specified.

Calculating the Change in Internal Energy and Temperature

Heat Added (Q) and Its Implications

Given:
- $Q = 2438\, \text{J}$

In an ideal monatomic gas undergoing an isochoric process, all the heat supplied goes into increasing the internal energy:

$$
Q = \Delta U
$$

Therefore:

$$
\Delta U = Q = 2438\, \text{J}
$$

The new internal energy:

$$
U_2 = U_1 + Q = 12901.09 + 2438 \approx 15339.09\, \text{J}
$$

Determining the Final Temperature

Since internal energy of an ideal monatomic gas depends only on temperature:

$$
U = \frac{3}{2} n R T
$$

Rearranged to solve for the new temperature T_2:

$$
T_2 = \frac{2 U_2}{3 n R}
$$

Plugging in the values:

$$
T_2 = \frac{2 \times 15339.09}{3 \times 8.314 \times 3}
$$

$$
T_2 = \frac{30678.18}{3 \times 8.314 \times 3}
$$

$$
T_2 = \frac{30678.18}{74.826}
$$

$$
T_2 \approx 410\, \text{K}
$$

Final Temperature after heat addition: approximately 410 K, indicating a temperature increase of:

$$
\Delta T = T_2 - T_1 = 410 - 345 = 65\, \text{K}
$$

Work Done During the Process

If the process involves volume change, work is performed by or on the gas. The work done (W) in a thermodynamic process is:

$$
W = P \Delta V
$$

However, in an isochoric process:

$$
\Delta V = 0 \Rightarrow W = 0
$$

Thus, no work is done in a constant volume process, and all the heat supplied increases the internal energy and temperature.

In contrast, if the process occurs at constant pressure, the work done is:

$$
W = n R \Delta T
$$

Calculating:

$$
W = 3 \times 8.314 \times 65 \approx 3 \times 8.314 \times 65 \approx 1619.13\, \text{J}
$$

Depending on the actual process, the work done can be significant.

Summary of Key Results

Parameter	Value
Initial internal energy U_1	~12,901 J
Heat added Q	2,438 J
Final internal energy U_2	~15,339 J
Initial temperature T_1	345 K
Final temperature T_2	~410 K
Temperature change ΔT	65 K
Work done (assuming isochoric)	0 J

Implications and Applications

Understanding how heat affects the internal energy and temperature of an ideal monatomic gas has multiple practical applications:

- Thermodynamic cycle analysis: Essential in designing engines and refrigerators.
- Calorimetry: Measuring heat transfer in gases.
- Material science: Studying gas behaviors under heating.

This scenario demonstrates fundamental principles such as energy conservation, heat transfer, and the dependence of internal energy on temperature in ideal gases.

Real-World Considerations and Limitations

While ideal gases provide a simplified model, real gases can deviate due to factors like intermolecular forces and non-negligible molecular volume. In practical applications:

- Non-ideal behaviors may alter energy and temperature calculations.

- Process constraints (constant pressure, volume, or adiabatic) must be explicitly specified to determine work and heat transfer accurately.
- Heat losses and environmental interactions can influence actual outcomes.

Despite these limitations, the ideal gas model remains a cornerstone of thermodynamics education and analysis.

Conclusion

Adding 2438 J of heat to three moles of an ideal monatomic gas initially at 345 K results in a significant temperature increase to approximately 410 K, assuming a constant volume process. The internal energy increases by the same amount of heat supplied, reflecting the direct proportionality between internal energy and temperature for ideal monatomic gases. This analysis exemplifies core thermodynamic principles, emphasizing the relationship between heat, internal energy, temperature, and work. Such understanding is critical in fields ranging from engineering to atmospheric science, underpinning the design and analysis of thermodynamic systems.

SEO Keywords: ideal monatomic gas, heat transfer, internal energy, temperature change, thermodynamics, heat capacity, work done, ideal gas law, energy transfer, thermodynamic process, heat addition, temperature increase, gas behavior, physics, engineering

Frequently Asked Questions

What is the initial temperature of the three moles of the ideal monatomic gas?

The initial temperature of the gas is 345 K.

How much heat energy is added to the gas in the process?

A total of 2438 J of heat energy is added to the gas.

What is the significance of adding heat to an ideal monatomic gas at constant volume?

Adding heat increases the internal energy and temperature of the gas, since for a monatomic ideal gas internal energy depends only on temperature.

How can we calculate the change in internal energy of the gas after heating?

The change in internal energy (ΔU) for a monatomic ideal gas is given by $\Delta U = (3/2)\ n\ R\ \Delta T$, where n is the number of moles, R is the gas constant, and ΔT is the change in temperature.

What is the value of the gas constant R used in calculations?

The universal gas constant R is approximately 8.314 J/(mol·K).

Can we determine the final temperature of the gas after heat addition?

Yes, by using the relation between heat added and change in internal energy, $\Delta Q = \Delta U$, we can find the change in temperature and thus the final temperature.

What is the formula to calculate the change in internal energy for the given amount of heat added?

$\Delta U = 2438$ J, since for an ideal gas, the heat added at constant volume equals the change in internal energy.

What is the final temperature of the gas after adding 2438 J of heat?

First, calculate $\Delta U = 2438$ J; then, use $\Delta U = (3/2)\ n\ R\ \Delta T$ to find ΔT. With n=3 mol, $\Delta T = (2/3)\ (\Delta U)\ /\ (n\ R) = (2/3)\ 2438\ /\ (3\ 8.314) \approx 62$ K. Therefore, the final temperature is approximately 345 K + 62 K = 407 K.

Is the process described an isochoric process, and why?

Yes, since heat is added to an ideal gas at constant volume (implied by the context), the process is isochoric, meaning volume remains constant.

Additional Resources

Thermodynamics and the Behavior of an Ideal Monatomic Gas Under Heat Addition

Understanding the fundamental principles of thermodynamics is essential when analyzing the behavior of gases subjected to various energy exchanges. In this detailed exploration, we examine the case of three moles of an ideal monatomic gas initially at a temperature of 345 K, which then experiences a heat addition of 2438 J. We will delve into the thermodynamic processes involved, calculate key parameters, and interpret the physical significance of each step.

Introduction to the System

The system under consideration involves three moles of an ideal monatomic gas—common examples include helium, neon, or argon—in a controlled environment. The initial conditions specify:

- Number of moles, $n = 3$ mol
- Initial temperature, $T_i = 345$ K
- Heat added, $Q = 2438$ J

This setup allows us to analyze the changes in internal energy, work done, temperature, and other thermodynamic properties resulting from the heat transfer.

Fundamentals of Ideal Monatomic Gases

Before diving into calculations, it's important to review the key properties of ideal monatomic gases:

- Internal Energy (U): For an ideal monatomic gas, the internal energy depends solely on temperature and is given by:

$$
U = \frac{3}{2} n R T
$$

where:
- R is the universal gas constant, $R = 8.314\, \mathrm{J\,mol^{-1}\,K}$
- T is temperature in Kelvin

- Heat Capacity at Constant Volume (C_V):

$$
C_V = \frac{3}{2} R \approx 12.471\, \mathrm{J\,mol^{-1}\,K}
$$

- Heat Capacity at Constant Pressure (C_P):

$$
C_P = C_V + R = \frac{5}{2} R \approx 20.785\, \mathrm{J\,mol^{-1}\,K}
$$

- Degrees of Freedom: Monatomic gases have 3 degrees of freedom (translational motion), which explains their specific heat capacities.

Initial Internal Energy of the Gas

Using the internal energy formula:

$$
U_i = \frac{3}{2} n R T_i
$$

Substituting the known values:

$$
U_i = \frac{3}{2} \times 3\, \mathrm{mol} \times 8.314\, \mathrm{J\,mol^{-1}\,K} \times 345\, \mathrm{K}
$$

Calculating step-by-step:

1. $\frac{3}{2} \times 3 = 4.5$
2. $8.314 \times 345 \approx 2866.53$
3. $U_i = 4.5 \times 2866.53 \approx 12899.89\, \mathrm{J}$

Initial internal energy:

$$
U_i \approx \boxed{12899.89\, \mathrm{J}}
$$

Heat Addition and Its Immediate Effects

Adding heat $Q = 2438\, \mathrm{J}$ to the system influences both the internal energy and the work done by or on the gas during the process. The key questions are:

- How much does the internal energy change?
- What is the resulting change in temperature?
- Does the process involve work done by the gas?
- What is the final state of the gas?

Determining the Nature of the Process

The problem does not specify whether the process occurs at constant volume, constant pressure, or involves some other constraints. To proceed, we analyze the most general case and then specify particular scenarios:

- If the process is at constant volume (V constant):

The heat added directly increases internal energy, with no work done ($W = 0$). The change in internal energy:

$$
\Delta U = Q
$$

The temperature change:

$$
\Delta T = \frac{\Delta U}{\frac{3}{2} n R}
$$

- If the process is at constant pressure (P constant):

The heat added is partitioned into increasing internal energy and doing work:

$$
Q = \Delta U + W
$$

with $W = P \Delta V$.

Since the problem involves a specified heat addition but no explicit constraint, we'll analyze both cases and then discuss implications.

Case 1: Constant Volume Process

Change in internal energy:

$$
\Delta U = Q = 2438\, \mathrm{J}
$$

Final internal energy:

$$
U_f = U_i + \Delta U = 12899.89 + 2438 = 15337.89\, \mathrm{J}
$$

\]

Final temperature:

$$
T_f = \frac{2}{3} \times \frac{U_f}{n R}
$$

Calculating:

$$
T_f = \frac{2}{3} \times \frac{15337.89}{3 \times 8.314} = \frac{2}{3} \times \frac{15337.89}{24.942} \approx \frac{2}{3} \times 615.0 \approx 410.0\, \mathrm{K}
$$

Temperature increase:

$$
\Delta T = T_f - T_i = 410 - 345 = 65\, \mathrm{K}
$$

Case 2: Constant Pressure Process

Heat added:

$$
Q = n C_P \Delta T
$$

Rearranged to find ΔT:

$$
\Delta T = \frac{Q}{n C_P} = \frac{2438}{3 \times 20.785} \approx \frac{2438}{62.355} \approx 39.1\, \mathrm{K}
$$

Final temperature:

$$
T_f = T_i + \Delta T = 345 + 39.1 \approx 384.1\, \mathrm{K}
$$

Change in internal energy:

$$
\Delta U = \frac{3}{2} n R \Delta T = 12899.89 \times \frac{\Delta T}{T_i} \text{ (or directly)}:
$$

\]

Alternatively, using:

\[
\Delta U = \frac{3}{2} n R (T_f - T_i) = 1.5 \times 3 \times 8.314 \times 39.1 \approx 3 \times 8.314 \times 39.1 \approx 3 \times 325.0 \approx 975\, \mathrm{J}
\]

Work done:

\[
W = Q - \Delta U \approx 2438 - 975 = 1463\, \mathrm{J}
\]

Interpretation:

In the constant pressure process, the gas does approximately 1463 J of work during expansion, and the internal energy increases by about 975 J, consistent with the heat added.

Work Done by the Gas and Energy Conservation

The amount of work done during the process depends on the process path:

- Constant volume: \(W = 0 \), all heat goes into internal energy.
- Constant pressure: \(W = P \Delta V \), and internal energy increases accordingly.

Since the problem does not specify constraints, understanding both scenarios provides comprehensive insight.

Final State Parameters and Physical Implications

Assuming the constant volume case as a baseline:

- Final temperature: approximately 410 K
- Increase in internal energy: about 2438 J
- Physical interpretation: The gas warms up, internal energy increases, and there is no volume change.

In the constant pressure case:

- Final temperature: approximately 384 K
- Work done: about 1463 J
- Physical interpretation: The gas expands, doing work on surroundings, with internal energy increasing less than the heat supplied.

Additional Thermodynamic Quantities

To deepen understanding, we calculate other relevant parameters:

Change in entropy (ΔS):

- For reversible heat transfer:

$$
\Delta S = n C_P \ln \frac{T_f}{T_i}
$$

or, for an ideal gas:

$$
\Delta S = n R \ln \frac{V_f}{V_i}
$$

depending on process details.

- In constant volume process:

$$
\Delta S = n C_V \ln \frac{T_f}{T_i}
$$

Calculations:

$$
\ln \frac{T_f}{T_i} = \ln \frac{410}{345} \approx \ln 1.188 \approx 0.173
$$

- Entropy change at constant volume:

\[

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