# 1. Determine The Discrete Fourier Transform. Square Your FinalAnswer.a. X(n) = 2n U(−n)b. X(n) = 0.25n

**1. Determine The Discrete Fourier Transform. Square Your FinalAnswer.a. X(n) = 2n U(−n)b. X(n) = 0.25n**

Understanding the Discrete Fourier Transform (DFT) is fundamental in signal processing, digital communications, and many engineering applications. When tasked with determining the DFT of a given sequence, especially those involving step functions or linear sequences, a structured approach helps simplify the process. In this article, we will explore how to determine the DFT for two specific sequences: X(n) = 2n U(−n) and X(n) = 0.25n, and then square the final answers as instructed.

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## Understanding the Discrete Fourier Transform (DFT)

The DFT converts a discrete-time sequence into its frequency domain representation. Given a sequence x[n], the DFT is defined as:

$$
X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn}
$$

where:
- $N$ is the length of the sequence,
- $k$ is the frequency index,
- $x[n]$ is the discrete-time sequence.

The inverse DFT allows reconstructing the sequence from its frequency domain representation.

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## Analyzing the Sequences

The sequences in question are:

1. $X(n) = 2n U(-n)$
2. $X(n) = 0.25n$

Let's analyze each one separately.

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# Sequence 1: $X(n) = 2n U(-n)$

## Understanding the Sequence

- Unit Step Function $U(-n)$: This function is 1 for $n \le 0$, and 0 for $n > 0$.
- Sequence Behavior: Since $X(n) = 2n$ when $n \le 0$, and 0 otherwise, the sequence is non-zero only for non-positive $n$.

In essence, $X(n)$ is a sequence that exists for $n \le 0$, with values decreasing as $n$ becomes more negative:

$$
X(n) = \begin{cases}
2n, & n \le 0 \\
0, & n > 0
\end{cases}
$$

---

## Expressing the Sequence for DFT Calculation

To compute the DFT, we need to consider the sequence over a finite length $N$. Let's assume a length $N$, and define the sequence over $n = 0, 1, 2, ..., N-1$. Since the sequence is non-zero for $n \le 0$, we need to handle negative $n$, which implies extending the sequence to negative indices.

In digital signal processing, sequences are often considered over a finite interval, with appropriate zero-padding if necessary.

Approach:

- Define the sequence over a symmetric interval, for example $n = -M, ..., 0, ..., M$, then analyze over a finite window.
- Alternatively, consider the periodic extension for DFT calculation.

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## Calculating the DFT of $X(n) = 2n U(-n)$

Assuming the sequence is defined for $n = 0, 1, ..., N-1$, with the understanding that for $n$ outside the sequence's domain, $x[n] = 0$.

If we interpret $X(n)$ as a finite sequence, then:

$$
x[n] = \begin{cases}
2n, & n \le 0 \\
0, & n > 0
\end{cases}

$$

but since $n \ge 0$ in the discrete-time domain, the only non-zero part is when $n = 0$, where $X(0) = 0$.

Alternatively, to properly analyze this, it's better to consider the sequence as an infinite one, then compute the DTFT (Discrete-Time Fourier Transform), which generalizes the DFT for infinite sequences.

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## Transforming the Sequence: DTFT Approach

Since the sequence is non-zero only for $n \le 0$, it resembles a left-sided sequence. The DTFT of such a sequence is given by:

$$
X(e^{j\omega}) = \sum_{n=-\infty}^{0} 2n e^{-j\omega n}
$$

Calculating this sum involves recognizing it as a weighted sum over negative $n$.

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## Computing the DTFT of $X(n) = 2n U(-n)$

Let’s set $m = -n$, so that when $n \le 0$, $m \ge 0$. Then:

$$
X(n) = 2n U(-n) \implies X(-m) = -2m
$$

The DTFT becomes:

$$
X(e^{j\omega}) = \sum_{m=0}^{\infty} (-2m) e^{j \omega m}
$$

which converges under certain conditions.

This sum resembles a generating function for a weighted geometric series, which can be summed using known series formulas.

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## Sum of the Series

The sum:

$$
S = \sum_{m=0}^{\infty} m r^{m}
$$

is known as:

$$
S = \frac{r}{(1 - r)^2}
$$

for $|r| < 1$.

In our case, $r = e^{j\omega}$, and since $|e^{j\omega}|=1$, the sum converges in the sense of the bilateral DTFT, considering the principal value.

Therefore,

$$
X(e^{j\omega}) = -2 \sum_{m=0}^{\infty} m e^{j \omega m} = -2 \frac{e^{j \omega}}{(1 - e^{j \omega})^2}
$$

---

## Final Expression for the DTFT of Sequence 1

$$
X(e^{j\omega}) = -2 \frac{e^{j \omega}}{(1 - e^{j \omega})^2}
$$

This expression characterizes the frequency response of the sequence $X(n)$.

---

## Square the Final Answer

As per the instructions, we are to square this final answer:

$$
\left( X(e^{j\omega}) \right)^2 = \left( -2 \frac{e^{j \omega}}{(1 - e^{j \omega})^2} \right)^2 = 4 \frac{e^{2j \omega}}{(1 - e^{j \omega})^4}
$$

This squared form could be useful for spectral analysis or further processing.

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## Sequence 2: $X(n) = 0.25 n$

## Understanding the Sequence

This is a linear sequence with amplitude proportional to $n$. For DFT computation, a finite-length sequence is assumed, say $n = 0, 1, ..., N-1$.

The sequence:

$$
x[n] = 0.25 n
$$

is a simple, linearly increasing sequence.

---

## Computing the DFT of $X(n) = 0.25 n$

The DFT is:

$$
X[k] = \sum_{n=0}^{N-1} 0.25 n e^{-j \frac{2\pi}{N} kn}
$$

Factor out 0.25:

$$
X[k] = 0.25 \sum_{n=0}^{N-1} n e^{-j \frac{2\pi}{N} kn}
$$

This sum is a standard form involving a weighted sum of complex exponentials.

---

## Summation of the Weighted Exponentials

The sum:

$$
S = \sum_{n=0}^{N-1} n r^{n}
$$

where $r = e^{-j \frac{2 \pi}{N} k}$.

The sum of a weighted geometric series:

$$
S = \frac{r (1 - N r^{N-1})}{(1 - r)^2} - \frac{(N-1) r^{N}}{(1 - r)}
$$

for $r \neq 1$.

Alternatively, for the case where $r \neq 1$:

\[
\sum_{n=0}^{N-1} n r^{n} = \frac{r \left[ 1 - (N) r^{N-1} + (N-1) r^{N} \right]}{(1 - r)^2}
\]

Calculating this sum gives the DFT component at each \( k \).

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### Final Expression for the DFT of Sequence 2

Therefore,

\[
X[k] = 0.25 \times \left[ \frac{r (1 - N r^{N-1})}{(

## Frequently Asked Questions

### What is the Discrete Fourier Transform (DFT) of the sequence X(n) = 2n U(-n)?

The DFT of $X(n) = 2n U(-n)$ can be computed by applying the definition of DFT, considering the finite length of the sequence limited by the unit step. Since $U(-n)$ is 1 for $n \leq 0$ and 0 otherwise, the sequence is non-zero for $n \leq 0$. Its DFT involves summing over these n values, resulting in a frequency domain representation that captures the spectrum of the causal part of the sequence. The exact expression requires summing n from $-\infty$ to 0, which simplifies to a finite sum if we consider a truncated sequence.

### How do you compute the DFT of X(n) = 0.25n?

To compute the DFT of $X(n) = 0.25n$, you substitute into the DFT formula: $X(k) = \Sigma X(n) e^{-j(2\pi/N)nk}$ over n. Since $X(n)$ is a linear function of n, the sum results in a weighted sum that can be evaluated analytically or numerically, depending on the length N. The DFT will reflect the linear trend in the frequency domain, producing a spectrum with specific characteristics related to the linear increase.

### Why is squaring the final answer important when determining the DFT of these sequences?

Squaring the final answer is important because it often relates to the power spectrum of the signal in the frequency domain. The magnitude squared of the DFT provides insight into the signal's energy distribution across frequencies, which is essential in many applications like spectral analysis and signal processing.

### What is the significance of the unit step function U(-n) in the sequence X(n) = 2n U(-n)?

The unit step function $U(-n)$ effectively limits the sequence to non-positive n values (n ≤ 0). This means the sequence is causal in the negative direction, which impacts its DFT by constraining the sum to that range. It influences the frequency content and symmetry of the spectrum.

### How does the sequence X(n) = 2n U(−n) affect the symmetry of its Fourier transform?

Since the sequence is non-zero only for n ≤ 0, its Fourier transform is generally asymmetric, reflecting the causal nature of the sequence. The symmetry properties depend on whether the sequence is real and even, but in this case, the presence of the step function breaks even symmetry, leading to an asymmetric spectrum.

### What challenges might arise when calculating the DFT of X(n) = 0.25n?

Calculating the DFT of a linearly increasing sequence like 0.25n can lead to divergence issues if the sequence is infinite, as the sum may not converge. For practical computation, the sequence is typically truncated to a finite length, which introduces approximation errors but makes the DFT computable.

### In what scenarios would you prefer to use the Discrete Fourier Transform over other spectral analysis methods?

The DFT is preferred when analyzing finite, discrete signals, especially when computational efficiency is needed, such as in digital signal processing. It provides a direct way to examine the frequency content of sampled signals, making it useful in applications like audio processing, communications, and image analysis.

### How does the linearity property of the DFT assist in computing the transforms of X(n) = 2n U(−n) and X(n) = 0.25n?

Linearity allows us to compute the DFT of each component separately and then sum the results. For sequences like 2n U(−n) and 0.25n, if they can be expressed as linear combinations of simpler sequences, this property simplifies the calculation process by breaking down complex sequences into manageable parts.

## Additional Resources

Discrete Fourier Transform (DFT): Unlocking the Frequency Domain of Discrete Signals

The Discrete Fourier Transform (DFT) is a cornerstone in digital signal processing, enabling engineers and analysts to analyze the frequency content of discrete signals efficiently. Whether for audio processing, image analysis, communications, or data compression, understanding the DFT's mechanics and applications is essential. In this article, we delve into the intricacies of the DFT by exploring two specific sequences: $X(n) = 2n \cdot U(-n)$ and $X(n) = 0.25n$. Our goal is to determine their DFTs, analyze their properties, and ultimately, present their squared final answers. Think of this as an expert feature tailored to deepen your understanding of the DFT's power and nuances.

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# Understanding the Discrete Fourier Transform (DFT)

Before tackling the specific sequences, it’s vital to comprehend what the DFT is, how it works, and why it’s so fundamental in digital signal analysis.

## What Is the DFT?

The DFT converts a finite sequence of equally spaced samples of a discrete-time signal into a sequence of complex numbers representing the signal's frequency components. Formally, for a sequence $x(n)$ of length $N$:

$$
X(k) = \sum_{n=0}^{N-1} x(n) e^{-j 2 \pi \frac{kn}{N}}, \quad k = 0, 1, ..., N-1
$$

where:
- $X(k)$ is the DFT output at frequency bin $k$,
- $x(n)$ is the input sequence,
- $j$ is the imaginary unit.

This transformation reveals the amplitude and phase of each frequency component, enabling spectral analysis.

## Key Properties of the DFT

Understanding properties such as linearity, symmetry, and periodicity helps interpret DFT results. Some important properties are:
- Linearity: The DFT of a sum is the sum of the DFTs.
- Symmetry: For real-valued signals, the DFT exhibits conjugate symmetry.
- Periodicity: The DFT outputs are periodic with period $N$.
- Time and Frequency Resolution: Determined by the sequence length $N$.

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# Analyzing the Sequence: $X(n) = 2n \cdot U(-n)$

The first sequence incorporates the unit step $U(-n)$, which is a step function that equals 1 for $n \leq 0$ and 0 otherwise. Thus, the sequence becomes:

\[
X(n) = 2n \quad \text{for} \quad n \leq 0, \quad \text{and} \quad 0 \quad

\text{for} \quad n > 0
\]

This is a non-zero sequence extending into negative \(n\), which introduces challenges when applying the DFT directly, as the DFT assumes finite-length sequences defined over \(n = 0, 1, ..., N-1\). To analyze this, we need to handle the sequence carefully.

## Reformulating the Sequence for DFT Analysis

Given the sequence's support on \(n \leq 0\), one common approach is to consider a finite-length segment and zero-pad as necessary. For simplicity, assume a finite segment where \(n\) runs from \(-M\) to 0, with \(M\) large enough to capture the essential behavior.

Suppose \(N\) is the total length, and we define:

\[
x(n) = \begin{cases}
2n, & n = -M, -M+1, ..., 0 \\
0, & \text{otherwise}
\end{cases}
\]

To compute the DFT, we need to map this into an array of length \(N\). Commonly, we use the following indexing:

\[
x(n) = x(n) \quad \text{for} \quad n=0, 1, ..., N-1
\]

by shifting the sequence:

\[
x(n) = 2(n - M) \quad \text{for} \quad n = M, M+1, ..., M+N-1
\]

and zero elsewhere. This shifting ensures the sequence is properly represented within the DFT window.

## Calculating the DFT of \(X(n)\)

The DFT becomes:

\[
X(k) = \sum_{n=0}^{N-1} x(n) e^{-j 2 \pi \frac{kn}{N}}
\]

Substituting the shifted sequence:

\[
X(k) = \sum_{n=M}^{M+N-1} 2(n - M) e^{-j 2 \pi \frac{kn}{N}}
\]

which simplifies to:

\[

X(k) = 2 \sum_{n=M}^{M+N-1} (n - M) e^{-j 2 \pi \frac{kn}{N}}
\]

The summation involves a linear term multiplied by a complex exponential, which is a classic sum in Fourier analysis, often tackled via generating functions or known summation formulas.

Key steps:
1. Recognize that the sum involves a linear term \( (n - M) \).
2. Use known formulas for geometric sums and their derivatives to compute the sum.
3. After computation, the result will be a complex function \(X(k)\), encoding the spectrum.

Note: Exact closed-form solutions depend on the sequence length \(N\) and the choice of \(M\). In practical applications, numerical methods or software tools (such as MATLAB or Python's NumPy) are used to evaluate these sums efficiently.

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## Analyzing the Sequence: \(X(n) = 0.25n\)

The second sequence is more straightforward, as it is a linear function over the entire domain. Assuming the sequence is defined for \(n=0, 1, ..., N-1\):

\[
X(n) = 0.25n
\]

This sequence is a simple ramp scaled by 0.25, which has a well-studied DFT.

### Calculating the DFT of \(X(n) = 0.25n\)

The DFT is:

\[
X(k) = \sum_{n=0}^{N-1} 0.25 n e^{-j 2 \pi \frac{kn}{N}}
\]

Factoring out constants:

\[
X(k) = 0.25 \sum_{n=0}^{N-1} n e^{-j 2 \pi \frac{kn}{N}}
\]

This sum resembles the Fourier transform of a linear sequence, which can be evaluated using known formulas or generating functions.

Approach:
- Recognize that this sum is a finite geometric series derivative.
- For \(k \neq 0\), the sum can be expressed as:

\[
\sum_{n=0}^{N-1} n r^n = r \frac{1 - (N) r^{N-1} + (N-1) r^N}{(1 - r)^2}
\]

where $r = e^{-j 2 \pi \frac{k}{N}}$.

- For $k=0$, the sum simplifies to:

$$
\sum_{n=0}^{N-1} n = \frac{(N-1)N}{2}
$$

which yields:

$$
X(0) = 0.25 \times \frac{(N-1)N}{2}
$$

Similarly, for $k \neq 0$, the sum involves complex geometric series calculations, leading to expressions involving the denominator $(1 - r)^2$.

Implications:
- The DFT of a ramp sequence exhibits a sinc-like pattern, with spectral components decreasing as $k$ increases.
- The spectrum contains both magnitude and phase information, revealing the linear trend in the original sequence.

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# Final Step: Squaring the Results

The problem states: "Square your final answer." This implies taking the magnitude squared of the DFT results, which corresponds to the power spectrum of the sequence.

## Power Spectrum Calculation

Given the DFT $X(k)$, the power spectrum is:

$$
|X(k)|^2
$$

For the sequence $X(n) = 0.25n$:

$$
|X(k)|^2 = \left| 0.25 \sum_{n=0}^{N-1} n e^{-j 2 \pi \frac{kn}{N}} \right|^2
$$

The exact magnitude depends on the sum's evaluation, but the key idea is that the power spectrum emphasizes the energy distribution across frequencies.

For the sequence $X(n) = 2n \cdot U(-n)$:

$$
|X(k)|^2 = \left| 2 \sum_{n=M}^{M+N-1} (n - M) e^{-j 2 \pi \frac{kn}{N}} \right|^2
$$

This squared magnitude reveals the spectral energy distribution and helps identify dominant frequency components, spectral leakage, and other important features.

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## Conclusion: The

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## 1 Determine The Discrete Fourier Transform Square Your Finalanswer A X N 2n U N B X N 0 25n

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