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1) Its Per-worker Production Function Is Y=f(k)=5k^{1/2} (5 Times The Square Root Of K). 2) The Marginal

Understanding the Per-Worker Production Function: Y = f(k) = 5k^{1/2}

Introduction to the Production Function

The per-worker production function, represented as $Y = f(k) = 5k^{1/2}$, describes how output per worker (Y) depends on the capital per worker (k). In economic analysis, this function plays a vital role in understanding how capital accumulation influences productivity and overall economic growth. The specific functional form, a square root, indicates diminishing returns to capital, a common feature in production models.

Functional Form Breakdown

- Y: Output per worker, also known as labor productivity.
- k: Capital per worker, representing the amount of capital available to each worker.
- $f(k) = 5k^{1/2}$: The production function indicates that as capital per worker increases, output per worker increases as well, but at a decreasing rate.

This particular form, with a coefficient of 5 and an square root dependence, signifies that the productivity gains from additional capital diminish as capital per worker increases. This phenomenon is consistent with empirical observations and theoretical models of production.

Implications of the Production Function

Diminishing Returns to Capital

Since the production function involves a square root, the marginal productivity of capital diminishes as k increases. Mathematically, the marginal product of capital (MPK) can be derived and analyzed to understand this behavior.

Marginal Product of Capital (MPK)

The MPK measures the additional output generated by an extra unit of capital, holding other factors constant. It is derived by taking the first derivative of the production function with respect to k:

$$
MPK = \frac{dY}{dk} = \frac{d}{dk} (5k^{1/2}) = 5 \times \frac{1}{2} k^{-1/2} = \frac{5}{2} \times \frac{1}{\sqrt{k}}
$$

This expression reveals that:
- The MPK decreases as k increases because it is inversely proportional to $\sqrt{k}$.
- When capital per worker is low, the marginal product is high, indicating significant productivity gains from additional capital.
- As k grows larger, the MPK diminishes, illustrating diminishing returns.

Graphical Representation

A graph of the production function $Y = 5k^{1/2}$ would show a concave curve, rising rapidly initially and then gradually flattening out. The slope at any point reflects the MPK, which declines as k increases.

Economic Significance and Applications

Steady-State Analysis

In growth models like the Solow model, the steady state occurs when capital accumulation equals depreciation and technological progress. Using the given production function, economists can analyze how different savings rates, depreciation, and technological advances influence the steady-state level of capital and output per worker.

Impact of Investment and Savings

Assuming a savings rate s, the amount of investment per worker is sY. Since $Y = 5k^{1/2}$, investment per worker becomes $s \times 5k^{1/2}$. Over time, this influences the evolution of capital per worker according to the following dynamic equation:

$$
\frac{dk}{dt} = s \times 5k^{1/2} - \delta k
$$

where:
- δ is the depreciation rate of capital.
- The balance point where $\frac{dk}{dt} = 0$ defines the steady-state capital per worker.

Growth and Convergence

This functional form suggests that countries or regions with low capital per worker will experience higher marginal returns and faster growth rates, aligning with the concept of convergence. Conversely, regions with high capital levels will grow more slowly due to diminishing marginal productivity.

Limitations and Considerations

Assumption of Constant Coefficients

The model assumes a fixed coefficient (5) in the production function, which simplifies analysis but may not capture real-world complexities such as technological change or varying efficiencies.

Ignoring Technological Progress

While the function effectively models capital's role, it does not explicitly incorporate technological progress, a key driver of sustained long-term growth.

Policy Implications

Understanding the diminishing returns to capital underscores the importance of technological innovation, human capital development, and productivity-enhancing policies to sustain growth beyond mere capital accumulation.

Conclusion

The production function $Y = 5k^{1/2}$ provides a foundational framework for analyzing how capital per worker influences output and productivity. Its mathematical properties, especially the diminishing marginal product of capital, reflect essential economic principles and inform policy decisions aimed at fostering sustainable economic growth. By examining this function and its implications, economists and policymakers can better understand the dynamics of capital accumulation, technological progress, and convergence among economies.

Frequently Asked Questions

What is the per-worker production function given in the problem?

The per-worker production function is $Y = f(k) = 5\sqrt{k}$, where Y is output per worker and k is capital per worker.

How do you interpret the function $Y = 5\sqrt{k}$ in economic terms?

This function indicates that output per worker increases with capital per worker, but at a decreasing rate, since the square root function grows more slowly as k increases.

What is the marginal product of capital (MPK) derived from the given production function?

The marginal product of capital is the derivative of Y with respect to k, which is MPK = dY/dk = 5 (1/2) k^(-1/2) = 2.5 / √k.

How does the marginal product of capital change as capital per worker increases?

As capital per worker k increases, the marginal product of capital decreases because MPK = 2.5 / √k, and √k increases, causing MPK to decline.

What does the decreasing marginal product imply for capital accumulation in this model?

It implies diminishing returns to capital: adding more capital yields smaller additional output per worker, which can influence long-term growth and savings behavior in the model.

Additional Resources

Understanding the Per-worker Production Function and Marginal Product in Economics

When analyzing how economies grow and firms operate, understanding the per-worker production function and the marginal product of capital is essential. These concepts help economists and business analysts evaluate productivity, efficiency, and the impact of different levels of capital on output. In this article, we'll delve into a specific case: the per-worker production function given by Y = f(k) = 5k^{1/2}, which is five times the square root of capital per worker. We'll explore its characteristics, implications, and the behavior of its marginal product to provide a comprehensive understanding.

The Per-Worker Production Function: Y = 5k^{1/2}

What is a Per-Worker Production Function?

A per-worker production function relates the output produced per worker (Y) to the amount of capital per worker (k). It reflects how much each worker can produce given a certain level of capital investment, holding other factors constant. This function is critical because it isolates productivity at the individual worker level, helping distinguish between aggregate output and individual contributions.

The Specific Function: Y = 5k^{1/2}

In this case, the production function is:

$$ Y = f(k) = 5k^{1/2} $$

which can also be written as:

$$ Y = 5 \sqrt{k} $$

This form indicates that:

- Output per worker increases with the square root of capital per worker.
- The constant 5 scales the overall level of productivity.
- The function exhibits diminishing returns to capital, meaning each additional unit of capital adds less to output than the previous one.

Characteristics of the Production Function

1. Shape and Nature

The function $Y = 5k^{1/2}$ is a concave, increasing function. It starts at zero when $k = 0$ and rises as k increases, but at a decreasing rate.

2. Diminishing Marginal Returns

Since the function involves a square root, the marginal product of capital diminishes as capital per worker increases. This is a core property of most production functions, reflecting real-world constraints where adding more capital yields progressively smaller increases in output.

Calculating the Marginal Product of Capital (MPk)

Definition of Marginal Product

The marginal product of capital (MPk) measures how much additional output is generated by adding one more unit of capital, holding other inputs constant. Mathematically, it's the derivative of the production function with respect to k:

$$
MP_k = \frac{dY}{dk}
$$

Deriving the Marginal Product

Given:

$$
Y = 5k^{1/2}
$$

Differentiate with respect to k:

$$
MP_k = \frac{d}{dk} (5k^{1/2}) = 5 \times \frac{1}{2} k^{-1/2} = \frac{5}{2}k^{-1/2}
$$

which simplifies to:

\[

$$
MP_k = \frac{5}{2 \sqrt{k}}
$$

Key Insights:

- The marginal product decreases as k increases.
- When k is small, MP_k is large.
- When k is large, MP_k approaches zero.

Interpreting the Marginal Product

Capital per Worker k	Marginal Product MP_k	Interpretation
Small k (e.g., near 0)	Very high	Adding initial units of capital significantly boosts output.
Moderate k	Moderate	Additional capital still increases output, but less dramatically.
Large k	Approaching 0	Additional capital yields minimal increases in output.

This decreasing trend reflects diminishing returns to capital: beyond a certain point, investing more in capital results in smaller incremental increases in output per worker.

Practical Implications

1. Economic Growth and Capital Accumulation

- Economies with low levels of capital per worker can experience rapid growth when they invest in capital, due to high marginal product.
- As capital accumulation continues, growth slows down because of diminishing marginal returns.

2. Optimal Capital Investment

- Firms and policymakers should consider the marginal product when deciding how much to invest.
- Excessive capital accumulation beyond a certain point yields negligible output gains, potentially leading to inefficient resource allocation.

Visualizing the Function and Marginal Product

A graph of $Y = 5k^{1/2}$:

- Starts at the origin (0,0).
- Concaves upward, illustrating diminishing returns.
- The slope (marginal product) decreases as k increases.

A graph of $MP_k = \frac{5}{2 \sqrt{k}}$:

- Starts at a very high value when k is near zero.
- Declines asymptotically towards zero as k increases.

Key Takeaways

- The per-worker production function $Y = 5k^{1/2}$ captures how individual output depends on capital per worker.
- The marginal product $MP_k = \frac{5}{2 \sqrt{k}}$ illustrates diminishing returns to capital.
- Both functions highlight that initial investments in capital dramatically increase productivity, but additional investments become less effective over time.
- Understanding these relationships is crucial for designing effective investment strategies and policies aimed at sustainable economic growth.

Final Thoughts

The analysis of the specific production function $Y = 5k^{1/2}$ and its marginal product provides a clear illustration of fundamental economic principles. It emphasizes the importance of balancing capital accumulation with productivity gains and serves as a foundational concept for more complex models of economic growth, such as the Solow growth model. Recognizing the nature of diminishing returns helps policymakers and business leaders make informed decisions about investment, technology, and resource management to foster long-term prosperity.

By mastering the concepts of the per-worker production function and marginal product, you gain vital tools for evaluating productivity and growth dynamics in various economic contexts.

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