(1 Point) Determine The Sum Of The Series N=1[infinity]6n(n + 2) If Possible. (if The Series Diverges,

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Introduction

Understanding infinite series is a fundamental aspect of mathematical analysis, particularly in the fields of calculus and mathematical series convergence. When we encounter a series such as $\sum_{n=1}^\infty 6n(n + 2)$, the first question that arises is whether this series converges or diverges. If it converges, we seek to determine its sum; if it diverges, then we conclude that it does not have a finite sum. This article delves into the process of analyzing the given series, exploring the methods of series summation, and ultimately determining whether the series converges or diverges, along with the steps involved in such an analysis.

Understanding the Series

The series in question is:

$$
\sum_{n=1}^\infty 6n(n + 2)
$$

This can be expanded as:

$$
6 \sum_{n=1}^\infty n(n + 2)
$$

Our goal is to analyze the behavior of the series:

$$
\sum_{n=1}^\infty 6n(n + 2)
$$

and determine whether this infinite sum converges to a finite value or diverges to infinity.

Step 1: Simplify the General Term

To analyze the series, it is helpful to simplify the general term:

$$
6n(n + 2)
$$

Expanding the expression:

$$
6n(n + 2) = 6n^2 + 12n
$$

Thus, the series can be rewritten as:

$$
\sum_{n=1}^\infty (6n^2 + 12n)
$$

which separates into two simpler series:

$$
\sum_{n=1}^\infty 6n^2 + \sum_{n=1}^\infty 12n
$$

or equivalently:

$$
6 \sum_{n=1}^\infty n^2 + 12 \sum_{n=1}^\infty n
$$

Now, the problem reduces to analyzing these two component series.

Step 2: Analyze the Component Series

The two series involved are:

1. $\sum_{n=1}^\infty n^2$
2. $\sum_{n=1}^\infty n$

Both are well-known divergent series.

Series 1: $\sum_{n=1}^\infty n$

The harmonic series of natural numbers:

$$
\sum_{n=1}^\infty n

$$
$$

diverges because the partial sums:

$$
S_N = 1 + 2 + 3 + \dots + N = \frac{N(N+1)}{2}
$$

grow without bound as $N \to \infty$. Hence, this series diverges to infinity.

Series 2: $\sum_{n=1}^\infty n^2$

Similarly, the sum of squares:

$$
\sum_{n=1}^\infty n^2
$$

also diverges because the partial sums:

$$
S_N = \sum_{n=1}^N n^2 = \frac{N(N+1)(2N+1)}{6}
$$

grow without bound as $N \to \infty$.

Step 3: Conclusion on the Series Convergence

Since both component series $\sum n$ and $\sum n^2$ diverge, their linear combinations also diverge. Specifically, the original series:

$$
\sum_{n=1}^\infty 6n(n + 2) = 6 \sum_{n=1}^\infty n^2 + 12 \sum_{n=1}^\infty n
$$

diverges to infinity.

Therefore, the series $\sum_{n=1}^\infty 6n(n + 2)$ diverges; it does not have a finite sum.

Understanding Divergence of the Series

The divergence of the series is primarily due to the fact that the individual terms grow quadratically and linearly, respectively, and their sum does not tend to a finite limit. The divergence can be formally confirmed through comparison tests or the nth-term test.

nth-term Test for Divergence

The nth-term test states that if:

$$
\lim_{n \to \infty} a_n \neq 0
$$

then the series $\sum a_n$ diverges.

Calculating the limit of the general term:

$$
a_n = 6n(n + 2) = 6n^2 + 12n
$$

As $n \to \infty$:

$$
a_n \to \infty
$$

which is not zero; hence, the series diverges.

Additional Insights and Implications

Understanding the divergence of this series has broader implications in various areas of mathematics and physics, especially where infinite sums are involved.

Implication for Series Summation

- Not all infinite series converge; recognizing divergence is crucial in avoiding incorrect assumptions about their sums.
- Divergence indicates that the partial sums grow without bound, and no finite sum exists.

Application of Tests

- The nth-term test is often the first step in analyzing series convergence.
- For more complex series, comparison tests, ratio tests, or integral tests can provide additional confirmation.

Summary

- The series in question: $\sum_{n=1}^\infty 6n(n + 2)$
- Simplified to: $\sum_{n=1}^\infty (6n^2 + 12n)$

- Both component series diverge to infinity.
- The original series diverges because its terms grow without bound.
- Conclusion: The series does not converge; it diverges to infinity.

Conclusion

In conclusion, the series $\sum_{n=1}^\infty 6n(n + 2)$ diverges, and thus, it does not have a finite sum. This analysis underscores the importance of examining the behavior of the general term and applying convergence tests to determine the nature of infinite series. Recognizing divergence is as vital as computing sums, especially in mathematical modeling and theoretical physics, where infinite series frequently describe various phenomena.

References:

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Frequently Asked Questions

What is the series to be evaluated in the problem?

The series is the sum from n=1 to infinity of 6n(n + 2).

How can we simplify the general term 6n(n + 2)?

Expanding the term gives 6n^2 + 12n.

What method should be used to determine if the series converges or diverges?

We can use the comparison test, ratio test, or limit comparison test to analyze convergence.

Does the series involve a quadratic term, and what does that imply for convergence?

Yes, the series involves n^2 terms, which suggests that the series behaves like a p-series with p=2, known to diverge.

Can the series be expressed as a sum of simpler series?

Yes, it can be written as 6∑ n^2 + 12∑ n, where both are standard series.

What is the behavior of the series as n approaches infinity?

Since the terms grow proportionally to n^2, the series diverges.

Is the series convergent or divergent?

The series diverges because the sum of quadratic terms tends to infinity.

What is the final conclusion about the sum of the series?

The series diverges; thus, it does not have a finite sum.

If the series diverges, what does that imply about summing from n=1 to infinity?

It implies that the sum tends to infinity and cannot be assigned a finite value.

Additional Resources

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In the realm of mathematical analysis, infinite series serve as fundamental tools for understanding functions, approximating values, and exploring the behaviors of sequences. Among these, series that converge to a finite sum are particularly noteworthy because they provide exact numerical insights. Conversely, divergent series, which do not settle to a finite limit, challenge mathematicians and analysts alike, prompting deeper investigation into their properties and implications. Today, we delve into a specific series: the sum from n=1 to infinity of 6n(n + 2). The central question is whether this series converges to a finite value or diverges, and if it converges, how that sum can be explicitly determined.

This exploration will entail a systematic approach—examining the structure of the series, simplifying its terms, applying convergence tests, and, ultimately, calculating its sum if it exists. Understanding the nature of this series not only sharpens our algebraic and analytical skills but also enhances our grasp of the broader principles governing infinite sums.

Understanding the Series: An Initial Look

Before diving into calculations, it’s essential to understand what the series entails. The series under consideration is:

$$ \sum_{n=1}^{\infty} 6n(n + 2) $$

This expression involves a quadratic term within each summand, multiplied by a coefficient of 6. At a glance, the series seems to grow quite rapidly because as n increases, $n(n + 2)$ behaves approximately like n^2, which suggests that the terms might not diminish to zero—a key requirement for convergence.

Key Points to Note:
- The summand: $6n(n + 2)$
- Behavior: As n increases, the term roughly behaves like $6n^2$
- Implication: If the terms do not tend to zero, the series cannot converge.

Simplifying the Series: Algebraic Manipulation

To analyze the series effectively, we start by simplifying the general term:

$$ 6n(n + 2) $$

Expanding this, we get:

$$ 6(n^2 + 2n) = 6n^2 + 12n $$

Thus, the series can be rewritten as:

$$ \sum_{n=1}^{\infty} (6n^2 + 12n) $$

This decomposition allows us to analyze the series as the sum of two simpler series:

$$ \sum_{n=1}^{\infty} 6n^2 + \sum_{n=1}^{\infty} 12n $$

or, equivalently,

$$ 6 \sum_{n=1}^{\infty} n^2 + 12 \sum_{n=1}^{\infty} n $$

This step is crucial because the sums of powers of natural numbers are well-known and have established formulas, which facilitate the study of convergence.

Convergence Analysis: Do the Series Converge?

The backbone of any analysis on infinite series is the convergence test. Here, we assess whether the sums:

$$ \sum_{n=1}^{\infty} 6n^2 \quad \text{and} \quad \sum_{n=1}^{\infty} 12n $$

converge or diverge.

Recall:
- The series $\sum_{n=1}^{\infty} n^p$ (for p > 0) diverges because its terms do not tend to zero fast enough.
- Specifically:
- $\sum n$ diverges (harmonic series).
- $\sum n^2$ diverges even more rapidly.

Applying the Term Test for Divergence: If the limit of the nth term as n approaches infinity does not tend to zero, the series diverges.

Calculating the terms' limit:

$$ \lim_{n \to \infty} 6n(n + 2) = \lim_{n \to \infty} (6n^2 + 12n) $$

Since this tends to infinity as n increases, the individual terms do not approach zero; rather, they grow without bound.

Conclusion:

- Because the terms of the series grow without bound, the series cannot converge.
- The divergence of the individual terms implies the entire series diverges.

Implications and Final Assessment

Given the analysis above, the series:

$$ \sum_{n=1}^{\infty} 6n(n + 2) $$

does not satisfy the necessary condition for convergence—that its terms tend to zero as n approaches infinity. Instead, the terms grow larger, diverging to infinity.

Therefore:

- The series diverges.
- Its sum does not exist in the finite sense.
- It is impossible to assign a finite value to this series.

What Does This Mean in Broader Context?

This conclusion aligns with fundamental principles of series analysis. For a series to converge, the individual terms must approach zero. Since the terms $6n(n + 2)$ grow quadratically, not only do they fail to diminish, but they grow without bound, making convergence impossible.

This insight also underscores the importance of the behavior of the summand as n increases. While some series with polynomial terms do converge—such as p-series with $p > 1$—others with lower powers or increasing functions diverge.

Summary and Key Takeaways

- The series in question is:

$$\sum_{n=1}^{\infty} 6n(n + 2)$$

- Simplified, it becomes:

$$\sum_{n=1}^{\infty} (6n^2 + 12n)$$

- The individual terms grow without bound as n approaches infinity.
- The terms do not tend to zero, violating a necessary condition for convergence.
- As a result, the series diverges; it does not have a finite sum.

In conclusion, while infinite series are powerful tools, not all series are summable. In this case, the growth rate of the terms prevents convergence, illustrating an essential principle in mathematical analysis: the behavior of individual terms dictates the possibility of summing an infinite sequence.

Final Reflection

Understanding why certain series diverge helps clarify the boundaries of mathematical analysis and informs practical applications where convergence is necessary—such as in series approximations, Fourier analysis, and signal processing. Recognizing divergent series is equally vital, reminding us that not all infinite sums lead to finite, meaningful results. The series $\sum_{n=1}^{\infty} 6n(n + 2)$ exemplifies how growth rates influence convergence, reinforcing core concepts in the study of infinite series.

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