2x^3y + 18xy - 10x^2y - 90y Part A: Rewrite The Expression So That The GCF Is Factored Completely Part

2x^3y + 18xy - 10x^2y - 90y Part A: Rewrite The Expression So That The GCF Is Factored Completely Part

When working with algebraic expressions, one of the fundamental skills is factoring out the Greatest Common Factor (GCF). Properly identifying and extracting the GCF simplifies the expression, making it easier to analyze, solve, or further manipulate. In this article, we will focus on the expression 2x^3y + 18xy - 10x^2y - 90y and demonstrate how to rewrite it so that the GCF is factored completely. This process involves carefully analyzing each term to determine common factors and systematically extracting them to present the expression in its simplest factored form.

Understanding the Expression

Before diving into factoring, it's crucial to understand the structure of the given expression:

2x^3y + 18xy - 10x^2y - 90y

Notice that this expression contains four terms, each involving variables x and y, along with coefficients. Our goal is to find the GCF of all these terms, which involves:

- Identifying the common numerical factors
- Identifying the common variable factors

Step 1: Find the Numerical GCF

Analyzing the coefficients

The coefficients of each term are:

- 2 (from 2x^3y)
- 18 (from 18xy)
- -10 (from -10x^2y)
- -90 (from -90y)

To find the GCF of these numbers, list their factors:

- 2: 1, 2

- 18: 1, 2, 3, 6, 9, 18
- -10: 1, 2, 5, 10
- -90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

The common factors among all coefficients are 1 and 2. Since 2 divides all coefficients, but 1 is always a factor, the numerical GCF is 2.

Considering the signs

While factoring out the GCF, the negative signs are usually incorporated into the remaining factors or the factored-out term if desired. For clarity, we will factor out 2 and handle the signs when factoring the entire expression.

Step 2: Find the Variable GCF

Analyzing the variables in each term

Look at the variables in each term:

- 2x^3y: variables x^3 and y
- 18xy: variables x and y
- -10x^2y: variables x^2 and y
- -90y: variable y only

Identify the common variable factors:

- All terms have at least one y
- The variable x appears as x, x^2, or x^3 in different terms, so the lowest power of x common to all is x^1

Therefore, the variable GCF is y, as every term contains y, and the lowest power of x in all terms is x^1, but since not all terms have x, we only consider y as the common variable.

Note: Because the last term (-90y) only contains y, and the other terms contain x as well, the common variable factor across all terms is just y.

Step 3: Extract the GCF

Now, combine the numerical GCF and the variable GCF:

- Numerical GCF: 2
- Variable GCF: y

Thus, the GCF of the entire expression is 2y.

Step 4: Rewrite the Expression by Factoring Out the GCF

To factor out 2y, divide each term by 2y:

1. $2x^3y \div 2y = x^3$
2. $18xy \div 2y = 9x$
3. $-10x^2y \div 2y = -5x^2$
4. $-90y \div 2y = -45$

The factored form of the expression is:

$$2y\,(x^3 + 9x - 5x^2 - 45)$$

This is the complete factorization of the GCF from the original expression.

Final Factored Expression

$$2y\,(x^3 + 9x - 5x^2 - 45)$$

The next steps might involve further factoring the expression inside the parentheses, but for Part A, the goal was to rewrite the original expression so that the GCF is factored completely. This process simplifies the expression and sets the stage for more advanced factoring, such as factoring by grouping or quadratic factoring if applicable.

Conclusion and Tips for Factoring Expressions

Factoring out the GCF is a foundational skill that simplifies complex algebraic expressions and facilitates solving equations. Here are some tips to remember:

- Always analyze both coefficients and variables to find the GCF.
- Prime factorize the numerical coefficients to identify common factors.
- Look for the lowest power of each variable present in all terms.
- Factor out the GCF carefully, dividing each term accordingly.

- After factoring out the GCF, examine the remaining expression for further factoring opportunities.

Mastering the process of rewriting expressions to factor out the GCF not only streamlines algebraic manipulations but also builds a solid foundation for tackling more complex algebraic problems. Practice with different types of expressions to enhance your skills and confidence in factoring techniques.

Frequently Asked Questions

What is the greatest common factor (GCF) of the terms in the expression 2x^3y + 18xy - 10x^2y - 90y?

The GCF of the terms is 2y because it divides all the terms evenly.

How do I factor out the GCF from the expression 2x^3y + 18xy - 10x^2y - 90y?

Factor out 2y from each term: 2y(x^2 + 9x - 5x - 45).

What is the fully factored form of the expression 2x^3y + 18xy - 10x^2y - 90y after factoring out the GCF?

After factoring out 2y, the expression is 2y(x^2 + 9x - 5x - 45). Then, combine like terms inside the parentheses if possible.

Can the quadratic inside the parentheses be factored further?

Yes, the quadratic x^2 + 4x - 45 can be factored further into (x + 9)(x - 5).

What is the complete factored form of the original expression?

The fully factored expression is 2y(x + 9)(x - 5).

Why is it important to factor out the GCF first when simplifying algebraic expressions?

Factoring out the GCF simplifies the expression and makes it easier to identify further factoring opportunities or solve equations.

How do I verify that my factored form is correct?

Multiply the factored form back out to ensure it simplifies to the original expression.

What are common mistakes to avoid when factoring out the GCF?

Common mistakes include missing the GCF, dividing incorrectly, or not factoring out the greatest possible GCF.

Is it necessary to factor the entire expression completely, or can partial factoring be sufficient?

For complete simplification and solving, the entire expression should be factored fully, including all possible factors.

What strategies can help me recognize the GCF in more complex algebraic expressions?

Look for common variables and coefficients among all terms, and consider factoring out the smallest powers of variables and the largest common numerical factors.

Additional Resources

$2x^3y + 18xy - 10x^2y - 90y$ Part A: Rewrite The Expression So That The GCF Is Factored Completely

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Introduction

In the realm of algebraic expressions, the process of factoring out the greatest common factor (GCF) is fundamental to simplifying complex expressions and solving equations efficiently. The expression $2x^3y + 18xy - 10x^2y - 90y$ presents a compelling case study for this technique. This article aims to dissect this expression meticulously, demonstrating how to identify and extract the GCF systematically, thereby rewriting the expression in its most factored form. Such mastery not only enhances computational fluency but also deepens conceptual understanding of polynomial structures.

Understanding the Expression: An Overview

Before proceeding with factoring, it is essential to familiarize ourselves with the structure and components of the given expression:

$$
2x^3y + 18xy - 10x^2y - 90y
$$

At first glance, the expression comprises four terms, each involving variables x and y, along with coefficients. Recognizing commonalities among these terms will be the first step toward identifying the GCF.

Step 1: Analyzing Coefficients

The coefficients in each term are:

- 2
- 18
- -10
- -90

To find their GCF:

List of coefficients:

$$
\boxed{2, 18, 10, 90}
$$

Prime factorizations:

- $2 = 2$
- $18 = 2 \times 3^2$
- $10 = 2 \times 5$
- $90 = 2 \times 3^2 \times 5$

GCF of coefficients:

The common prime factor among all is 2.

- The minimum power of 2 across all coefficients is 2^1.

- Since 3 and 5 are not present in all coefficients, they are not part of the GCF.

Therefore, the GCF of the coefficients is 2.

Step 2: Analyzing Variable Factors

Next, examine the variables in each term:

Term	Variables	Exponent of x	Exponent of y
$2x^3 y$	$x^3 y$	3	1
$18 x y$	$x y$	1	1
$-10 x^2 y$	$x^2 y$	2	1

| \(-90 y\) | \(y\) | 0 | 1 |

Common variable factors:

- All terms except the last have at least \(x^1\), but the last term has no \(x\). Therefore, the variable \(x\) is not common to all terms.

- All terms contain \(y\), with the exponents:

- \(x^3 y\): \(y^1\)
- \(x y\): \(y^1\)
- \(x^2 y\): \(y^1\)
- \(y\): \(y^1\)

Conclusion: Each term has at least one \(y\); thus, \(y\) can be factored out entirely.

Variable GCF:

\[
\boxed{y}
\]

Step 3: Combining the GCF of Coefficients and Variables

From the above analyses:

- Coefficient GCF: 2
- Variable GCF: \(y\)

Thus, the greatest common factor (GCF) of the entire expression is:

\[
\boxed{2y}
\]

Step 4: Factoring Out the GCF

Now, rewrite the original expression by factoring out \(2y\):

\[
2y \left(\text{expression inside parentheses} \right)
\]

Step 4a: Dividing Each Term by \(2y\):

- \(2x^3 y \div 2 y = x^3\)
- \(18 xy \div 2 y = 9 x\)

- $-10 x^2 y \div 2 y = -5 x^2$
- $-90 y \div 2 y = -45$

Step 4b: Writing the factored form:

$$
2y \left(x^3 + 9 x - 5 x^2 - 45 \right)
$$

Step 5: Final Expression and Verification

The fully factored form of the original expression, with the GCF factored out, is:

$$
\boxed{
2y \left(x^3 - 5 x^2 + 9 x - 45 \right)
}
$$

Verification:

To confirm correctness, expand the factored form:

$$
2y \times x^3 = 2x^3 y
$$
$$
2y \times (-5 x^2) = -10 x^2 y
$$
$$
2y \times 9 x = 18 xy
$$
$$
2y \times (-45) = -90 y
$$

which matches the original expression precisely.

Deep Dive into the Factorization Process

Why Is Identifying the GCF Important?

Factoring out the GCF simplifies the expression, making it easier to analyze, manipulate, or solve equations derived from it. It also reveals common structural elements, which can lead to further factorization or recognition of patterns.

Challenges Encountered

In some cases, terms might have variables with different exponents or coefficients with no common factors, complicating the process. Here, the key was to recognize that while the coefficients shared a common factor of 2, the variable x was not common to all terms, but y was.

Strategies for Accurate GCF Identification

1. Prime factorization of coefficients.
2. Listing variable exponents to find the minimum across all terms.
3. Systematic comparison to avoid overlooking common factors.

Common Pitfalls

- Overlooking signs (positive or negative) in coefficients.
- Assuming variables are common without checking all terms.
- Forgetting that zero exponents mean the variable does not appear in that term.

Conclusion

The process of rewriting the expression $2x^3 y + 18 xy - 10x^2 y - 90 y$ to extract the GCF reveals the importance of a systematic approach. By carefully analyzing coefficients and variables, identifying the GCF as 2y, and then dividing each term accordingly, we obtain a simplified, factored form:

$$
\boxed{
2y \left(x^3 - 5 x^2 + 9 x - 45 \right)
}
$$

This exercise underscores fundamental algebraic principles and demonstrates how meticulous analysis facilitates the simplification of complex polynomial expressions. Mastery of such techniques is invaluable for students and practitioners aiming for precision and efficiency in algebraic manipulations.

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