(3 Points) Evaluate Dxdx2ex/2ln(u)du 8. (3 Points) Compute The Average Value Of F(x)=x+tan(x) Over The

Introduction

The realm of calculus encompasses a wide array of techniques and concepts, each serving as fundamental tools for analyzing and solving complex mathematical problems. In this article, we delve into two specific tasks that exemplify the depth and utility of calculus: evaluating a definite integral involving exponential and logarithmic functions, and computing the average value of a given function over a specified interval. These problems not only reinforce core principles such as substitution and the Mean Value Theorem for integrals but also highlight the importance of strategic problem-solving approaches in calculus.

Part 1: Evaluating the Integral Dxdx2ex/2ln(u)du from 0 to 8

Understanding the Integral

The integral in question appears to be written as:

- ∫ from 0 to 8 of Dxdx 2e^{x/2} ln(u) du

However, this notation seems ambiguous or potentially contains typographical errors. A plausible interpretation is that the integral involves the differential dx, with the integrand being 2e^{x/2} ln(u), integrated with respect to u over some bounds. Alternatively, it could be a double integral involving variables x and u. For clarity, assume the integral is of the form:

$$\int_0^8 2e^{x/2} \ln(u)\, du$$

with the understanding that x is treated as a parameter during integration with respect to u, or vice versa. To proceed systematically, we need to clarify the structure of the integral.

Clarifying the Integral Expression

Suppose the integral is a double integral:

$$\iint_D 2e^{x/2} \ln(u)\, dx\, du$$

over some domain D. Alternatively, perhaps it's a single integral with respect to a variable u, with the integrand involving x as a parameter. Given the notation, a common scenario is an integral involving substitution to evaluate integrals with exponential and logarithmic functions.

Evaluating the Integral Step-by-Step

Step 1: Recognize the integrand

- The integrand is $2e^{x/2} \ln(u)$. If integrating with respect to u, then $e^{x/2}$ acts as a constant.
- If integrating with respect to x, then ln(u) acts as a constant.

Step 2: Decide on the order of integration

- Assuming the integral is with respect to u, from 0 to 8, with x fixed, then:

$$I(x) = \int_0^8 2e^{x/2} \ln(u)\, du$$

which simplifies to:

$$I(x) = 2e^{x/2} \int_0^8 \ln(u)\, du$$

Step 3: Evaluate the integral $\int \ln(u)\, du$

Recall that:

$$\int \ln(u)\, du = u \ln(u) - u + C$$

Step 4: Compute definite integral from 0 to 8

At u = 8:

$$8 \ln(8) - 8$$

At u approaching 0, ln(u) approaches negative infinity, but since the integral is from 0 to 8, and ln(u) is improper at 0, we need to consider the limit:

$$\lim_{a\to0^{+}} \int_{a}^8 \ln(u)\, du = \lim_{a\to0^{+}} [u \ln(u) - u]_{a}^{8}$$

Calculating at u = 8:

$$8 \ln(8) - 8$$

Calculating at u = a:

$$a \ln(a) - a$$

And taking the limit as a approaches 0+:

$$\lim_{a\to0^{+}} a \ln(a) - a$$

Since $a \ln(a) \to 0$ as $a \to 0+$, and $a \to 0$, the entire expression approaches 0. Therefore, the definite integral is:

$$\int_0^8 \ln(u)\, du = [8 \ln(8) - 8] - 0 = 8 \ln(8) - 8$$

Step 5: Final expression for I(x)

$$I(x) = 2e^{x/2} (8 \ln(8) - 8)$$

Summary of the Evaluation

The integral evaluates to:

$$\iint_D 2e^{x/2} \ln(u)\, du\, dx = 2e^{x/2} (8 \ln(8) - 8)$$

This result encapsulates the evaluation process, assuming the initial integral was intended as described. The key steps involved recognizing the integral of ln(u), handling the improper limit at 0, and applying properties of exponential and logarithmic functions.

Part 2: Computing the Average Value of F(x) = x + tan(x) Over an Interval

Understanding the Concept of Average Value

The average value of a continuous function f(x) over the interval [a, b] is given by the formula:

$$f_{avg} = (1/(b - a)) \int_a^b f(x)\, dx$$

This concept is central in analyzing the overall behavior of functions and provides insight into their mean levels over specific ranges.

Applying the Formula to F(x) = x + tan(x)

Step 1: Identify the interval

- The problem statement appears incomplete regarding the specific interval. For illustration, assume the interval is [a, b].
- Suppose, for example, the interval is $[0, \pi/4]$, a common choice involving tangent functions.

Step 2: Set up the integral

Average value = (1/(b - a)) ∫_a^b (x + tan(x)) dx

Step 3: Compute the integral

Integral of x:

∫ x dx = (1/2) x^2 + C

Integral of tan(x):

∫ tan(x) dx = -ln|cos(x)| + C

Combined integral:

∫ (x + tan(x)) dx = (1/2) x^2 - ln|cos(x)| + C

Part 3: Evaluating the definite integral

Suppose we choose $[a, b] = [0, \pi/4]$, then:

$\int_0$^{π/4} (x + tan(x)) dx = [(1/2) x^2 - ln|cos(x)|]_0^{π/4}

Calculating at the upper limit $x = \pi/4$:

(1/2)(π/4)^2 - ln|cos(π/4)| = (1/2)(π^2/16) - ln(√2/2)

which simplifies to:

$(\pi^2/32) - \ln(\sqrt{2}/2)$

Calculating at the lower limit x=0:

$(1/2)(0)^2 - \ln|\cos(0)| = 0 - \ln(1) = 0$

Therefore, the definite integral is:

$(\pi^2/32) - \ln(\sqrt{2}/2) - 0 = (\pi^2/32) - \ln(\sqrt{2}/2)$

Note that:

$\ln(\sqrt{2}/2) = \ln(\sqrt{2}) - \ln(2) = (1/2) \ln(2) - \ln(2) = - (1/2) \ln(2)$

Thus, the integral evaluates to:

$(\pi^2/32) + (1/2) \ln(2)$

Finally, the average value over [0, π/4] is:

$f_{avg} = (1/(\pi/4 - 0)) [(\pi^2/32) + (1/2) \ln(2)] = (4/\pi) [(\pi^2/32) + (1/2) \ln(2)]$

Simplify:

$f_{avg} = (4/\pi) (\pi^2/32) + (4/\pi) (1/2)$

Frequently Asked Questions

How do you evaluate the integral ∫ 3x 2e^x / (2 ln(u)) du?

To evaluate ∫ 3x 2e^x / (2 ln(u)) du, first simplify the integrand if possible, then identify suitable substitution or methods such as integration by parts or substitution to handle the exponential and logarithmic terms. Clarify the variables involved, especially u, and proceed accordingly.

What are the common methods to evaluate integrals involving exponential and logarithmic functions?

Common methods include substitution (e.g., u = ln(x)), integration by parts, and recognizing standard integral forms involving e^x and ln(x).

How do you compute the average value of the function F(x) = x + tan(x) over an interval?

The average value of F(x) over [a, b] is given by (1/(b - a)) ∫[a to b] F(x) dx. Compute the definite integral of F(x) over the interval and divide by the length of the interval.

What is the formula for the average value of a function over an interval?

The average value of a function F(x) over [a, b] is (1/(b - a)) ∫[a to b] F(x) dx.

How do you evaluate ∫ x + tan(x) dx for the average value calculation?

You compute the indefinite integral of x + tan(x), then evaluate it at the endpoints of the interval, subtract to find the definite integral, and finally divide by the interval length.

What challenges might arise when integrating F(x) = x + tan(x)?

Integrating tan(x) involves logarithmic functions, specifically ln|sec(x)|, and combining it with x requires careful handling. The indefinite integral of tan(x) is ln|sec(x)|, which can introduce complexity depending on the bounds.

Why is it important to understand the properties of exponential and logarithmic functions in integral evaluation?

Because many integrals involve these functions, understanding their properties helps simplify integrals, recognize standard forms, and choose appropriate substitution methods.

Can you provide the steps to evaluate the integral ∫ 3x 2e^x / (2 ln(u)) du?

First, clarify the relationship between x and u. If u is related to x through a substitution such as u = ln(x), then rewrite the integral in terms of u, simplify, and apply standard integration techniques. Otherwise, specify the variables to proceed properly.

What is the significance of computing the average value of a function in calculus?

The average value gives a representative value of the function over an interval, useful in applications like physics and engineering to understand the overall behavior of the function.

Are there standard integral formulas for functions like x + tan(x)?

Yes, the indefinite integral of x is (x^2)/2, and the integral of tan(x) is -ln|cos(x)|. Combining these gives the indefinite integral: (x^2)/2 - ln|cos(x)| + C.

Additional Resources

Understanding and Solving Integral and Average Value Problems: A Comprehensive Guide

When tackling advanced calculus problems, particularly those involving integrals and the computation of average values of functions, a thorough understanding of fundamental concepts is essential. In this article, we explore two significant topics: evaluating the integral $\int 3\,dx \times 2e^{x/2} \ln(u) \, du$ and computing the average value of the function $f(x) = x + \tan(x)$ over a specified interval. These problems not only test your grasp of integration techniques but also deepen your understanding of how integrals relate to real-world applications such as average values.

Part 1: Evaluating the Integral $\int 3\,dx \times 2e^{x/2} \ln(u) \, du$

Understanding the Problem

At first glance, the integral appears complex because it involves multiple variables and functions: an exponential function, a logarithmic function, and a constant multiplier. The key to solving such an integral is to recognize the structure of the expression and identify the appropriate techniques, such as substitution or integration by parts.

However, the expression as provided contains some ambiguity, primarily regarding the variables of integration. To clarify, let's assume the integral is:

$$
\int 3 \, dx \times 2 e^{x/2} \ln(u) \, du
$$

which suggests a product of two integrals:

$$
\left(\int 3\, dx \right) \times \left(\int 2 e^{x/2} \ln(u) \, du \right)
$$

Alternatively, if the integral is meant to be a single integral involving both x and u, it's crucial to understand the relationship between these variables. For this guide, we'll focus on the most common interpretation:

Approach 1: Separate Integrals (Product of Integrals)

If the integral is a product:

$$
\left(\int 3\, dx \right) \times \left(\int 2 e^{x/2} \ln(u) \, du \right)
$$

then:

- The first integral is straightforward: $\int 3\, dx = 3x + C_1$

- The second integral involves u and x. Since x appears in the exponential, but the integral is w.r.t. u, the variable x is treated as a constant during the integration over u.

Approach 2: Single Integral with Both Variables

If the integral involves both variables simultaneously, possibly as a double integral or an integral with respect to a parameter, the problem becomes more complex, requiring substitution techniques.

Step-by-Step Solution (Assuming Separate Integrals)

Step 1: Integrate $3\, dx$

$$
\int 3\, dx = 3x + C_1
$$

This is straightforward.

Step 2: Focus on $\int 2 e^{x/2} \ln(u) \, du$

Since this integral is with respect to u, and x is treated as a constant, the integral simplifies to:

$$
2 e^{x/2} \int \ln(u) \, du
$$

The factor $2 e^{x/2}$ is a constant with respect to u.

Step 3: Evaluate $\int \ln(u) \, du$

Recall the standard integral:

$$
\int \ln(u) \, du = u \ln(u) - u + C
$$

Step 4: Write the combined integral

$$
2 e^{x/2} [u \ln(u) - u] + C
$$

Final Expression:

$$
\boxed{
(3x + C_1) \times 2 e^{x/2} [u \ln(u) - u] + C
}
$$

Part 2: Computing The Average Value of $f(x) = x + \tan(x)$ Over an Interval

Understanding the Concept of Average Value

The average value of a continuous function $f(x)$ over an interval $[a, b]$ is given by the formula:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
$$

This concept is vital in understanding the behavior of functions over intervals, especially in applications such as physics, economics, and engineering, where average quantities provide meaningful insights.

Step 1: Define the Interval

Suppose we are asked to find the average value of $f(x) = x + \tan(x)$ over the interval $[a, b]$. For illustrative purposes, let's assume:

$$
a = 0, \quad b = \frac{\pi}{4}
$$

which is a common interval where $\tan(x)$ is well-behaved and integrable.

Step 2: Set Up the Integral

The average value becomes:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} (x + \tan(x)) \, dx
$$

For our example:

$$
f_{avg} = \frac{1}{\frac{\pi}{4} - 0} \int_{0}^{\pi/4} (x + \tan(x)) \, dx
$$

which simplifies to:

$$
f_{avg} = \frac{4}{\pi} \int_{0}^{\pi/4} (x + \tan(x)) \, dx
$$

Step 3: Evaluate the Integral

The integral can be separated into two parts:

$$
\int_{0}^{\pi/4} x \, dx + \int_{0}^{\pi/4} \tan(x) \, dx
$$

Part A: $\int x \, dx$

$$
\int x \, dx = \frac{x^2}{2}
$$

Evaluated from 0 to $\pi/4$:

$$
\left[\frac{x^2}{2} \right]_0^{\pi/4} = \frac{(\pi/4)^2}{2} - 0 = \frac{\pi^2/16}{2} = \frac{\pi^2}{32}
$$

Part B: $\int \tan(x) \, dx$

Recall:

$$
\int \tan(x) \, dx = -\ln |\cos(x)| + C
$$

Evaluate from 0 to $\pi/4$:

$$
[-\ln |\cos(x)|]_0^{\pi/4} = -\ln |\cos(\pi/4)| + \ln |\cos(0)| = -\ln \left(\frac{\sqrt{2}}{2} \right) + \ln (1)
$$

Since $\cos(\pi/4) = \frac{\sqrt{2}}{2}$ and $\cos(0) = 1$:

$$
-\ln \left(\frac{\sqrt{2}}{2} \right) + 0 = - \left(\ln \sqrt{2} - \ln 2 \right) = - \left(\frac{1}{2} \ln 2 - \ln 2 \right) = - \left(- \frac{1}{2} \ln 2 \right) = \frac{1}{2} \ln 2
$$

Step 4: Calculate the Average

Sum of the integrals:

$$
\frac{\pi^2}{32} + \frac{1}{2} \ln 2
$$

Multiply by the coefficient $\frac{4}{\pi}$:

$$
f_{avg} = \frac{4}{\pi} \left(\frac{\pi^2}{32} + \frac{1}{2} \ln 2 \right)
$$

Simplify:

$$
f_{avg} = \frac{4}{\pi} \times \frac{\pi^2}{32} + \frac{4}{\pi} \times \frac{1}{2} \ln 2
$$

$$
= \frac{4 \pi^2}{32 \pi} + \frac{2}{\pi} \ln 2
$$

$$
= \frac{\pi}{8} + \frac{2}{\pi} \ln 2
$$

Final Remarks and Practical Tips

- Always clarify variables and limits: When dealing with multiple variables or ambiguous expressions, ensure the problem's variables and limits are well-defined to choose the correct integration technique.

- Use substitution when appropriate: Recognize when substitution or integration by parts can simplify integrals, especially those involving exponential, logarithmic, or trigonometric functions.

- Understanding average value: Remember that the average value provides a mean level of the function over an interval and is computed via the integral divided by the interval length.

- Check for domain restrictions: Functions like $\tan(x)$ have discontinuities; choose intervals carefully to avoid undefined points.

Summary

In this guide, we've explored the process of evaluating a complex integral involving exponential and logarith

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