

3 Vectors, A , B , And C Are Described As Follows: $A = 2\hat{x} + 3\hat{y}$ $B = 4\hat{x} + 2\hat{y}$ $C = 1\hat{x} + 7\hat{y}$ Find

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Understanding vectors is fundamental in physics, engineering, and mathematics. They are essential tools for representing quantities that have both magnitude and direction. In this article, we will explore the vectors A, B, and C, analyze their components, and demonstrate how to perform various operations involving these vectors. By the end of this comprehensive guide, you'll have a clear understanding of vector addition, subtraction, scalar multiplication, and how to find the magnitude and direction of these vectors.

Introduction to Vectors in Cartesian Coordinates

Vectors are mathematical entities that are characterized by their magnitude and direction. They are often represented in Cartesian coordinates as ordered pairs or triples, depending on the dimensionality.

What is a Vector?

- A vector is a quantity with both magnitude and direction.
- Common examples include velocity, force, displacement, and acceleration.
- Vectors are denoted with an arrow or boldface notation, e.g., $\vec{A}$, $\vec{B}$, $\vec{C}$.

Vector Components

- In two-dimensional space, vectors are expressed as components along the X and Y axes:

$$\vec{V} = V_x \hat{i} + V_y \hat{j}$$

- Here, V_x and V_y are the components along the X and Y axes, respectively.

Analyzing the Given Vectors

Let's delve into the specific vectors provided:

Vector A

$$\vec{A} = 2 \hat{i} + 3 \hat{j}$$

- Components:

$$A_x = 2$$

$$A_y = 3$$

Vector B

$$\vec{B} = 4 \hat{i} + 2 \hat{j}$$

- Components:

$$B_x = 4$$

$$B_y = 2$$

Vector C

$$\vec{C} = 1 \hat{i} + 7 \hat{j}$$

- Components:

$$C_x = 1$$

$$C_y = 7$$

Calculating Magnitudes of the Vectors

The magnitude (length) of a vector in 2D space is calculated using the Pythagorean theorem:

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2}$$

Magnitude of Vector A

$$|\vec{A}| = \sqrt{(2)^2 + (3)^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.605$$

Magnitude of Vector B

$$|\vec{B}| = \sqrt{(4)^2 + (2)^2} = \sqrt{16 + 4} = \sqrt{20} \approx 4.472$$

Magnitude of Vector C

$$|\vec{C}| = \sqrt{(1)^2 + (7)^2} = \sqrt{1 + 49} = \sqrt{50} \approx 7.071$$

Finding the Direction of the Vectors

The direction of a vector can be described by the angle it makes with the positive X-axis, calculated via the tangent function:

$$\theta = \arctan\left(\frac{V_y}{V_x}\right)$$

Direction of Vector A

$$\theta_A = \arctan\left(\frac{3}{2}\right) \approx 56.31^\circ$$

Direction of Vector B

$$\theta_B = \arctan\left(\frac{2}{4}\right) = \arctan(0.5) \approx 26.57^\circ$$

Direction of Vector C

$$\theta_C = \arctan\left(\frac{7}{1}\right) \approx 81.87^\circ$$

Vector Operations

Understanding how to manipulate vectors is crucial. Let's explore basic vector operations with A, B, and C.

1. Vector Addition

Adding vectors involves summing their corresponding components:

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Example:

$$\vec{A} + \vec{B} = (2 + 4)\hat{i} + (3 + 2)\hat{j} = 6\hat{i} + 5\hat{j}$$

Resulting Vector:

- Components: $(6, 5)$
- Magnitude:

$$|\vec{A} + \vec{B}| = \sqrt{6^2 + 5^2} = \sqrt{36 + 25} = \sqrt{61} \approx 7.81$$

- Direction:

$$\theta_{A+B} = \arctan\left(\frac{5}{6}\right) \approx 39.81^\circ$$

2. Vector Subtraction

Subtracting vectors involves subtracting their components:

$$\vec{A} - \vec{C} = (A_x - C_x)\hat{i} + (A_y - C_y)\hat{j}$$

Example:

$$\vec{A} - \vec{C} = (2 - 1)\hat{i} + (3 - 7)\hat{j} = 1\hat{i} - 4\hat{j}$$

Resulting Vector:

- Components: $(1, -4)$

- Magnitude:

$$|\vec{A} - \vec{C}| = \sqrt{1^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17} \approx 4.123$$

- Direction:

$$\theta_{A-C} = \arctan\left(\frac{-4}{1}\right) \approx -75.96^\circ$$

(or equivalently, $180^\circ - 75.96^\circ = 104.04^\circ$) measured from the positive X-axis counterclockwise)

3. Scalar Multiplication

Multiplying a vector by a scalar scales its magnitude while preserving its direction:

$$k \vec{V} = k V_x \hat{i} + k V_y \hat{j}$$

Example:

Multiply vector C by 3:

$$3 \vec{C} = 3 \times 1 \hat{i} + 3 \times 7 \hat{j} = 3 \hat{i} + 21 \hat{j}$$

- Magnitude:

$$|3 \vec{C}| = 3 \times |\vec{C}| \approx 3 \times 7.071 = 21.213$$

Applications of Vector Operations in Real-World Problems

Vectors are not just theoretical; they have practical applications across various fields. Here are some examples:

Physics

- Calculating net force when multiple forces act on an object.
- Determining displacement in navigation and robotics.
- Analyzing motion in multiple dimensions.

Engineering

- Designing structures where forces and stresses are represented as vectors.
- Signal processing involving vector transformations.

Computer Graphics

- Representing movements, rotations, and transformations in 2D and 3D spaces.
- Rendering objects with correct orientation and positioning.

Advanced Vector Calculations

Beyond basic operations, vectors can be involved in more complex calculations:

Dot Product

- The dot product of vectors A and B:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

- Useful for finding the angle between vectors:

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

Cross Product (In 3D)

- While not directly applicable in 2D, for three-dimensional vectors, the cross product yields a vector perpendicular to the plane containing the original vectors.

Conclusion

Vectors A, B, and C serve as fundamental examples for understanding vector components,

magnitudes, directions, and operations. By analyzing these vectors, performing addition, subtraction, and scalar multiplication, and calculating their magnitudes and directions, you gain valuable insights into how vectors function and their significance in real-world applications. Mastery of vector operations is essential for students and professionals working in physics, engineering, computer science, and related fields.

Key Takeaways

- Vectors are characterized by their magnitude and direction.
- Components along axes determine the vector

Frequently Asked Questions

What are the components of vectors A, B, and C?

Vector A has components (2, 3), vector B has components (4, 2), and vector C has components (1, 7).

How do you find the magnitude of vector A?

The magnitude of A is $\sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.61$.

What is the dot product of vectors A and B?

$A \cdot B = (2)(4) + (3)(2) = 8 + 6 = 14$.

How can you determine if vectors A and C are orthogonal?

Calculate $A \cdot C = (2)(1) + (3)(7) = 2 + 21 = 23$. Since the result is not zero, A and C are not orthogonal.

What is the cross product of vectors A and B in 2D?

In 2D, the cross product magnitude is $(2)(2) - (3)(4) = 4 - 12 = -8$. The scalar value indicates the area and orientation.

How do you find the angle between vectors A and B?

Use the formula $\cos \theta = (A \cdot B) / (|A| |B|)$. Here, $\cos \theta = 14 / (\sqrt{13} \sqrt{20}) \approx 14 / (3.61 \cdot 4.47) \approx 14 / 16.14 \approx 0.866$. Therefore, $\theta \approx 30^\circ$.

What is the projection of vector C onto vector A?

Projection of C onto A = $[(C \cdot A) / |A|^2] A$. $C \cdot A = (1)(2) + (7)(3) = 2 + 21 = 23$. $|A|^2 = 13$. So,

projection = $(23/13)$ A = $(23/13)(2, 3)$.

Can vectors A, B, and C be coplanar?

Yes, in 2D all vectors lie in the same plane. To check if they are linearly independent, compute the scalar triple product; in 2D, since all are in the same plane, they are coplanar by definition.

Additional Resources

3 Vectors, A, B, and C, Are Described as Follows: $A = 2\hat{X} + 3\hat{Y}$, $B = 4\hat{X} + 2\hat{Y}$, $C = 1\hat{X} + 7\hat{Y}$. This set of vectors provides a foundational understanding of vector representation in two-dimensional space, which is crucial for various fields such as physics, engineering, computer graphics, and mathematics. Analyzing these vectors allows us to explore their properties, relationships, and applications. In this article, we will delve deeply into each vector's components, their geometric interpretations, operations like addition and scalar multiplication, and their significance in real-world scenarios.

Understanding the Vectors: A, B, and C

The vectors A, B, and C are expressed in terms of the standard basis vectors $\hat{X}$ and $\hat{Y}$, which represent unit vectors along the x-axis and y-axis, respectively. Their components are given as follows:

- $A = 2\hat{X} + 3\hat{Y}$
- $B = 4\hat{X} + 2\hat{Y}$
- $C = 1\hat{X} + 7\hat{Y}$

Each vector can be visualized as an arrow originating from the origin (0,0) to the point defined by its components in the coordinate plane. Understanding these components allows us to analyze their magnitude, direction, and relationships.

Component Analysis of Vectors A, B, and C

Vector A

- Components: (2, 3)
- Geometric interpretation: An arrow pointing to the point (2, 3) in the xy-plane.
- Magnitude: Calculated as $\sqrt{(2^2 + 3^2)} = \sqrt{(4 + 9)} = \sqrt{13} \approx 3.6055$.
- Direction: The angle θ with respect to the x-axis is given by $\theta = \arctangent(3/2) \approx 56.31^\circ$.

Vector B

- Components: (4, 2)
- Geometric interpretation: Points to (4, 2).
- Magnitude: $\sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} \approx 4.4721$.
- Direction: $\theta = \arctangent(2/4) = \arctangent(0.5) \approx 26.57^\circ$.

Vector C

- Components: (1, 7)
- Geometric interpretation: Points to (1, 7).
- Magnitude: $\sqrt{1^2 + 7^2} = \sqrt{1 + 49} = \sqrt{50} \approx 7.071$.
- Direction: $\theta = \arctangent(7/1) \approx 81.87^\circ$.

Operations on Vectors: Addition, Subtraction, and Scalar Multiplication

Understanding how these vectors interact through various operations is fundamental in applications such as physics (force analysis) and computer graphics (object transformations).

Vector Addition

Adding vectors combines their components:

$$- A + B = (2 + 4, 3 + 2) = (6, 5)$$

Geometrically, this results in a vector pointing to (6, 5), which can be visualized by placing the tail of B at the tip of A or vice versa.

$$- A + C = (2 + 1, 3 + 7) = (3, 10)$$

$$- B + C = (4 + 1, 2 + 7) = (5, 9)$$

Features:

- The resultant vector's magnitude can be calculated similarly.
- Vector addition is commutative and associative.

Vector Subtraction

Subtracting vectors involves component-wise subtraction:

$$- A - B = (2 - 4, 3 - 2) = (-2, 1)$$

$$- A - C = (2 - 1, 3 - 7) = (1, -4)$$

$$- B - C = (4 - 1, 2 - 7) = (3, -5)$$

Applications:

- Determining relative displacement.
- Calculating net forces.

Scalar Multiplication

Multiplying a vector by a scalar scales its magnitude:

- For example, $3A = (32, 33) = (6, 9)$
- The magnitude of $3A$ is 3 times that of A (~ 10.816).

Features:

- Changes the vector's length but not its direction unless the scalar is negative, which reverses direction.

Dot Product and Its Significance

The dot product (or scalar product) between vectors reveals information about the angle between them:

- $A \cdot B = (2)(4) + (3)(2) = 8 + 6 = 14$
- $A \cdot C = (2)(1) + (3)(7) = 2 + 21 = 23$
- $B \cdot C = (4)(1) + (2)(7) = 4 + 14 = 18$

Features:

- If the dot product is positive, vectors point roughly in the same direction.
- Zero dot product indicates orthogonality (perpendicular vectors).
- Useful in calculating angles between vectors:

$$\theta = \arccos[(A \cdot B) / (|A| |B|)]$$

Cross Product in 2D: An Overview

Although the cross product is primarily defined in three dimensions, in 2D, the scalar cross product (or the z-component of the 3D cross product) provides useful information about the relative orientation of vectors:

$$- A \times B = (2)(2) - (3)(4) = 4 - 12 = -8$$

Similarly,

- $A \times C = (2)(7) - (3)(1) = 14 - 3 = 11$

- $B \times C = (4)(7) - (2)(1) = 28 - 2 = 26$

Features:

- The sign indicates the relative rotation (positive for counterclockwise, negative for clockwise).
- The magnitude gives the area of the parallelogram spanned by the vectors.

Applications and Relevance of These Vectors

The analysis of vectors A, B, and C extends beyond theoretical mathematics into practical applications across various fields.

Physics and Engineering

- Force analysis: Vectors represent forces acting on an object. Summing vectors gives the net force.
- Motion: Displacement vectors determine the position change over time.
- Torque: Cross products help calculate torque in mechanical systems.

Computer Graphics and Robotics

- Object transformations: Scaling, rotation, and translation often involve vector operations.
- Pathfinding: Vectors help model movement and orientation.
- Robotics: Directional vectors determine the position and movement of robotic arms.

Mathematical Modeling and Data Analysis

- Vectors model multi-dimensional data points.
- Dot and cross products assist in understanding relationships between data vectors.

Pros and Cons of Vector Representation

Pros:

- Provides a clear geometric understanding of quantities like force, velocity, and displacement.
- Facilitates algebraic manipulation of geometric problems.
- Enables efficient computation of angles, projections, and areas.
- Essential in multidimensional modeling and simulations.

Cons:

- Limited to linear spaces; complex non-linear relationships require advanced tools.

- Visualization can become challenging in higher dimensions.
- Requires understanding of both algebraic and geometric concepts.

Conclusion

The vectors A, B, and C, with their explicit component forms, serve as fundamental building blocks for understanding multi-dimensional quantities. Their properties, such as magnitude, direction, and relationships via addition, subtraction, dot, and cross products, underpin numerous scientific and engineering applications. Mastery over these basic vector operations enables professionals and students alike to analyze complex systems effectively. Whether calculating forces, designing computer graphics, or exploring mathematical theories, vectors remain an indispensable tool in the modern scientific toolkit. Their versatility and clarity in representing spatial information make them a cornerstone concept deserving of thorough understanding and continuous exploration.

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