(a) Assume That F(x) Is A Function Defined By F(x) X-3x+1 2x - 1 For 2 X 3. Prove That F(r) Is Bounded

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Introduction

In mathematical analysis, understanding whether a function is bounded within a specific interval is fundamental. Bounded functions have maximum and minimum values within that interval, which simplifies many applications across calculus, optimization, and applied mathematics. This article explores the function $F(x)$, defined by the expression $F(x) = x - 3x + 1 + 2x - 1$, for x in the interval $[2, 3]$, and aims to prove that $F(x)$ is bounded on this domain.

Understanding the Function $F(x)$

Definition of $F(x)$

Given the function:

$$
F(x) = x - 3x + 1 + 2x - 1
$$

for x in the interval $[2, 3]$.

Simplification of $F(x)$

Before proceeding to analyze the boundedness, it's crucial to simplify the expression:

$$
F(x) = x - 3x + 1 + 2x - 1
$$

Combine like terms:

- Combine the x-terms: $x - 3x + 2x = (1 - 3 + 2)x = 0x$
- Combine the constants: $1 - 1 = 0$

Thus, the simplified form:

$$
F(x) = 0x + 0 = 0
$$

\]

Analysis of \(F(x)\)

Constant Function

The simplified form shows that:

\[
F(x) = 0 \quad \text{for all } x \in [2, 3]
\]

This indicates that \(F(x)\) is a constant function over the interval.

Implication for Boundedness

Since \(F(x)\) is constant and equal to zero everywhere in the interval:

- The function takes the value 0 at every point \(x \in [2, 3]\).
- Therefore, the maximum value of \(F(x)\) on \([2, 3]\) is 0.
- Similarly, the minimum value of \(F(x)\) on \([2, 3]\) is also 0.

Formal Proof That \(F(x)\) Is Bounded on \([2, 3]\)

Definition of Boundedness

A function \(f(x)\) is bounded on an interval \([a, b]\) if there exist real numbers \(M\) and \(m\) such that:

\[
m \leq f(x) \leq M \quad \text{for all } x \in [a, b]
\]

In this case, since \(F(x) = 0\) for all \(x\), the constants \(m = 0\) and \(M = 0\) satisfy these conditions.

Step-by-Step Proof

1. Calculate \(F(x)\): As shown, \(F(x) = 0\) everywhere.

2. Identify bounds: Since \(F(x) = 0\) for all \(x\), the maximum and minimum of \(F(x)\) over \([2, 3]\) are both 0.

3. Show boundedness: For all \(x \in [2, 3]\):

\[
0 \leq F(x) \leq 0

\]

which implies $F(x)$ is bounded, with bounds $m = 0$ and $M = 0$.

4. Conclusion: The function $F(x)$ is bounded on $[2, 3]$.

Additional Insights and Implications

Significance of Constant Functions

Constant functions are trivially bounded because they do not vary; their range consists of a single value. This makes analysis straightforward compared to more complex functions.

Real-World Applications

Understanding the boundedness of functions like $F(x)$ has applications in:

- Physics: where constant forces or quantities are involved.
- Engineering: in control systems where stability depends on bounded signals.
- Economics: for modeling constant returns or stable variables.

Extending the Concept

The process used here can be generalized to analyze the boundedness of other functions, especially polynomials or piecewise functions, by:

- Simplifying the function.
- Finding critical points.
- Evaluating the function at endpoints and critical points.
- Confirming the maximum and minimum values.

Summary

In this article, we examined the function $F(x) = x - 3x + 1 + 2x - 1$ over the interval $[2, 3]$. The key steps included simplifying the expression, recognizing that $F(x)$ is constant and equal to zero, and demonstrating that the function is bounded by the constants 0 and 0. This straightforward proof underscores the importance of algebraic simplification in analyzing the properties of functions. Since constant functions are inherently bounded, $F(x)$ is bounded on the given interval, satisfying the criteria for boundedness with ease.

Frequently Asked Questions (FAQs)

1. How do I determine if a function is bounded?

To determine if a function is bounded on an interval:

- Simplify the function if possible.
- Find critical points by differentiating (if applicable).
- Evaluate the function at critical points and endpoints.
- Identify the maximum and minimum values from these evaluations.
- If these are finite, the function is bounded.

2. Why is the boundedness of a function important?

Boundedness is crucial because it guarantees that the function's values stay within finite limits, which is essential for convergence, stability analysis, and ensuring physical or real-world constraints are respected.

3. Can a constant function be unbounded?

No, constant functions are always bounded because they do not vary; their range contains only a single value.

Conclusion

The analysis of $F(x)$ over $[2, 3]$ demonstrates that algebraic simplification can reveal fundamental properties like boundedness. Recognizing that $F(x) = 0$ simplifies the process of proving boundedness, making it a clear and straightforward example for students and practitioners alike. This approach underscores the importance of understanding the structure of functions in mathematical analysis and their applications across various disciplines.

For further reading on bounded functions and analysis techniques, consider exploring calculus textbooks or online resources dedicated to real analysis.

Frequently Asked Questions

What is the given function F(x) in the problem statement?

The function is defined as $F(x) = (x - 3x + 1) / (2x - 1)$ for x in the interval [2, 3].

How can we simplify the function F(x) for easier analysis?

Simplify numerator: $x - 3x + 1 = -2x + 1$. So, $F(x) = (-2x + 1) / (2x - 1)$.

Is F(x) defined at all points in the interval [2, 3]?

Yes, except at points where the denominator is zero. Since $2x - 1 = 0$ when $x = 0.5$, which is outside [2, 3], F(x) is defined on the entire interval [2, 3].

What is the main goal to prove that F(r) is bounded on [2, 3]?

To show that there exist real numbers M and m such that $m \leq F(x) \leq M$ for all x in [2, 3], proving F(x) is bounded on that interval.

How can we analyze the boundedness of F(x) in the interval [2, 3]?

By evaluating the maximum and minimum values of F(x) on [2, 3], which can be done by examining critical points and the endpoints of the interval.

What are the critical points of F(x) within the interval [2, 3]?

Find where the derivative F'(x) = 0 or is undefined. For F(x) = (-2x + 1)/(2x - 1), differentiate to find critical points inside [2, 3].

What is the derivative F'(x) of the function F(x)?

Actually, upon simplification, the numerator becomes 0, indicating F(x) is a constant function, which is trivially bounded.

Based on the derivative analysis, what conclusion can we draw about the boundedness of F(x)?

Since F(x) simplifies to a constant function over [2, 3], it is bounded, with both maximum and minimum equal to that constant value.

Additional Resources

Assume That F(x) Is A Function Defined By F(x) = (x - 3x + 1) / (2x - 1) for 2 < x < 3: Prove That F(x) Is Bounded

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Introduction

Mathematical functions often serve as the backbone of advanced analysis, modeling phenomena across sciences, engineering, and economics. A fundamental question in the study of functions is their boundedness within a given domain, which provides insights into their stability, limits, and continuity. This article delves into a specific function defined by a rational expression, aiming to rigorously establish its boundedness over the interval $2 < x < 3$.

The function under consideration is:

$$
F(x) = \frac{x - 3x + 1}{2x - 1}
$$

which simplifies to:

$$
F(x) = \frac{-2x + 1}{2x - 1}
$$

for all $x \in (2, 3)$. The primary goal is to prove that $F(x)$ is bounded on this interval, meaning there exist real numbers M and m such that:

$$
m \leq F(x) \leq M \quad \text{for all } x \in (2, 3)
$$

This investigation is not merely an exercise in algebraic manipulation but provides an essential understanding of the nature of rational functions, their limits, and their behavior within specified domains.

---

1. Analyzing the Function $F(x)$

1.1 Simplification of the Expression

Given:

$$
F(x) = \frac{x - 3x + 1}{2x - 1}
$$

Simplify numerator:

$$
x - 3x + 1 = -2x + 1
$$

Thus,

$$
F(x) = \frac{-2x + 1}{2x - 1}
$$

Note that the domain restriction arises from the denominator:

$$
2x - 1 \neq 0 \Rightarrow x \neq \frac{1}{2}
$$

Since our interval of interest is $(2, 3)$, which does not include $x = \frac{1}{2}$, $F(x)$ is well-defined throughout the interval.

1.2 Basic Properties

- Continuity: $F(x)$ is continuous for all $x \in (2, 3)$, as the denominator does not vanish there.
- Rationality: $F(x)$ is a rational function, which can exhibit asymptotic behavior at points where the denominator approaches zero.

---

2. Behavior of $F(x)$ on $(2, 3)$

2.1 Critical Points and Monotonicity

To understand whether $F(x)$ is bounded, we analyze its critical points, where the derivative $F'(x)$ equals zero or does not exist. Critical points often correspond to local maxima or minima, which are crucial in establishing bounds.

2.2 Derivative Calculation

Compute $F'(x)$ using the quotient rule:

$$
F(x) = \frac{N(x)}{D(x)} = \frac{-2x + 1}{2x - 1}
$$

where:

$$
N(x) = -2x + 1,\quad D(x) = 2x - 1
$$

Derivative:

$$
F'(x) = \frac{N'(x) D(x) - N(x) D'(x)}{[D(x)]^2}
$$

Calculate derivatives:

$$
N'(x) = -2,\quad D'(x) = 2
$$

Substitute:

$$
F'(x) = \frac{(-2)(2x - 1) - (-2x + 1)(2)}{(2x - 1)^2}
$$

Simplify numerator:

$$
= \frac{-2(2x - 1) + 2(2x - 1)}{(2x - 1)^2}
$$

$$
= \frac{-4x + 2 + 4x - 2}{(2x - 1)^2}
$$

$$
= \frac{0}{(2x - 1)^2} = 0
$$

---

3. Implications of the Derivative Analysis

The derivative $F'(x)$ simplifies to zero across the domain $(2, 3)$, indicating that:

$$
\boxed{
F(x) \text{ is constant on } (2, 3)
}
$$

3.1 Validity of the Conclusion

Given that $F'(x) = 0$ everywhere in the interval, the function must be constant. To verify, we can compute $F(x)$ at any point within the interval and confirm the value remains unchanged.

---

4. Verifying the Constancy of $F(x)$

Choose a convenient point, for example, $x = 2.5$:

$$
F(2.5) = \frac{-2(2.5) + 1}{2(2.5) - 1} = \frac{-5 + 1}{5 - 1} = \frac{-4}{4} = -1
$$

Similarly, test at $x = 2.9$:

$$
F(2.9) = \frac{-2(2.9) + 1}{2(2.9) - 1} = \frac{-5.8 + 1}{5.8 - 1} = \frac{-4.8}{4.8} = -1
$$

At $x = 2.1$:

$$
F(2.1) = \frac{-2(2.1) + 1}{2(2.1) - 1} = \frac{-4.2 + 1}{4.2 - 1} = \frac{-3.2}{3.2} = -1
$$

The function value is consistently -1 across multiple points within $(2, 3)$, confirming that:

$$
F(x) \equiv -1 \quad \text{for all } x \in (2, 3)

\]

---

5. Conclusion: Boundedness of \(F(x) \)

Since \(F(x) \) is constant and equal to \(-1\) over the entire domain \((2, 3) \), it is trivially bounded:

\[
\boxed{
\text{For all } x \in (2, 3), \quad F(x) = -1
}
\]

This implies that:

\[
\boxed{
\text{Boundedness:} \quad \text{The function } F(x) \text{ is bounded by } -1
}
\]

More precisely, the bounds are:

\[
m = M = -1
\]

making the function not only bounded but also constant within the interval.

---

6. Broader Implications and Mathematical Significance

6.1 Rational Functions and Constant Behavior

The case of \(F(x) \) being constant highlights an interesting property of rational functions: when the numerator and denominator are proportional across the domain, the function simplifies to a constant. Recognizing such properties is essential for mathematicians and analysts when evaluating the behavior of complex functions.

6.2 Application in Analysis and Modeling

Understanding the boundedness of functions is vital in stability analysis, limit evaluation, and in the context of continuous mappings between spaces. The result here demonstrates how algebraic simplification can lead to profound insights about the nature of a function.

6.3 Educational Value

This exploration serves as a pedagogical example for students learning about derivatives, rational functions, and boundedness. It emphasizes the importance of derivative tests, algebraic simplification, and verification through substitution.

Final Remarks

The detailed analysis of the function $F(x) = \frac{-2x + 1}{2x - 1}$ over the interval $(2, 3)$ reveals a surprisingly simple yet instructive result: $F(x)$ is a constant function equal to -1, and thus, it is inherently bounded on this domain. The initial complexity of the function masks its elegant behavior, illustrating the power of calculus and algebraic techniques in uncovering fundamental properties.

The key takeaway is that rigorous analysis—through derivative computation, critical point examination, and substitution—can definitively establish boundedness, an essential aspect in mathematical analysis and its applications.

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