(a) Estimate The Area Under The Graph Of F(x) = 5 Cos(x) From X = 0 To X = /2 Using Four Approximating

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Understanding how to approximate the area under a curve is a fundamental concept in calculus, particularly in the study of definite integrals. When an exact calculation of the area is difficult or impossible to perform analytically, mathematicians and students often rely on numerical methods such as Riemann sums, trapezoidal rules, and Simpson's rule to estimate the value with reasonable accuracy.

In this article, we'll focus on estimating the area under the curve of the function $f(x) = 5 \cos(x)$ over the interval from $x=0$ to $x=\pi/2$. Specifically, we will use four approximations—meaning we will partition the interval into four subintervals and apply different numerical methods to estimate the integral.

This task not only enhances understanding of integral approximation but also provides insight into how the choice of method and the number of subintervals can influence the accuracy of the estimate. Whether you're a student preparing for exams, a teacher designing instructional content, or a curious learner, this comprehensive guide aims to clarify the process step-by-step.

Understanding the Problem and Its Context

Before diving into calculations, it's essential to understand the problem's components thoroughly:

- Function to integrate: $f(x) = 5 \cos(x)$

- Interval of integration: from $x=0$ to $x=\pi/2$

- Number of subintervals: 4

- Objective: To estimate the area under the curve using four different approximations.

The definite integral

$$
\int_{0}^{\pi/2} 5 \cos(x) \, dx
$$

represents the exact area under the curve between these two points. Since the cosine function is well-understood and its integral is straightforward, we can compare our approximations to the exact value later to evaluate their accuracy.

Partitioning the Interval

The first step in numerical approximation involves dividing the interval $[0, \pi/2]$ into subintervals of equal length.

1. Calculate the width of each subinterval (Δx):

$$
\Delta x = \frac{b - a}{n} = \frac{\pi/2 - 0}{4} = \frac{\pi/2}{4} = \frac{\pi}{8}
$$

2. Determine the subinterval endpoints:

- $x_0 = 0$
- $x_1 = x_0 + \Delta x = \pi/8$
- $x_2 = x_1 + \Delta x = \pi/4$
- $x_3 = x_2 + \Delta x = 3\pi/8$
- $x_4 = x_3 + \Delta x = \pi/2$

This partitioning provides the framework for applying various approximation methods.

Calculating Function Values at Key Points

To perform the approximations, we need the function values at specific points within each subinterval.

- $f(x) = 5 \cos(x)$

Calculate at the endpoints and midpoints as necessary for different methods.

x	$f(x) = 5 \cos(x)$	Approximate value
0	$5 \cos(0) = 5 \times 1 = 5$	5
$\pi/8$	$5 \cos(\pi/8) \approx 5 \times 0.9239 \approx 4.619$	4.619
$\pi/4$	$5 \cos(\pi/4) = 5 \times \frac{\sqrt{2}}{2} \approx 5 \times 0.7071 \approx 3.535$	3.535
$3\pi/8$	$5 \cos(3\pi/8) \approx 5 \times 0.3827 \approx 1.913$	1.913
$\pi/2$	$5 \cos(\pi/2) = 5 \times 0 = 0$	0

These values are fundamental for implementing the approximation techniques.

Approximating the Area Using Numerical Methods

Several methods exist for estimating the integral, each with varying degrees of complexity and accuracy. We'll explore four approaches:

1. Left Riemann Sum
2. Right Riemann Sum
3. Midpoint Rule
4. Trapezoidal Rule

Let's examine each in detail.

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1. Left Riemann Sum

The Left Riemann Sum uses the left endpoints of each subinterval to evaluate the function and multiply by the subinterval width.

Formula:

$$
\text{Approximate Area} = \sum_{i=0}^{n-1} f(x_i) \times \Delta x
$$

Implementation:

Using the values:

$$
A_{L} = \left[f(x_0) + f(x_1) + f(x_2) + f(x_3) \right] \times \Delta x
$$

$$
A_{L} = (5 + 4.619 + 3.535 + 1.913) \times \frac{\pi}{8}
$$

Calculate the sum:

$$
\text{Sum} = 5 + 4.619 + 3.535 + 1.913 = 15.067
$$

Multiply:

$$
A_{L} = 15.067 \times \frac{\pi}{8} \approx 15.067 \times 0.3927 \approx 5.913
$$

Estimated area using Left Riemann Sum: approximately 5.913 square units.

---

2. Right Riemann Sum

The Right Riemann Sum uses the right endpoints:

$$
A_{R} = \sum_{i=1}^{n} f(x_i) \times \Delta x
$$

Values at the right endpoints:

$$
f(x_1) = 4.619, \quad f(x_2) = 3.535, \quad f(x_3) = 1.913, \quad f(x_4) = 0
$$

Sum:

$$
4.619 + 3.535 + 1.913 + 0 = 10.067
$$

Multiply:

$$
A_{R} = 10.067 \times \frac{\pi}{8} \approx 10.067 \times 0.3927 \approx 3.951
$$

Estimated area using Right Riemann Sum: approximately 3.951 square units.

---

3. Midpoint Rule

The Midpoint Rule evaluates the function at the midpoints of each subinterval:

- Midpoints:

$$
x_{1/2} = \frac{x_0 + x_1}{2} = \frac{0 + \pi/8}{2} = \pi/16
$$

$$
x_{3/2} = \frac{\pi/8 + \pi/4}{2} = 3\pi/16
$$

$$
x_{5/2} = \frac{\pi/4 + 3\pi/8}{2} = 5\pi/16
$$

$$
x_{7/2} = \frac{3\pi/8 + \pi/2}{2} = 7\pi/16
$$

Calculate function values at these midpoints:

x	$f(x) = 5 \cos(x)$	Approximate value
$\pi/16$	$5 \cos(\pi/16) \approx 5 \times 0.9808 \approx 4.904$	4.904
$3\pi/16$	$5 \times 0.8315 \approx 4.157$	4.157
$5\pi/16$	$5 \times 0.5557 \approx 2.778$	2.778
$7\pi/16$	$5 \times 0.1951 \approx 0.975$	0.975

Apply the midpoint rule:

$$
A_{M} = \left(4.904 + 4.157 + 2.778 + 0.975 \right) \times \frac{\pi}{8}
$$

Sum:

$$
4.904 + 4.157 + 2.778 + 0.975 = 12.814
$$

Multiply:

$$
A_{M} = 12.814 \times 0.3927 \approx 5.038
$$

Estimated area using Midpoint Rule: approximately 5.038 square units.

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4. Trapezoidal Rule

The Trape

Frequently Asked Questions

What is the main goal when estimating the area under the graph of F(x) = 5 Cos(x) from x = 0 to x = π/2 using four approximations?

The main goal is to approximate the definite integral of $F(x) = 5 \cos(x)$ over the interval $[0, \pi/2]$ by dividing it into four subintervals and applying numerical methods such as Riemann sums, trapping, or Simpson's rule to estimate the area.

How do you determine the width of each subinterval when dividing the interval from 0 to π/2 into four parts?

The width of each subinterval, Δx, is calculated by dividing the total interval length by the number of subintervals: $\Delta x = (\pi/2 - 0) / 4 = \pi/8$.

Which numerical approximation methods can be used with four subintervals to estimate the area under F(x) = 5 Cos(x)?

Common methods include the Left Riemann Sum, Right Riemann Sum, Trapezoidal Rule, and Simpson's Rule, each providing different levels of accuracy based on how they approximate the area.

How is the Trapezoidal Rule applied to estimate the area in this problem?

Using the Trapezoidal Rule, the area is estimated by summing the areas of trapezoids formed over each subinterval: Area $\approx (\Delta x/2) [f(x0) + 2f(x1) + 2f(x2) + 2f(x3) + f(x4)]$, where $x0=0$ and $x4=\pi/2$.

What is the significance of choosing four subintervals for approximation in this context?

Using four subintervals balances computational simplicity and accuracy, providing a reasonable estimate of the integral without excessive calculations, especially useful in educational or quick estimation scenarios.

How can the accuracy of the approximation be improved beyond four subintervals?

Accuracy can be improved by increasing the number of subintervals, applying higher-order methods like Simpson's rule, or using adaptive techniques that refine the partition where the function varies rapidly.

What is the expected value of the area under the graph of F(x)

= 5 Cos(x) from 0 to π/2, and how close are the approximations with four subintervals likely to be?

The exact area is the definite integral of 5 Cos(x) from 0 to π/2, which equals 5 sin(x) evaluated from 0 to π/2, giving 5 (1 - 0) = 5. Approximations with four subintervals will be close but may slightly underestimate or overestimate depending on the method used.

Additional Resources

Estimate The Area Under The Graph Of F(x) = 5 Cos(x) From X = 0 To X = π/2 Using Four Approximating Methods

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Introduction

Calculus, as a cornerstone of mathematical analysis, offers various methods to estimate the area under a curve — a concept rooted in the fundamental idea of integration. When the function in question is complex or when an exact integral is challenging to compute, approximation techniques become invaluable tools. This article aims to provide a comprehensive review of how to estimate the area under the graph of $F(x) = 5 \cos(x)$ from $x = 0$ to $x = \frac{\pi}{2}$ using four well-established approximating methods: the Left Riemann Sum, Right Riemann Sum, Midpoint Rule, and Trapezoidal Rule.

By exploring these approaches in detail, including their theoretical foundations, step-by-step calculations, and comparative accuracy, we offer insights into practical applications of numerical integration. This detailed analysis not only aids students and educators in understanding approximation methods but also illustrates their relevance in real-world scenarios where exact solutions are either unavailable or computationally expensive.

Background: The Function and Interval

The function under consideration:

$$
F(x) = 5 \cos(x)
$$

is a trigonometric function scaled by a factor of 5. Its graph oscillates between 5 and -5, with the cosine function being positive on the interval $[0, \frac{\pi}{2}]$. The interval $[0, \frac{\pi}{2}]$ spans from the point where $\cos(0) = 1$ to where $\cos(\frac{\pi}{2}) = 0$.

The exact value of the definite integral:

$$
\int_{0}^{\frac{\pi}{2}} 5 \cos(x) \, dx
$$

can be computed analytically:

$$
= 5 \int_{0}^{\frac{\pi}{2}} \cos(x) \, dx = 5 \left[\sin(x) \right]_0^{\frac{\pi}{2}} = 5 (1 - 0) = 5
$$

This exact value provides a benchmark against which the approximations will be compared.

The Rationale for Numerical Approximation

While the exact integral is straightforward, understanding the approximation techniques is essential, especially when dealing with functions that are difficult or impossible to integrate analytically. Approximation techniques like Riemann sums and the Trapezoidal Rule allow us to estimate the area under the curve with controlled accuracy, depending on the number of subdivisions (or intervals) used.

In this article, we employ four methods with four subdivisions ($n=4$). This choice balances computational simplicity with meaningful insight into the behavior of each method.

Setting Up the Approximation: Subdividing the Interval

The interval $[0, \frac{\pi}{2}]$ has a length:

$$
\Delta x = \frac{\frac{\pi}{2} - 0}{4} = \frac{\pi}{8}
$$

The partition points are:

$$
x_0 = 0, \quad x_1 = \frac{\pi}{8}, \quad x_2 = \frac{\pi}{4}, \quad x_3 = \frac{3\pi}{8}, \quad x_4 = \frac{\pi}{2}
$$

Corresponding function values are:

$$
F(x_i) = 5 \cos(x_i)
$$

which will be used in the approximations.

---

1. Left Riemann Sum

Concept: Uses the left endpoints of each subinterval to estimate the area. It tends to underestimate when the function is decreasing and overestimate when increasing.

Calculation:

$$
L_4 = \sum_{i=0}^{3} F(x_i) \times \Delta x
$$

Values:

- $F(x_0) = 5 \cos(0) = 5 \times 1 = 5$
- $F(x_1) = 5 \cos(\pi/8) \approx 5 \times 0.9239 \approx 4.6195$
- $F(x_2) = 5 \cos(\pi/4) \approx 5 \times 0.7071 \approx 3.5355$
- $F(x_3) = 5 \cos(3\pi/8) \approx 5 \times 0.3827 \approx 1.9135$

Sum:

$$
L_4 = (5 + 4.6195 + 3.5355 + 1.9135) \times \frac{\pi}{8}
$$

Compute sum:

$$
\text{Sum} = 5 + 4.6195 + 3.5355 + 1.9135 = 15.068
$$

Approximate area:

$$
L_4 \approx 15.068 \times \frac{\pi}{8} \approx 15.068 \times 0.3927 \approx 5.918
$$

Observation: The Left Riemann Sum overestimates the true area (which is 5), due to the decreasing nature of $\cos(x)$ in this interval.

---

2. Right Riemann Sum

Concept: Uses the right endpoints; tends to overestimate when the function is decreasing.

Calculation:

$$
R_4 = \sum_{i=1}^{4} F(x_i) \times \Delta x
$$

Values:

- $F(x_1) \approx 4.6195$

- $F(x_2) \approx 3.5355$
- $F(x_3) \approx 1.9135$
- $F(x_4) = 5 \cos(\pi/2) = 0$

Sum:

$$
R_4 = (4.6195 + 3.5355 + 1.9135 + 0) \times \frac{\pi}{8} = 10.0685 \times 0.3927 \approx 3.951
$$

Observation: The Right Riemann Sum underestimates the true area, consistent with the decreasing nature of $F(x)$.

---

3. Midpoint Rule

Concept: Uses the midpoint of each subinterval, often providing a more accurate estimate because it considers the function's value at the center of each interval.

Calculation:

Midpoints:

$$
x_{i+0.5} = x_i + \frac{\Delta x}{2}
$$

Values:

- $x_{0.5} = \frac{\pi}{16} \approx 0.1963$

$$
F(x_{0.5}) = 5 \cos\left(\frac{\pi}{16}\right) \approx 5 \times 0.9808 \approx 4.904
$$

- $x_{1.5} = \frac{3\pi}{16} \approx 0.5890$

$$
F(x_{1.5}) \approx 5 \times 0.8315 \approx 4.158
$$

- $x_{2.5} = \frac{5\pi}{16} \approx 0.9817$

$$
F(x_{2.5}) \approx 5 \times 0.5557 \approx 2.778
$$

- $x_{3.5} = \frac{7\pi}{16} \approx 1.3744$

$$

$$
F(x_{3.5}) \approx 5 \times 0.3827 \approx 1.913
$$

Sum:

$$
4.904 + 4.158 + 2.778 + 1.913 = 13.753
$$

Approximate:

$$
\text{Midpoint Estimate} = 13.753 \times \frac{\pi}{8} \approx 13.753 \times 0.3927 \approx 5.396
$$

This estimate is closer to the exact value of 5, illustrating the Midpoint Rule's superior accuracy among the three Riemann sum methods.

---

4. Trapezoidal Rule

Concept: Approximates the area under the curve by trapezoids, averaging the function values at the endpoints of each subinterval.

Calculation:

$$
T_4 = \frac{\Delta x}{2} \left[F(x_0) + 2 \sum_{i=1}^{3} F(x_i) + F(x_4) \right]
$$

Sum of interior points:

$$
F(x_1) + F(x_2) + F(x_3) \approx 4.6195 + 3.5355 + 1.9135 = 10.0685
$$

Plugging in:

$$
T_4 = \frac{\pi/8}{2} \left[5 + 2 \times 10.0685 + 0 \right] = \frac{\pi/8}{2} (5 + 20.137) = \frac{\pi/8}{2} \times 25.137
$$

Simplify:

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A Estimate The Area Under The Graph Of F X 5 Cos X From X 0 To X 2 Using Four Approximating

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