(a). Evaluate The Polynomial: Y=x^3 -5x^2 + 6x + 0.55 At X=1.37. Use 3-digit Arithmetic With Chopping.

(a). Evaluate The Polynomial: Y=x^3 -5x^2 + 6x + 0.55 At X=1.37. Use 3-digit Arithmetic With Chopping.

Evaluating polynomials accurately is a fundamental aspect of numerical analysis, especially when computational tools are constrained by limited precision. In this article, we will focus on computing the value of the polynomial $Y = x^3 - 5x^2 + 6x + 0.55$ at $x = 1.37$, employing 3-digit arithmetic with chopping. This approach simulates the effects of limited precision arithmetic common in numerical computations, helping to understand the potential errors and their implications in real-world applications.

Understanding the Polynomial and Its Evaluation

Before diving into the computation, it is essential to understand the structure of the polynomial and the significance of using 3-digit chopping arithmetic.

Details of the Polynomial

- The polynomial in question is a cubic expression:

$$
Y = x^3 - 5x^2 + 6x + 0.55
$$

- The variable x is given as 1.37.

- The degree of the polynomial (degree 3) influences how errors propagate during evaluation, especially with limited precision.

Why Use 3-Digit Chopping Arithmetic?

- Chopping refers to truncating numbers to a specified number of significant digits, which can introduce truncation errors.
- Using 3-digit arithmetic mimics the limitations of some computational hardware or software that cannot handle more than three significant figures.
- Understanding how these limitations affect the calculation of polynomial values is crucial for applications requiring high precision.

Step-by-Step Evaluation Using 3-Digit Chopping Arithmetic

The process involves calculating each term of the polynomial at $x = 1.37$, with intermediate results truncated to three significant digits.

Step 1: Representing the Given Value

- The value of x is 1.37.
- In 3-digit chopping, it remains 1.37, as it already has three significant digits.

Step 2: Calculating x^2

- First, compute x^2:

$$
x^2 = 1.37 \times 1.37
$$

- Perform the multiplication:

$$
1.37 \times 1.37 = 1.8769
$$

- Chopping to 3 significant digits:

$$
1.8769 \rightarrow 1.87
$$

- Therefore, the chopped x^2 is 1.87.

Step 3: Calculating x^3

- To find x^3, multiply x^2 by x:

$$
x^3 = (x^2) \times x = 1.87 \times 1.37
$$

- Multiplying:

$$
1.87 \times 1.37 = 2.5649
$$

- Chopping to 3 significant digits:

$$
2.5649 \rightarrow 2.56
$$

- The chopped x^3 is 2.56.

Step 4: Calculating $-5x^2$

- Multiply x^2 by -5:

$$
-5 \times 1.87 = -9.35
$$

- No further chopping needed here, as the result already has 3 significant digits: -9.35.

Step 5: Calculating $6x$

- Multiply 6 by x:

$$
6 \times 1.37 = 8.22
$$

- The result, 8.22, already has 3 significant digits.

Step 6: Combining All Terms

Now, sum all the computed terms:

$$
Y = x^3 + (-5x^2) + 6x + 0.55
$$

Substituting the chopped intermediate results:

$$
Y = 2.56 + (-9.35) + 8.22 + 0.55
$$

Perform the addition step-by-step:

1. $2.56 + (-9.35) = -6.79$
2. $-6.79 + 8.22 = 1.43$
3. $1.43 + 0.55 = 1.98$

Thus, the final evaluated value of the polynomial at $x=1.37$ using 3-digit chopping arithmetic is approximately 1.98.

Discussion of Results and Errors

While the above steps provide an approximate value, understanding the implications of chopping errors is vital.

Propagation of Errors in Chopping Arithmetic

- Each arithmetic operation introduces a truncation error, which can accumulate.
- In this example, the initial multiplication to find x^2 introduces a small error (from 1.8769 to 1.87).
- Subsequent calculations build upon these truncated values, potentially amplifying inaccuracies.

Comparison to Exact Computation

- The exact value of the polynomial at $x=1.37$ can be computed with full precision:

$$
y_{exact} = (1.37)^3 - 5(1.37)^2 + 6(1.37) + 0.55
$$

- Calculating:

$$
(1.37)^3 \approx 2.571
$$
$$
(1.37)^2 \approx 1.877
$$
$$
-5 \times 1.877 \approx -9.385
$$
$$
6 \times 1.37 = 8.22
$$

- Summing:

$$
2.571 - 9.385 + 8.22 + 0.55 = 1.956
$$

- The exact value is approximately 1.956, whereas our chopped approximation yields 1.98.

- The difference (~0.024) illustrates the impact of limited precision and chopping errors.

Conclusion and Practical Implications

This detailed evaluation underscores the significance of precision in numerical computations. Using only three significant digits with chopping introduces errors that, while small in this case, can become substantial in more sensitive calculations or with more complex polynomials.

Key Takeaways:
- Chopping truncates numbers to a set number of significant digits, which can lead to cumulative errors.
- When evaluating polynomials or performing iterative calculations, understanding error propagation is crucial.
- For high-precision applications, techniques such as rounding or increased significant digits are recommended over simple chopping.

Practical Applications:
- Engineering calculations where hardware limitations restrict precision.
- Numerical simulations requiring error analysis.
- Educational purposes to illustrate the importance of precision in computational mathematics.

By mastering these concepts, students and professionals can better design algorithms that mitigate errors, ensuring more reliable and accurate computational results in various scientific and engineering fields.

Frequently Asked Questions

How do you evaluate the polynomial Y = x^3 - 5x^2 + 6x + 0.55 at x = 1.37 using 3-digit chopping arithmetic?

First, approximate each calculation by chopping to 3 significant digits. Calculate x^3, x^2, then multiply and subtract accordingly, truncating after each step to ensure 3-digit precision. For example, x^3 ≈ 2.57, x^2 ≈ 1.87, then compute the polynomial step-by-step with chopping.

What is the step-by-step process to evaluate Y at x=1.37 with 3-digit chopping?

1. Calculate x^3: 1.37^3 ≈ 2.58 (chopped from 2.571). 2. Calculate 5x^2: 5 1.87 ≈ 9.35, chopped to 9.35. 3. Calculate 6x: 6 1.37 ≈ 8.22, chopped to 8.22. 4. Sum: 2.58 - 9.35 + 8.22 + 0.55 ≈ 2.00 after chopping each intermediate result.

Why is chopping used in evaluating the polynomial, and how does it affect accuracy?

Chopping simulates limited precision arithmetic by truncating numbers to 3 significant digits, which can introduce errors and reduce accuracy. It helps understand how finite precision affects computation results, especially in numerical methods.

What is the approximate value of Y when x=1.37 using 3-digit chopping?

Using 3-digit chopping, the approximate value of Y is about 2.00.

How does the use of chopping compare to rounding in numerical evaluation?

Chopping truncates numbers to a fixed number of significant digits, potentially introducing more error than rounding, which adjusts the number to the nearest value. Chopping may lead to underestimation or overestimation in calculations.

Can the errors caused by chopping be minimized, and how?

Errors can be minimized by using higher precision, applying rounding instead of chopping, or implementing numerical methods that reduce accumulated errors. Consistent use of rounding tends to produce more accurate results than chopping.

What is the importance of understanding finite precision arithmetic in evaluating polynomials?

Understanding finite precision arithmetic is crucial because it helps predict and control errors in numerical computations, ensuring more reliable and accurate results in scientific and engineering applications.

Additional Resources

Evaluating the Polynomial \(Y = x^3 - 5x^2 + 6x + 0.55\) at \(x=1.37\) Using 3-Digit Arithmetic with Chopping

---

Introduction

The process of evaluating polynomials at specific points is fundamental in numerical analysis, engineering computations, and scientific simulations. Accurate and efficient methods are vital, especially when working with limited precision arithmetic—such as 3-digit arithmetic, which is common in hardware-limited systems or for simplified computational models.

In this detailed analysis, we examine the evaluation of the polynomial:

$$
Y = x^3 - 5x^2 + 6x + 0.55
$$

at the point $x=1.37$, utilizing 3-digit arithmetic with chopping. Chopping refers to truncating a number to a fixed number of digits without rounding, which can lead to significant differences compared to exact calculations. Our goal is to understand how this approach influences the accuracy of the result, dissect the step-by-step evaluation process, and discuss the implications of such limited precision.

Background: Polynomial Evaluation and Numerical Precision

Polynomial Evaluation Methods

Evaluating a polynomial can be performed straightforwardly by substituting the value of x and computing each term separately. However, for higher-degree polynomials or in computational contexts, algorithms such as Horner's method are preferred due to their numerical stability and efficiency.

In our case, the polynomial is of degree 3, so we can evaluate it directly:

$$
Y = x^3 - 5x^2 + 6x + 0.55
$$

or use Horner's form:

$$
Y = (((x) \times x + 0) \times x + (-5)) \times x + (6x + 0.55)
$$

However, since the polynomial is straightforward, direct substitution suffices, especially when illustrating the effects of limited precision.

Significance of 3-Digit Arithmetic with Chopping

Standard floating-point arithmetic often employs 16, 32, or 64 bits, corresponding to high precision. Here, we intentionally restrict ourselves to 3-digit arithmetic, meaning each calculated intermediate and final result is truncated to three significant digits.

Chopping differs from rounding: it truncates the number just after the third significant digit, disregarding any remaining digits. This process can introduce cumulative errors, especially in iterative calculations or when dealing with small differences.

Understanding the impact of such limited precision helps in designing robust algorithms and in recognizing the potential pitfalls in hardware with constrained precision.

Step-by-Step Evaluation of Y at $x=1.37$

Step 1: Set the value of x

Given:

$$
x = 1.37
$$

Since we're using 3-digit chopping, we must represent x with three significant digits.

Initial x:

$$
x = 1.37 \quad \text{(already 3 significant digits, no chopping needed)}
$$

Step 2: Compute x^2

Calculate:

$$
x^2 = (1.37)^2
$$

Exact calculation:

$$
1.37 \times 1.37 = 1.8769
$$

Chopping to 3 significant digits:

- The first three significant digits are 1.87.
- Truncate beyond that; discard 6 and 9.

Result:

$$
x^2 \approx \boxed{1.87}
$$

Step 3: Compute x^3

Calculate:

$$
x^3 = x \times x^2 = 1.37 \times 1.87
$$

Exact calculation:

$$
1.37 \times 1.87 \approx 2.5649
$$

Chopping to 3 significant digits:

- The first three significant digits are 2.56.
- Discard 4 and 9.

Result:

$$
x^3 \approx \boxed{2.56}
$$

Step 4: Compute $-5x^2$

Multiply:

$$
-5 \times 1.87
$$

Exact calculation:

$$
-5 \times 1.87 = -9.35
$$

Chopping to 3 significant digits:

- First three digits: -9.35 (already 4 digits, but the number is already in 3 significant digits).

- Since the number has only four characters, and the last digit is 5, but we're chopping, the fractional part remains as is.

Result:

$$
-5x^2 \approx \boxed{-9.35}
$$

Step 5: Compute $6x$

Calculate:

$$
6 \times 1.37 = 8.22
$$

Exact calculation:

$$
6 \times 1.37 = 8.22
$$

(Already 3 significant digits, no chopping needed.)

Result:

$$
6x \approx \boxed{8.22}
$$

Step 6: Sum all components: $x^3 + (-5x^2) + 6x + 0.55$

Now, assemble the parts step-by-step, applying chopping at each step.

Step 6.1: Sum x^3 and $-5x^2$

$$
2.56 + (-9.35) = -6.79
$$

Exact calculation:

$$
2.56 - 9.35 = -6.79
$$

- The sum is exactly -6.79. No further chopping needed here since the number already has 3 significant digits.

Result:

$$
\text{Sum}_1 \approx \boxed{-6.79}
$$

Step 6.2: Add $6x$

$$
-6.79 + 8.22 = 1.43
$$

Exact calculation:

$$
-6.79 + 8.22 = 1.43
$$

- Already 3 digits; no chopping necessary.

Result:

$$
\text{Sum}_2 \approx \boxed{1.43}
$$

Step 6.3: Add the constant 0.55

$$
1.43 + 0.55 = 1.98
$$

Exact calculation:

$$
1.43 + 0.55 = 1.98
$$

- Already with 3 significant digits.

Final evaluated value:

$$
Y \approx \boxed{1.98}
$$

Analytical Summary of the Chopping Process and Error Analysis

Observation of the Chopping Impact

Throughout the calculation, each intermediate step was truncated to 3 significant digits. This process introduces truncation errors at every stage, which can accumulate or cancel depending on the operations.

- Initial approximations of x^2 and x^3 already deviate from their exact values:
- Exact $x^2 \approx 1.8769$, chopped to 1.87.
- Exact $x^3 \approx 2.5649$, chopped to 2.56.

- These small deviations propagate into the subsequent calculations, affecting the final value.

Error Propagation

The initial error in x^2:

$$
\text{Error}_x^2 = 1.8769 - 1.87 = 0.0069
$$

Similarly, for x^3:

$$
\text{Error}_{x^3} = 2.5649 - 2.56 = 0.0049
$$

When multiplying by constants, errors are scaled:

- For $-5x^2$:

$$
-5 \times 0.0069 \approx -0.0345
$$

- For $6x$:

$$
6 \times 0.01 \approx 0.06
$$

Adding these errors cumulatively impacts the final sum, leading to a result that may differ from the exact value computed with full precision.

Comparing to Exact Calculation

Calculating the exact value of the polynomial at $x=1.37$:

$$
Y_{\text{exact}} = (1.37)^3 - 5(1.37)^2 + 6(1.37) + 0.55
$$

$$
= 2.5649 - 5 \times 1.8769 + 8.22 + 0.55
$$

$$
= 2.5649 - 9.3845 + 8.22 + 0.55
$$

$$
= (2.5649 - 9.3845) + 8.77
$$

$$
= -6.8196 + 8.77 = 1.9504
$$

Our approximated value via chopping:

$$
\boxed{1.98}
$$

is close but exhibits a small discrepancy (~0.03), attributable to the limited precision.

Implications and Significance of Chopping in Numerical Computations

Accuracy Limitations

Chopping to 3 significant digits introduces truncation errors, which can be especially problematic when evaluating polynomials with small differences or near

A Evaluate The Polynomial Y X 3 5x 2 6x 0 55 At X 1 37 Use 3 Digit Arithmetic With Chopping

A Evaluate The Polynomial Y X 3 5x 2 6x 0 55 At X 1 37 Use 3 Digit Arithmetic With Chopping

Related Articles

- When A Moving Object Reflects Sound Back To A Stationary Source Of Frequency F, Two Doppler Shifts Of
- What Is The Meaning Of The Word Gravely As It Is Used In This Sentence? Peter Was Not Gravely Hurt In
- What Could You Include To Give A Sense Of Scale In A Photograph Of A Mountain? The Sky Clouds Another

Back to Home