(a) Find The Slope Of The Curve Y=x-7x At The Given Point P(2.-6) By Finding The Limiting Value Of The

(a) Find The Slope Of The Curve Y = x - 7x At The Given Point P(2, -6) By Finding The Limiting Value Of The

Understanding how to find the slope of a curve at a specific point is a fundamental concept in calculus. The slope at a point on a curve provides valuable information about the rate of change of the function at that particular location. In this article, we will focus on the problem: *Find the slope of the curve y = x - 7x at the given point P(2, -6) by finding the limiting value of the difference quotient.* We will walk through the process step-by-step, illustrating the method of using the definition of the derivative through limits, which is essential for understanding the principles of calculus.

Understanding the Problem: The Curve and the Point

Before diving into calculations, it’s important to clarify what the problem entails.

The Given Function

- The function is y = x - 7x.
- Simplifying the function gives y = -6x.
- This is a linear function with a constant slope.

The Given Point

- P(2, -6) indicates that when x = 2, y = -6.
- Confirming the point on the function: substituting x = 2,
y = -6 2 = -12, which does not match the y-coordinate given (-6).
- This suggests a potential typo or mistake in the original problem statement or the point provided.
- For the sake of clarity, we will proceed assuming the point is P(2, -12), which lies on y = -6x.

Note: If the original point P(2, -6) is intended, the function y = x - 7x simplifies to y = -6x, which at x=2 yields y = -12, not -6. So, either the function or the point needs correction. For consistency, we will assume the point is P(2, -12).

Step-by-Step Solution: Finding the Slope Using Limits

The slope of a curve at a point is given by the derivative, which can be defined as the limit of the difference quotient as h approaches zero.

The Difference Quotient

The difference quotient is expressed as:

$$ \frac{f(x+h) - f(x)}{h} $$

To find the slope at x = 2, we evaluate:

$$ \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} $$

where f(x) = -6x.

Calculating the Difference Quotient

- First, compute f(2 + h):

$$ f(2 + h) = -6(2 + h) = -12 - 6h $$

- Next, evaluate f(2):

$$ f(2) = -6 \times 2 = -12 $$

- Now, find the numerator of the difference quotient:

$$ f(2 + h) - f(2) = (-12 - 6h) - (-12) = -12 - 6h + 12 = -6h $$

- The difference quotient becomes:

$$ \frac{-6h}{h} $$

- Simplify:

$$ -6 $$

- Finally, take the limit as h approaches 0:

$$ \lim_{h \to 0} -6 = -6 $$

This shows that the derivative of y = -6x at x=2 (and indeed at any x) is -6.

Conclusion: The Slope at the Given Point

The calculated limit confirms that the slope of the curve y = -6x at any point, including the point P(2, -12), is -6. This is consistent with the fact that the function y = -6x is linear with a constant slope of -6 everywhere on its domain.

Key Takeaways

- The derivative of a function at a point can be found as the limit of the difference quotient as h approaches zero.
- For linear functions like y = -6x, the slope is constant and equals the coefficient of x.
- Understanding the limit process is fundamental for analyzing more complex functions where the slope varies at different points.

Additional Notes on Calculus and Derivatives

Why Use Limits to Find Derivatives?

- Limits provide a precise way to define the instantaneous rate of change.
- The process captures the idea of approaching a point infinitely closely, which is essential for accurate slope calculation at that point.

Applications of Finding Slopes of Curves

- Optimization problems in economics and engineering.
- Analyzing motion in physics (velocity as the slope of a position-time graph).
- Graphing functions accurately by understanding their tangent lines.

Summary

In this article, we examined how to find the slope of the curve y = -6x at a specific point P(2, -12) (assuming correction from the original problem). By applying the limit definition of the derivative, we calculated the difference quotient and found that the slope at x=2 is -6. This example underscores the fundamental role of limits in calculus for determining the rate of change and understanding the behavior of functions at specific points.

Whether dealing with simple linear functions or complex nonlinear curves, mastering the process of finding derivatives through limits is essential for solving numerous problems across science and engineering disciplines. Remember, the key is to carefully compute the difference quotient and evaluate the limit as h approaches zero, which reveals the true nature of the curve's slope at any given point.

Frequently Asked Questions

What is the first step to find the slope of the curve y = x - 7x at point P(2, -6)?

The first step is to simplify the given function y = x - 7x to y = -6x and then find its derivative to determine the slope.

How do you find the derivative of y = x - 7x?

Since y = x - 7x, simplifying gives y = -6x. The derivative dy/dx = -6, which is constant everywhere on the curve.

Given the function y = -6x, what is the slope at any point, including P(2, -6)?

The slope at any point on the line y = -6x is -6, including at P(2, -6).

Why is the limiting process necessary in finding the slope of the curve at a point?

The limiting process helps find the instantaneous rate of change (slope) at a point by considering the limit of the average rate of change as the interval approaches zero.

How do you set up the limit to find the slope of y = x - 7x at point P(2, -6)?

You set up the limit of $(y - y_0) / (x - x_0)$ as x approaches $x_0 = 2$, where $y_0 = -6$, to find the derivative at that point.

What is the limiting value of the difference quotient for y = x - 7x at P(2, -6)?

Since y = -6x, the difference quotient simplifies to -6, and the limit as x approaches 2 is -6.

Is the slope of the curve at P(2, -6) equal to the derivative of y = -6x?

Yes, because the derivative of the simplified function y = -6x is -6, which gives the slope at P(2, -6).

Can the slope be different at different points for the function y = x - 7x?

No, because the function simplifies to a straight line y = -6x, which has a constant slope of -6 everywhere.

What is the importance of the point P(2, -6) in calculating the slope?

The point P(2, -6) confirms the specific location on the curve where the slope is being calculated, but since the function is linear, the slope is the same at all points.

What is the final answer for the slope of the curve y = x - 7x at P(2, -6)?

The slope at P(2, -6) is -6, which is the derivative of the simplified function y = -6x.

Additional Resources

Find The Slope Of The Curve Y = x - 7x At The Given Point P(2, -6) By Finding The Limiting Value Of The Derivative

Understanding the slope of a curve at a particular point is fundamental in calculus, as it provides insights into the behavior of functions, their increasing or decreasing tendencies, and the nature of their tangents at specific points. In this context, the task involves determining the slope of the curve defined by the function Y = x - 7x at the point P(2, -6) through the process of calculating the limiting value of the derivative. This approach emphasizes the foundational concept of derivatives as limits of average rates of change and highlights the importance of limit processes in calculus.

Understanding the Problem Statement

Before diving into the solution, it’s essential to clarify what the problem entails. The goal is to find the instantaneous rate of change of the given function at a specific point. The function provided appears to be Y = x - 7x, but this simplifies further:

- Simplification of the given function:

Y = x - 7x = -6x

This simplified form indicates the function is a straight line with a slope of -6. However, the problem seems to be designed for illustrative purposes—perhaps it intends to demonstrate the process of calculating derivatives via limits, regardless of the simplicity of the function.

The point P(2, -6) lies on this line because substituting x=2 yields Y = -6(2) = -12, yet the point is given as (2, -6). This suggests a possible typo or misstatement, or perhaps the function was intended to be different (say, Y = x - 7x + some constant). For the sake of clarity and educational value, let's assume the function was meant to be Y = x - 7x + c, or perhaps simply Y = x - 7x, and the point is P(2, -12).

Alternatively, if the given point is P(2, -6), then the function might be Y = -3x, which would make P(2,

-6) valid, since:

Y = -3(2) = -6.

To proceed meaningfully, we'll assume the function is Y = -3x, matching the point P(2, -6). The problem then becomes calculating the slope of Y = -3x at P(2, -6), which is straightforward as the derivative of a linear function is constant.

But to emphasize the process of finding derivatives via limits, we'll consider a more general function, say Y = x - 7x, and demonstrate how to find the derivative at a point using the limit process, regardless of the simplicity of the function.

Fundamentals of Derivatives and Limits

Calculus defines the derivative of a function at a point as the instantaneous rate of change or the slope of the tangent to the curve at that point. Mathematically, the derivative f'(x) at a point x = a is given by:

$$ f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} $$

This limit, called the difference quotient, measures how much the function changes over an infinitesimally small interval h. As h approaches zero, the average rate of change approaches the exact slope at a.

Step-by-Step Solution Approach

1. Express the Function

Given the original function:

$$ Y = x - 7x $$

which simplifies to:

$$ Y = -6x $$

Since it's linear, the derivative is constant; however, to showcase the limit process, we'll proceed with the difference quotient method.

2. Find the Difference Quotient

The slope at $x = 2$ is given by:

$$ \text{Slope} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} $$

Calculating $f(2+h)$:

$$ f(2+h) = -6(2+h) = -12 - 6h $$

Calculating $f(2)$:

$$ f(2) = -6(2) = -12 $$

Now, substitute into the difference quotient:

$$ \frac{f(2+h) - f(2)}{h} = \frac{(-12 - 6h) - (-12)}{h} = \frac{-12 - 6h + 12}{h} = \frac{-6h}{h} $$

Simplify:

$$ \frac{-6h}{h} = -6 \quad \text{(for } h \neq 0) $$

3. Take the Limit as h Approaches Zero

Since the expression simplifies to -6 regardless of h, the limit is straightforward:

$$ \lim_{h \to 0} -6 = -6 $$

Thus, the derivative $f'(2)$, or the slope at $x=2$, is -6.

4. Interpret the Result

The slope of the curve at the point P(2, -6) is -6. This indicates the function decreases at a constant rate of 6 units vertically for every 1 unit increase horizontally, consistent with the linear nature of the function.

Key Features and Insights

- Constant Derivative for Linear Functions: For any linear function $Y = mx + c$, the derivative (slope) is simply m. In this case, $Y = -6x$, so the slope is -6 at all points.

- Limit Process Validity: The limit method used here confirms the derivative as the limit of the difference quotient, reinforcing the foundational concept of derivatives in calculus.

- Applicability to Non-Linear Functions: While this example deals with a linear function, the limit process applies equally to non-linear functions, where the derivative varies with x.

Alternative Approach: Using Power Rule

Since Y = -6x, the derivative can be directly obtained using the power rule:

$$\frac{d}{dx} (-6x) = -6$$

This confirms our limit-based calculation, emphasizing that for simple functions, direct differentiation is straightforward. However, understanding the limit process is crucial for handling more complex functions.

Features, Pros, and Cons of Limit-Based Derivative Calculation

Features:

- Fundamental to understanding the concept of instantaneous rate of change.
- Provides a rigorous foundation for differentiability.
- Applicable to all types of functions, including non-linear and complex functions.

Pros:

- Offers deep insight into the behavior of functions at specific points.
- Essential for theoretical development in calculus.
- Enables precise calculation of derivatives even for functions without simple rules.

Cons:

- Can be computationally intensive for complicated functions.
- Limited practicality for quick calculations compared to shortcut rules.
- Requires careful handling of limits, especially for functions with indeterminate forms.

Conclusion

The process of finding the slope of a curve at a given point through the limiting value of the difference quotient is a cornerstone of calculus. In the example discussed, the function Y = -6x exhibits a constant slope of -6, which is confirmed both through the limit process and direct differentiation. This approach not only solidifies understanding of derivatives as limits but also underscores their universal applicability across various functions. Whether dealing with simple lines or complex curves, mastering the limit definition of derivatives equips students and practitioners with a powerful tool to analyze and interpret the behavior of functions precisely and rigorously.

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