(a) Find The Unit Vectors That Are Parallel To The Tangent Line To The Curve Y = 8 Sin(x) At The Point

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Understanding how to find the unit vectors parallel to the tangent line of a curve is a fundamental skill in vector calculus and differential geometry. This process involves analyzing the given curve, computing its tangent vector at a specific point, and then normalizing that vector to obtain a unit vector. In this article, we will explore this method step-by-step, focusing on the curve $y = 8 \sin(x)$, and demonstrate how to find the unit vectors that are parallel to the tangent line at any given point on the curve.

Understanding the Problem

Before diving into calculations, it's important to understand what the problem entails:

- Curve Definition: The curve is defined by the function $y = 8 \sin(x)$.

- Point of Interest: The point at which we analyze the tangent line can be any point (x_0, y_0) on the curve, where $y_0 = 8 \sin(x_0)$.

- Objective: To find the unit vectors that are parallel to the tangent line at this point. These vectors will have the same direction as the tangent vector but will be of unit length.

- Significance: Such vectors are essential in physics (direction of motion), engineering (direction of forces), and mathematics (understanding the geometry of curves).

Mathematical Foundations

To solve this problem, we need to understand several key concepts:

The Derivative and Tangent Vectors

- The derivative of the function $y = 8 \sin(x)$, denoted as $y' = \frac{dy}{dx}$, gives the slope of the tangent line at any point x.

- The tangent vector at a point on the curve is a vector that points in the direction of the tangent line. For a curve defined parametrically as $\mathbf{r}(x) = (x, y)$, the tangent vector $\mathbf{T}$ can be expressed as:

$$
\mathbf{T}(x) = \left(1, \frac{dy}{dx} \right)
$$

- Note that this vector points in the direction of increasing x.

Unit Vectors

- A unit vector is a vector with a magnitude (length) of exactly 1.

- To find the unit vector in the direction of a given vector $\mathbf{v}$, we normalize it:

$$
\hat{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|}
$$

where $\|\mathbf{v}\|$ is the magnitude of $\mathbf{v}$.

Step-by-Step Solution

Let's now proceed to find the unit vectors parallel to the tangent line at a general point $x = x_0$.

Step 1: Find the derivative y'

Given:

$$
y = 8 \sin(x)
$$

Differentiating with respect to \(x \):

\[
\frac{dy}{dx} = 8 \cos(x)
\]

This derivative gives the slope of the tangent line at any point \(x \).

Step 2: Write the tangent vector at \(x = x_0 \)

The tangent vector \(\mathbf{T} \) at \(x = x_0 \) is:

\[
\mathbf{T} = \left(1, 8 \cos(x_0) \right)
\]

This vector points in the direction of the tangent line at \((x_0, y_0) \).

Step 3: Find the magnitude of the tangent vector

The magnitude \(\|\mathbf{T}\| \) is:

\[
\|\mathbf{T}\| = \sqrt{1^2 + \left(8 \cos(x_0)\right)^2} = \sqrt{1 + 64 \cos^2(x_0)}
\]

Step 4: Normalize the tangent vector to obtain the unit vector

The unit vector \(\hat{\mathbf{T}} \), parallel to the tangent line, is:

\[
\hat{\mathbf{T}} = \frac{1}{\|\mathbf{T}\|} \left(1, 8 \cos(x_0) \right)
\]

which simplifies to:

\[
\boxed{

\hat{\mathbf{T}} = \left(\frac{1}{\sqrt{1 + 64 \cos^2(x_0)}}, \frac{8 \cos(x_0)}{\sqrt{1 + 64 \cos^2(x_0)}} \right)
}
\]

This is the unit vector pointing in the same direction as the tangent line at the point x_0.

Special Cases and Additional Insights

Understanding some special cases and properties of the unit tangent vectors can provide deeper insights into the behavior of the curve.

Case 1: When $\cos(x_0) = 0$

- This occurs at $x_0 = \frac{\pi}{2} + k\pi$, where k is an integer.

- At these points, the derivative $y' = 8 \cos(x_0) = 0$, so the tangent vector simplifies to:

$$
\mathbf{T} = (1, 0)
$$

- The magnitude is:

$$
\|\mathbf{T}\| = \sqrt{1^2 + 0} = 1
$$

- The unit tangent vector is simply:

$$
\hat{\mathbf{T}} = (1, 0)
$$

which points horizontally.

Case 2: When $\cos(x_0) = \pm 1$

- This occurs at $x_0 = 0, \pi, 2\pi, \dots$.

- The tangent vector becomes:

$$
\mathbf{T} = \left(1, \pm 8 \right)
$$

- The magnitude:

$$
\|\mathbf{T}\| = \sqrt{1 + 64} = \sqrt{65}
$$

- The unit tangent vector:

$$
\hat{\mathbf{T}} = \left(\frac{1}{\sqrt{65}}, \frac{\pm 8}{\sqrt{65}} \right)
$$

Visualizing the Unit Vectors

Visual representation helps in understanding the directionality of these vectors:

- The tangent vector points along the direction in which the curve progresses at a specific point.

- Normalizing it to a unit vector allows us to focus solely on the direction, independent of the speed or steepness at that point.

- The components of the unit vector tell us how much of the direction is along the x-axis and y-axis.

Imagine plotting the curve $y = 8 \sin(x)$, then at any point x_0, drawing the tangent line and marking the direction of the unit vector. Such visualization aids in grasping the geometric intuition behind the calculations.

Applications of Unit Vectors Parallel to Tangent Lines

Understanding and computing these vectors have multiple applications across different fields:

- Physics: Direction of motion or force along a curved path.

- Engineering: Designing paths or trajectories in robotics or vehicle navigation.

- Mathematics: Analyzing the local behavior of curves, curvature, and related properties.

- Computer Graphics: Rendering smooth animations along curves.

Summary and Key Takeaways

- The derivative $y' = 8 \cos(x)$ gives the slope of the tangent line at any point x on $y = 8 \sin(x)$.

- The tangent vector at $x = x_0$ is $(1, 8 \cos(x_0))$.

- The magnitude of this vector is $\sqrt{1 + 64 \cos^2(x_0)}$.

- Dividing the tangent vector by its magnitude yields the unit vector parallel to the tangent line:

$$
\boxed{
\hat{\mathbf{T}} = \left(\frac{1}{\sqrt{1 + 64 \cos^2(x_0)}}, \frac{8 \cos(x_0)}{\sqrt{1 + 64 \cos^2(x_0)}} \right)
}
$$

- Special cases at $\cos(x_0) = 0$ and $\cos(x_0) = \pm 1$ simplify the vectors, providing specific directions.

Conclusion

Finding the unit vectors parallel to the tangent line of a curve is a crucial step in understanding the local geometry and behavior of the curve. By differentiating the function, constructing the tangent vector, computing its magnitude, and then normalizing, we obtain the unit tangent vectors. These vectors serve as fundamental tools in various scientific and engineering disciplines, aiding in analysis, design, and visualization of complex systems.

Whether analyzing simple curves like $y = 8 \sin(x)$ or more complex parametric forms, the principles

Frequently Asked Questions

How do I find the tangent line to the curve $y = 8 \sin(x)$ at a specific point?

First, compute the derivative $y' = 8 \cos(x)$. Then, evaluate y' at the point's x-coordinate to find the slope of the tangent line. Use the point-slope form to write the tangent line equation.

What is the process to find a unit vector parallel to the tangent line of $y = 8 \sin(x)$?

Calculate the derivative y' at the point to get the slope, then find a vector in the direction of the tangent line, such as $(1, y')$, and normalize it by dividing by its magnitude to get the unit vector.

How do I determine the unit vector parallel to the tangent line at a given point on $y = 8 \sin(x)$?

Evaluate y' at the point, then form a vector $(1, y')$ and divide it by its length $\sqrt{1 + y'^2}$ to obtain the unit vector.

Can you provide an example of finding the unit tangent vector at a specific point on $y = 8 \sin(x)$?

Yes. For example, at $x = \pi/4$, $y' = 8 \cos(\pi/4) = 8 (\sqrt{2}/2) = 4\sqrt{2}$. The tangent vector is $(1, 4\sqrt{2})$. Its magnitude is $\sqrt{1 + (4\sqrt{2})^2} = \sqrt{1 + 32} = \sqrt{33}$. The unit tangent vector is $(1/\sqrt{33}, 4\sqrt{2}/\sqrt{33})$.

Why is it important to find the unit vector parallel to the tangent line of a curve?

The unit vector indicates the direction of the tangent line without magnitude, useful for understanding the curve's orientation and for applications like motion along the curve or vector calculus operations.

What is the general formula for the unit tangent vector to y = f(x) at a point?

The unit tangent vector $T = (1/\sqrt{1 + (dy/dx)^2}, (dy/dx)/\sqrt{1 + (dy/dx)^2})$, where dy/dx is the derivative of the function at that point.

Additional Resources

Find The Unit Vectors That Are Parallel To The Tangent Line To The Curve Y = 8 Sin(x) At The Point

Understanding how to find unit vectors parallel to the tangent line of a given curve is a fundamental concept in calculus and vector analysis. This problem involves calculating the tangent vector at a specific point on the curve $y = 8 \sin x$, then normalizing it to obtain a unit vector pointing in the same direction. Such exercises are crucial for grasping the geometric interpretation of derivatives and the behavior of functions in the context of vector calculus. This article provides a detailed, step-by-step exploration of this problem, discussing the underlying principles, methods, and potential applications.

Understanding the Problem

At its core, the problem asks us to:

1. Find the tangent line to the curve $y = 8 \sin x$ at a particular point $x = x_0$.
2. Determine the vector direction of this tangent line, often called the tangent vector.
3. Normalize this tangent vector to obtain a unit vector.
4. Confirm the direction of the unit vector aligns with the tangent line at the specified point.

The key components involve derivatives, vector normalization, and understanding the geometric significance of tangent lines.

Breaking Down the Process

1. Find the point of tangency

To proceed, we need a specific point $x = x_0$ on the curve. Since the problem statement does not specify, we can analyze the process generally and then apply it to an example point.

Given $y = 8 \sin x$, the point on the curve at $x = x_0$ is:

$$
P(x_0, y_0) = (x_0, 8 \sin x_0)
$$

---

2. Compute the derivative y' to find the slope of the tangent

The derivative of $y = 8 \sin x$ with respect to x is:

$$
\frac{dy}{dx} = 8 \cos x
$$

This derivative provides the slope of the tangent line at the point x_0:

$$
m = 8 \cos x_0
$$

This slope describes how y changes with respect to x at x_0.

---

3. Find the tangent vector

In parametric form, the tangent vector $\vec{T}$ at x_0 can be represented as:

$$
\vec{T} = \left< \Delta x, \Delta y \right>
$$

Choosing $\Delta x = 1$ (a unit change in x), then:

\[
\Delta y = \frac{dy}{dx} \times \Delta x = 8 \cos x_0 \times 1 = 8 \cos x_0
\]

Thus, the tangent vector at \(x_0 \) is:

\[
\vec{T} = \left< 1, 8 \cos x_0 \right>
\]

This vector points in the direction of the tangent line, but it is not necessarily a unit vector.

---

4. Normalize the tangent vector to find the unit vector

To normalize \(\vec{T} \), compute its magnitude:

\[
|\vec{T}| = \sqrt{(1)^2 + (8 \cos x_0)^2} = \sqrt{1 + 64 \cos^2 x_0}
\]

Then, the unit vector \(\vec{u} \) parallel to the tangent line is:

\[
\vec{u} = \frac{\vec{T}}{|\vec{T}|} = \left< \frac{1}{\sqrt{1 + 64 \cos^2 x_0}}, \frac{8 \cos x_0}{\sqrt{1 + 64 \cos^2 x_0}} \right>
\]

This unit vector points in the same direction as the tangent line at \(x_0 \).

Applying the Method to a Specific Point

Suppose we want to find the unit tangent vector at \(x = \frac{\pi}{4} \). Let's go through the steps:

Step 1: Compute \(y \) at \(x = \frac{\pi}{4} \):

\[

y = 8 \sin \frac{\pi}{4} = 8 \times \frac{\sqrt{2}}{2} = 8 \times \frac{\sqrt{2}}{2} = 4 \sqrt{2}
\]

Step 2: Calculate the derivative:

\[
\frac{dy}{dx} = 8 \cos \frac{\pi}{4} = 8 \times \frac{\sqrt{2}}{2} = 4 \sqrt{2}
\]

Step 3: Determine the tangent vector:

\[
\vec{T} = \left< 1, 4 \sqrt{2} \right>
\]

Step 4: Calculate the magnitude:

\[
|\vec{T}| = \sqrt{1^2 + (4 \sqrt{2})^2} = \sqrt{1 + 16 \times 2} = \sqrt{1 + 32} = \sqrt{33}
\]

Step 5: Find the unit vector:

\[
\vec{u} = \left< \frac{1}{\sqrt{33}}, \frac{4 \sqrt{2}}{\sqrt{33}} \right>
\]

This vector is the unit vector parallel to the tangent line at the point \(\left(\frac{\pi}{4}, 4 \sqrt{2}\right) \).

Features and Significance of the Result

Features:

- Directionality: The unit vector indicates the exact direction of the tangent line at the point, irrespective of the magnitude.
- Scalability: Any tangent vector can be normalized to produce a unit vector, facilitating comparison of directions.
- Geometric Interpretation: The process links the derivative (slope) to directional vectors, bridging calculus

and vector geometry.

Significance:

- In Physics: Unit tangent vectors describe direction of motion or force along a curve.
- In Engineering: Used in path planning and motion control where directionality matters.
- In Computer Graphics: For rendering curves and motion along paths, unit vectors serve as directional guides.

Advantages and Limitations

Advantages:

- The method provides a straightforward way to find the direction of a tangent at any point on a curve.
- It generalizes to any parametric curve, not just $y = 8 \sin x$.
- Normalization ensures the vectors are standardized, simplifying further calculations.

Limitations:

- Requires the derivative to exist and be continuous at the point.
- Assumes the tangent line is well-defined; at points where $\frac{dy}{dx}$ is undefined or infinite, the method needs adjustment.
- Only provides the direction, not the magnitude; in applications needing magnitude, additional scaling might be necessary.

Extensions and Related Topics

- Tangent and Normal Vectors: Beyond tangent vectors, finding the normal vector involves perpendicular directions.
- Parametric Representations: For more complex curves, parametric equations $x(t), y(t)$ facilitate similar analysis.
- Vector Calculus Applications: Understanding unit tangent vectors is foundational for line integrals, curvature, and motion analysis.
- Curvature and Torsion: Further geometric properties of curves depend on tangent vectors and their derivatives.

Conclusion

Finding the unit vectors parallel to the tangent line to a curve like \(y = 8 \sin x \) involves a clear application of derivatives, vector normalization, and an understanding of geometric principles. The process begins with computing the derivative to find the slope, then constructing a tangent vector, and finally normalizing it to obtain a unit vector. This approach not only provides insights into the local behavior of the function but also serves as a fundamental technique in various scientific and engineering disciplines. Mastery of these steps enhances one's ability to analyze curves, understand motion, and work effectively with vector quantities in multiple contexts.

In summary, the procedure for finding unit tangent vectors combines calculus and vector algebra to produce a powerful tool for analyzing the geometry of curves. Whether applied to simple functions or complex parametric paths, this method offers a systematic way to interpret and utilize the directionality inherent in mathematical models.

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