-Determining Vector And Parametric Equations Of The Plane That Contains The Points A(1,2,-1), B(2,1,1)

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Understanding how to find the vector and parametric equations of a plane that passes through given points is a fundamental concept in vector calculus and analytic geometry. This process involves translating geometric information into algebraic representations, enabling precise calculations and further analysis. In this article, we will explore step-by-step methods to determine the equations of a plane containing points A(1, 2, -1) and B(2, 1, 1), along with detailed explanations, illustrative examples, and tips for mastering this important topic.

Introduction to Planes in 3D Space

A plane in three-dimensional space is a flat, two-dimensional surface extending infinitely in all directions. It can be uniquely defined using various methods, including:

- A point on the plane and a normal vector
- Three non-collinear points lying on the plane
- A point and two direction vectors lying on the plane

For the problem at hand, since we are given two points, we need to determine the plane's equation by identifying a point on the plane and the normal vector to the plane.

Understanding the Basic Concepts

Points and Vectors

- Points: Locations in space, represented as coordinates (x, y, z). For example, point A has coordinates (1, 2, -1).
- Vectors: Quantities with both magnitude and direction, represented in 3D as . The vector from point P to point Q is obtained by subtracting their coordinates: PQ = Q - P.

Plane Equation Fundamentals

The standard form of a plane's equation is:

$$ \mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0 $$

where:

- n is the normal vector to the plane.
- r is the position vector of a generic point on the plane (x, y, z).
- r_0 is the position vector of a known point on the plane (e.g., point A or B).

This can be expanded to the scalar equation:

$$ a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 $$

where n = .

Step-by-Step Methodology to Find the Plane Equation

Step 1: Identify Known Points and Vectors

Given points:

- A(1, 2, -1)
- B(2, 1, 1)

First, find the vector AB:

$$ \mathbf{AB} = \mathbf{B} - \mathbf{A} = (2 - 1, 1 - 2, 1 - (-1)) = (1, -1, 2) $$

This vector is a direction vector lying on the plane, connecting points A and B.

Step 2: Find a Third Point or Use Additional Vectors

To define a unique plane, we need a normal vector n. The normal vector is perpendicular to any two vectors lying on the plane. Since we only have one vector AB, we need another vector lying on the plane.

Approach:

- Find a third point C not on the line AB and on the plane, or
- Use known points to generate an additional vector that, together with AB, span the plane.

Suppose we choose an arbitrary point C not collinear with A and B. For simplicity, if the problem only provides points A and B, we need to assume the plane passes through these points and contains at least one more point, or we can consider an auxiliary point to define the plane.

In many cases, if only two points are given, the plane is not uniquely determined unless additional information is provided. However, in this problem, the assumption is that the plane contains points A and B, and perhaps the problem intends for us to find the plane passing through these points with an arbitrary third point on the plane.

Alternatively, if the context suggests the plane contains the points A and B and is not specified otherwise, the problem might be incomplete. But typically, in such problems, the second point is used to find the direction vector, and then the normal vector is derived from the cross product with another vector.

Since only two points are given, and the goal is to find the plane containing these points, a common approach is:

- Find a vector perpendicular (normal) to the plane by incorporating an additional known point or assumption.
- Or, if the problem is designed for the plane passing through points A and B, and the origin, then the third point can be taken as the origin or any other point not collinear with A and B.

Assumption: Let's assume the plane contains points A, B, and the origin (0, 0, 0). Then, the vectors:

- AB = (1, -1, 2)
- AO = (0 - 1, 0 - 2, 0 - (-1)) = (-1, -2, 1)

are both on the plane, and their cross product will give the normal vector.

Step 3: Calculate the Normal Vector via Cross Product

Compute:

$$ \mathbf{n} = \mathbf{AB} \times \mathbf{AO} $$

$$
\begin{aligned}
\mathbf{n} &=
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & -1 & 2 \\
-1 & -2 & 1 \\
\end{vmatrix} \\
&= \mathbf{i}((-1)(1) - 2(-2)) - \mathbf{j}(1(1) - 2(-1)) + \mathbf{k}(1(-2) - (-1)(-1)) \\
&= \mathbf{i}(-1 + 4) - \mathbf{j}(1 + 2) + \mathbf{k}(-2 - 1) \\
&= \mathbf{i}(3) - \mathbf{j}(3) + \mathbf{k}(-3) \\
&= (3, -3, -3)
\end{aligned}
$$

Simplify:

$$ \mathbf{n} = (1, -1, -1) $$

This vector is perpendicular to the plane.

Note: The choice of the third point is flexible, but the key is that the vectors used are not collinear, and their cross product provides the normal vector.

Step 4: Write the Plane Equation

Using point A(1, 2, -1) and normal vector n = (1, -1, -1):

$$
\begin{aligned}
& a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \\
& 1(x - 1) - 1(y - 2) - 1(z + 1) = 0
\end{aligned}
$$

Expanding:

$$
(x - 1) - (y - 2) - (z + 1) = 0
$$

Simplify:

$$
x - 1 - y + 2 - z - 1 = 0
$$

$$
x - y - z + 0 = 0
$$

Thus, the scalar equation of the plane is:

$x - y - z = 0$

Parametric Equations of the Plane

Once the scalar (general) equation of the plane is known, the parametric equations can be derived to describe points on the plane in terms of parameters.

Step 1: Find Two Direction Vectors

- Use vectors lying on the plane. For example:

\[
\mathbf{AB} = (1, -1, 2)
\]
and
\[
\mathbf{AC} = \text{another vector on the plane}
\]

Suppose we choose point C as (0, 0, 0), which is not on the plane as per previous assumptions, but for parametric equations, you can choose any two vectors lying on the plane.

From the previous calculation, AB is on the plane.

- Find another vector AD, for instance, from point A to some point D satisfying the plane equation. Let's pick point D(0, 1, 0):

Check if D lies on the plane:

\[
x - y - z = 0 \Rightarrow 0 - 1 - 0 = -1 \neq 0
\]

No, D does not lie on the plane. Instead, choose a point that satisfies the plane equation:

\[
x - y - z = 0
\]

For example, select x = 0, then:

\[
0 - y - z = 0 \Rightarrow y + z = 0
\]

Choose y = 0, then z = 0, so point D(0, 0, 0) is on the plane only if the equation is satisfied, which it is:

\[
0 - 0

Frequently Asked Questions

How do I find the vector equation of the plane passing through points A(1,2,-1) and B(2,1,1)?

First, find the direction vector between the points, which is B - A = (1, -1, 2). To define the plane, you need a normal vector; you can find it using the cross product of two vectors lying on the plane. For example, choosing vectors AB and AC (with another point C), or using known points, and then write the vector equation as r = r0 + s v1 + t v2, where r0 is a point on the plane, and v1, v2 are direction vectors.

What is the process to derive the parametric equations of the plane containing points A and B?

Identify two independent direction vectors on the plane, such as vectors AB and an additional vector from a third point. Then, select a point on the plane (for example, point A) as the position vector. The parametric equations are then written as: x = x0 + s dx, y = y0 + s dy, z = z0 + s dz, where (x0, y0, z0) is point A, and (dx, dy, dz) is a direction vector.

How can I determine the normal vector of the plane containing points A and B?

To find the normal vector, you need at least two vectors lying on the plane. If you have a third point C, find vectors AB and AC, then compute their cross product: n = AB × AC. This normal vector n is perpendicular to the plane and can be used to write the Cartesian or parametric equations.

Can you provide an example of the parametric equations for the plane through points A(1,2,-1) and B(2,1,1)?

Yes. First, find a direction vector, for example, AB = (1, -1, 2). Choose point A as a reference point. The parametric equations are: x = 1 + s, y = 2 - s, z = -1 + 2s, where s is a parameter. To fully describe the plane, include a second direction vector and parameter t if needed.

How do I verify that a point lies on the plane defined by points A and B?

After deriving the plane's equation, substitute the coordinates of the point into the equation. If the equation holds true (i.e., both sides are equal), then the point lies on the plane. Alternatively, verify that the vectors from the point to points A and B are coplanar.

Why is it important to find both the vector and parametric equations of a plane in 3D geometry?

Finding both forms provides flexibility in problem-solving. The vector equation offers a compact way to represent the plane using vectors, which is useful for calculations involving directions and normals. The parametric equations allow you to describe all points on the plane explicitly, facilitating visualization, intersection calculations, and solving geometric problems.

Additional Resources

Determining Vector and Parametric Equations of the Plane That Contains the Points A(1, 2, -1) and B(2, 1, 1)

Understanding the geometric and algebraic representations of planes in three-dimensional space is fundamental in advanced mathematics, physics, engineering, and computer graphics. The process of deriving the vector and parametric equations of a plane that contains specific points offers insight into how spatial relationships are represented analytically. This article provides a detailed exploration of how to determine these equations, focusing on the points A(1, 2, -1) and B(2, 1, 1). It aims to serve as both an instructional guide and a comprehensive review for students, educators, and professionals interested in three-dimensional analytical geometry.

Foundations of Plane Equations in 3D Space

Before delving into the specific case involving points A and B, it’s essential to comprehend the general principles that govern the equation of a plane in three-dimensional space.

General Equation of a Plane

A plane in three-dimensional Euclidean space can be expressed in various forms, but the most common are:

- Scalar (Cartesian) Equation:

$$
Ax + By + Cz + D = 0
$$

where A, B, C are the coefficients of the normal vector to the plane, and D is a scalar.

- Vector Equation:

$$
\vec{r} = \vec{r}_0 + s \vec{u} + t \vec{v}
$$

where $\vec{r}_0$ is a position vector to a known point on the plane, and $\vec{u}$ and $\vec{v}$ are two non-parallel vectors lying on the plane.

- Parametric Equations:

Explicitly expressing the coordinates as functions of parameters s, t:

$$
\begin{cases}
x = x_0 + s u_x + t v_x \\
y = y_0 + s u_y + t v_y \\
z = z_0 + s u_z + t v_z
\end{cases}
$$

Understanding the relationships among these forms allows us to derive the necessary equations systematically.

Step 1: Analyzing the Given Points

The points provided are:

- $A(1, 2, -1)$
- $B(2, 1, 1)$

These serve as foundational references from which the plane’s properties can be deduced.

Identifying the Direction Vector

A key step in defining the plane is to find vectors lying within it. Given two points on the plane, the vector connecting them is:

$$
\vec{AB} = \vec{B} - \vec{A} = (2 - 1, 1 - 2, 1 - (-1)) = (1, -1, 2)
$$

This vector points from A to B, lying entirely within the plane.

Step 2: Establishing a Third Point or Vector

To fully determine the plane, we need at least two non-parallel vectors lying on it. One vector is established by points A and B. To define the plane explicitly, we require either:

- A third point C not on the line AB, or
- A second vector $\vec{AC}$ that is not parallel to $\vec{AB}$.

In this case, since only two points are provided, the general approach is to consider a point A (or B) as a reference and find the normal vector to the plane.

Step 3: Deriving the Normal Vector to the Plane

The normal vector $\vec{n}$ to the plane is perpendicular to any vectors lying within the plane. Since $\vec{AB}$ is within the plane, we need at least one more vector that lies within the plane and is not parallel to $\vec{AB}$.

However, with only two points, the plane is not uniquely determined without additional information.

But, in many problems, the plane is assumed to be the one passing through points A and B and an arbitrary point C not on AB.

Common assumption: The plane contains points A and B and is determined by these points along with an additional point or a known direction.

In this context, if no third point is specified, the typical approach is to find the plane that passes through the line AB and is perpendicular to a known vector or plane condition.

But, considering the common scenario, the most straightforward method is to specify that the plane contains A, B, and any other point C that is not collinear with A and B.

Since no such third point is given, the problem likely aims to find the vector and parametric equations of the plane passing through points A and B and an arbitrary point C, or to find the set of all points satisfying an equation derived from these points.

Step 4: Constructing the Equation of the Plane

Given only points A and B, the key is to understand that the plane is not uniquely determined unless a third point or additional information is specified.

However, if the problem's context is to find the plane that contains these points and any other point C such that the position vectors satisfy certain conditions, then the general approach involves:

- Choosing a third point $C(x, y, z)$ that satisfies the plane equation.
- Using the vector $\vec{AB}$ and a second vector $\vec{AC}$ (or $\vec{BC}$) to find the normal vector.

Step 5: Constructing the Vector Equation of the Plane

Suppose we select a third point $C(x, y, z)$ on the plane, then the position vector of C is $\vec{r} = (x, y, z)$.

The vectors lying in the plane are:

- $\vec{AB} = (1, -1, 2)$

- $\vec{AC} = (x - 1, y - 2, z + 1)$

The normal vector $\vec{n}$ can be obtained via the cross product:

$$
\vec{n} = \vec{AB} \times \vec{AC}
$$

which, when computed, gives a vector perpendicular to both, hence normal to the plane.

Calculating the Normal Vector

$$
\vec{n} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & -1 & 2 \\
x - 1 & y - 2 & z + 1
\end{vmatrix}
$$

Expanding the determinant:

$$
\vec{n} = \mathbf{i}((-1)(z + 1) - 2(y - 2)) - \mathbf{j}(1(z + 1) - 2(x - 1)) + \mathbf{k}(1(y - 2) - (-1)(x - 1))
$$

Simplify each component:

- i-component:

$$
(-1)(z + 1) - 2(y - 2) = -z - 1 - 2y + 4 = -z - 2y + 3
$$

- j-component:

$$
1(z + 1) - 2(x - 1) = z + 1 - 2x + 2 = z - 2x + 3
$$

- k-component:

$$
1(y - 2) - (-1)(x - 1) = y - 2 + x - 1 = x + y - 3
$$

Thus, the normal vector:

$$
\vec{n} = (-z - 2y + 3, - (z - 2x + 3), x + y - 3)
$$

Since the normal vector depends on the coordinates of $C(x, y, z)$, for the plane's explicit equation, we need a point through which the plane passes, such as A, and the normal vector.

Step 6: Deriving the Scalar Equation of the Plane

Using the point-normal form:

$$
\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0
$$

where $\vec{r}_0$ is the position vector of point $A(1, 2, -1)$:

$$
\vec{r}_0 = (1, 2, -1)
$$

and $\vec{r} = (x, y, z)$. The

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