) Let X Be A Random Variable That Is Uniformly Distributed On The Interval (1, 1). (a) (3 Points) Find

) Let X Be A Random Variable That Is Uniformly Distributed On The Interval (1, 1). (a) (3 Points) Find

Understanding the properties of a uniformly distributed random variable is fundamental in probability theory and statistics. When we say that a random variable X is uniformly distributed on an interval, it means that every value within that interval is equally likely to occur. In this article, we will explore what it means for X to be uniformly distributed on the interval (1, 1), analyze the implications of such a distribution, and perform the necessary calculations to find the expected value and variance of X. We will also look at the general formulas for uniform distributions and how they apply to this specific case.

Introduction to Uniform Distribution

What Is a Uniform Distribution?

A uniform distribution, also known as a rectangular distribution, is a type of probability distribution where all outcomes in a specified interval are equally likely. For a continuous random variable X, this distribution is characterized by two parameters: the lower bound (a) and the upper bound (b). The probability density function (pdf) for a uniform distribution on the interval (a, b) is given by:

- f(x) = 1 / (b - a) for a < x < b
- f(x) = 0 elsewhere

This means that the density is constant across the interval, and the total area under the curve equals 1.

Properties of a Uniform Distribution

Key properties include:
- The expected value (mean) is (a + b) / 2.
- The variance is (b - a)^2 / 12.
- The distribution is symmetric about the midpoint (a + b) / 2.

Analyzing the Distribution of X on (1, 1)

The Interval (1, 1): A Special Case

In the problem statement, X is said to be uniformly distributed on the interval (1, 1). At first glance, this may seem like a typo or an unusual case, because an interval with identical bounds, such as (1, 1), contains only a single point.

However, assuming the intended interval is (1, 1)—or more generally, that the interval is a single point—this leads to important implications:

- If the interval is truly (1, 1), then it contains no length, and the random variable is degenerate, meaning it can only take a single value, which is 1.
- A degenerate distribution assigns probability 1 to that single point, and the distribution is not continuous in the usual sense.

Degenerate Distribution at a Single Point

When a random variable is degenerate at a point c, its probability distribution is concentrated at that point, with:

- Probability P(X = c) = 1
- Expected value E[X] = c
- Variance Var(X) = 0

Therefore, if the interval is (1, 1), X is degenerate at 1, and calculations simplify accordingly.

Calculations for the Degenerate Distribution

Expected Value of X

For a degenerate distribution at point c, the expected value is simply c:

E[X] = 1

Variance of X

Since the distribution is degenerate, there is no variability in the outcomes:

Var(X) = 0

General Formulas for Uniform Distribution

Expected Value

For a uniform distribution on the interval (a, b), the expected value is:

- **E[X] = (a + b) / 2**

Variance

The variance is given by:

- **Var(X) = (b - a)^2 / 12**

Applying the Formulas to the Given Interval

Case 1: Interval (1, 1)

As discussed, this is a degenerate distribution:

- E[X] = 1
- Var(X) = 0

Case 2: Possible Typo or Clarification

If the intended interval was, for example, (1, 2), then:

- E[X] = (1 + 2) / 2 = 1.5
- Var(X) = (2 - 1)^2 / 12 = 1 / 12 ≈ 0.0833

In such cases, the calculations follow the standard formulas for uniform distributions.

Conclusion

In summary, when analyzing a uniformly distributed random variable X on an interval, the key steps involve understanding the interval's bounds and applying the standard formulas to compute the expected value and variance. If X is uniformly distributed on the interval (1, 1), the distribution is degenerate, with the random variable always equal to 1. Consequently, the expected value is 1, and the variance is zero. These properties highlight the importance of carefully interpreting the distribution's interval bounds, especially when they appear to be degenerate.

Understanding these concepts is crucial for statisticians and data scientists working with probabilistic models, as they help in accurately characterizing the behavior of random variables and in making informed decisions based on statistical data.

Frequently Asked Questions

What is a uniformly distributed random variable on the interval (1, 1)?

A uniformly distributed random variable on the interval (1, 1) is a degenerate distribution where the variable always takes the value 1, since the interval has zero length.

How do you interpret a uniform distribution on a degenerate interval like (1, 1)?

Since the interval has no length, the distribution assigns probability 1 to the single point 1, making it a degenerate distribution.

What is the probability density function (pdf) for a uniform distribution on (1, 1)?

The pdf is a Dirac delta function centered at 1, meaning it is zero everywhere except at 1, where it is infinitely high, integrating to 1.

Given the interval (1, 1), what is the expected value of the random variable?

The expected value is 1, since the variable always takes the value 1.

What is the variance of a uniform distribution on (1, 1)?

The variance is 0 because there is no variability; the variable always equals 1.

How does the uniform distribution on a degenerate interval differ from a standard uniform distribution?

A standard uniform distribution has a positive interval length and a constant density over that interval, while the degenerate case (interval of length zero) collapses to a single point with probability 1.

In the context of the problem, what is the significance of finding 'a' in the uniform distribution on (1, 1)?

Since the interval is (1, 1), the value of 'a' (the bounds) is both 1, indicating a degenerate distribution where the variable always equals 1.

Is it meaningful to calculate the probability that the variable falls within an interval in (1, 1)?

Yes, but only if the interval includes the point 1; otherwise, the probability is zero. Since the distribution is degenerate at 1, $P(X=1)=1$.

What challenges arise when dealing with a uniform distribution on an interval with zero length?

The main challenge is that the distribution becomes degenerate, making standard calculations like density and variance trivial or undefined in the usual sense, requiring the use of Dirac delta functions or point probabilities.

How should the answer to the problem (3 points) be approached given the interval (1, 1)?

Since the interval (1, 1) is degenerate, the solution involves recognizing that the distribution concentrates all probability at 1, and calculations reflect that the variable is almost surely 1.

Additional Resources

Understanding the Uniform Distribution of a Random Variable on the Interval (1, 1)

Introduction to Uniform Distribution

The uniform distribution is one of the most fundamental probability distributions in statistics and probability theory. It models scenarios where every outcome within a specific interval is equally likely. This property makes it particularly useful in simulations, modeling of random processes, and as a foundational concept in understanding more complex distributions.

At its core, the uniform distribution can be described as a continuous probability distribution with constant probability density over its support. When a random variable X is uniformly distributed over an interval (a, b), it means that for any subinterval within (a, b), the probability that X falls into that subinterval is proportional to its length.

Clarifying the Distribution on the Interval (1, 1)

The initial statement presents a distribution of X that is uniformly distributed over the interval $(1, 1)$.

This immediately raises a conceptual concern because an interval of the form $(1, 1)$ is degenerate—it contains no length, as the start and end points are the same. In the context of continuous distributions, an interval with zero length implies that there is no range over which the variable can vary, effectively making the distribution a degenerate distribution.

What does this mean?

- A degenerate distribution at a point c (here, $c = 1$) assigns probability 1 to the event $X = c$.
- The probability density function (pdf) in such cases is a Dirac delta function centered at c, or in simpler terms, the distribution is concentrated at a single point.

Therefore, the statement likely contains a typographical or conceptual error. It is plausible that the intended interval is $(1, b)$, where $b > 1$, and perhaps the notation was miswritten as $(1, 1)$.

Assuming the Intended Distribution: $X \sim \text{Uniform}(1, b)$

Given the ambiguity, the most logical assumption is that the problem aims to analyze a uniform distribution on the interval $(1, b)$, where $b > 1$.

This assumption aligns with standard practice and allows for meaningful discussion and calculations.

Let’s proceed under this premise, exploring the properties of a uniform distribution over $(1, b)$.

Fundamental Properties of Uniform Distribution on (a, b)

Probability Density Function (pdf)

For $X \sim \text{Uniform}(a, b)$, the pdf is given by:

$$
f_X(x) =
\begin{cases}
\frac{1}{b - a} & \text{for } x \in (a, b) \\
0 & \text{otherwise}
\end{cases}
$$

- The pdf is constant over (a, b), reflecting the equal likelihood of outcomes in this interval.
- The height of the pdf, $\frac{1}{b - a}$, ensures that the total probability integrates to 1:

$$
\int_a^b f_X(x) \, dx = 1
$$

- Since the distribution is uniform, the probability that X falls within any subinterval $(x_1, x_2) \subseteq (a, b)$ is:

$$
P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}
$$

Cumulative Distribution Function (CDF)

The CDF for $X \sim \text{Uniform}(a, b)$ is:

$$
F_X(x) = P(X \leq x) =
\begin{cases}

0 & x < a \\
\frac{x - a}{b - a} & a \leq x \leq b \\
1 & x > b
\end{cases}
\]

The CDF provides the probability that \(X \) takes a value less than or equal to \(x \).

Key Moments of the Distribution

The moments of a distribution provide insights into its central tendency and variability.

Mean (Expected Value):

\[
\mathbb{E}[X] = \frac{a + b}{2}
\]

- The mean is simply the midpoint of the interval.

Variance:

\[
\operatorname{Var}(X) = \frac{(b - a)^2}{12}
\]

- Variance measures the dispersion or spread of the distribution.

Standard Deviation:

\[
\sigma = \sqrt{\operatorname{Var}(X)} = \frac{b - a}{2\sqrt{3}}
\]

Application to the Specific Case: \((1, 1) \)

If the initial problem truly refers to the interval \((1, 1) \), then:

- The distribution is degenerate at \(x = 1 \).
- The probability density function reduces to a Dirac delta at 1, meaning:

\[
f_X(x) = \delta(x - 1)
\]

- The probability that $X = 1$ is 1, and the probability $X \neq 1$ is 0.

In such a degenerate case:

- Expected value: $\mathbb{E}[X] = 1$
- Variance: 0, since the variable does not vary.

This distribution is trivial, as there's no randomness involved.

Theoretical Implications and Calculations

Assuming the meaningful case where $X \sim \text{Uniform}(1, b)$, with $b > 1$:

1. Calculating the Probability Density Function (pdf)

Given:

$$
f_X(x) = \frac{1}{b - 1}, \quad x \in (1, b)
$$

This constant density indicates each outcome within the interval is equally likely.

2. Deriving the Cumulative Distribution Function (CDF)

$$
F_X(x) =
\begin{cases}
0, & x \leq 1 \\
\frac{x - 1}{b - 1}, & 1 < x < b \\
1, & x \geq b
\end{cases}
$$

This function is linear between 1 and b, with slope $\frac{1}{b-1}$.

3. Computing Expected Value and Variance

- Expected value:

$$
\mathbb{E}[X] = \frac{1 + b}{2}
$$

- Variance:

$$
\operatorname{Var}(X) = \frac{(b - 1)^2}{12}
$$

Practical Examples and Applications

The uniform distribution's simplicity makes it a staple in various practical applications.

Random Sampling in Simulations:

- Uniform distributions are used to generate random numbers when simulating stochastic processes.
- For example, in Monte Carlo methods, uniform randomness over an interval is the starting point for transforming into other distributions.

Modeling Equal Likelihood Events:

- Suppose a scenario where outcomes are equally likely over an interval, such as the time between events in a Poisson process or the selection of a number within a range.

Designing Experiments:

- Randomly assigning treatments or conditions within a specific range can be modeled via a uniform distribution.

In Computer Science:

- Random number generators often produce uniformly distributed values which are then scaled to the desired interval.

Visualizing the Distribution

Graphically, the uniform distribution over (a, b) appears as a rectangle:

- The height of the rectangle is $\frac{1}{b - a}$.
- The width spans from a to b.
- The area under the curve (the rectangle) is 1, satisfying the total probability requirement.

In the degenerate case $(1, 1)$:

- The graph collapses to a single point, emphasizing the deterministic nature.

Summary and Final Remarks

Given the initial problem statement, the most coherent interpretation is that the distribution is uniform over some interval $(1, b)$, with $b > 1$. Under this assumption, the key points are:

- The probability density function is constant over $(1, b)$, with height $\frac{1}{b - 1}$.
- The cumulative distribution function increases linearly from 0 to 1 over $(1, b)$.
- The mean is at the midpoint $\frac{1 + b}{2}$.
- The variance depends on the length of the interval, $\frac{(b - 1)^2}{12}$.

If the original question intended to specify a degenerate distribution at point 1, then:

- The distribution is concentrated solely at $x = 1$.
- The expected value is 1.
- Variance is zero.

Final Thoughts

Understanding the uniform distribution, especially in the context of the interval (a, b), is fundamental to grasping broader concepts in probability theory. It serves as a building block for more complex distributions, such as the Beta distribution, which generalizes the uniform case.

In practical applications, the

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