Triangle ABC Has Perimeter 20 Cm.AB = 7 Cm.BC = 4 Cm.By Calculation, Deduce Whether Triangle ABC Is A

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Introduction

Understanding the properties of triangles is fundamental in geometry, especially when it comes to verifying whether a given set of side lengths can form a valid triangle. In this article, we explore a specific problem involving triangle ABC, where the perimeter and two side lengths are provided. The key question is: Given that triangle ABC has a perimeter of 20 cm, with side AB measuring 7 cm and side BC measuring 4 cm, can we determine whether triangle ABC is an isosceles, equilateral, or scalene triangle?

This problem not only emphasizes the importance of the triangle inequality theorem but also reinforces techniques for calculating unknown side lengths and classifying triangles based on their side lengths. Through detailed calculations and logical reasoning, we will arrive at a conclusion about the nature of triangle ABC.

Understanding the Given Data

Before delving into calculations, let's organize the provided information:

- Perimeter of triangle ABC: 20 cm
- Side AB: 7 cm
- Side BC: 4 cm

Our goal is to find the length of side AC and then analyze the triangle's properties based on side lengths.

Step 1: Finding the Length of Side AC

Since the perimeter (the sum of all three sides) is known, we can compute the length of side AC.

Perimeter formula:

\[

$$
AB + BC + AC = \text{Perimeter}
$$

Plugging in the known values:

$$
7\, \text{cm} + 4\, \text{cm} + AC = 20\, \text{cm}
$$

Simplify:

$$
11\, \text{cm} + AC = 20\, \text{cm}
$$

Solve for AC:

$$
AC = 20\, \text{cm} - 11\, \text{cm} = 9\, \text{cm}
$$

Result:
Side AC = 9 cm

Step 2: Verifying the Validity of Triangle ABC

To confirm whether a triangle with sides 7 cm, 4 cm, and 9 cm is valid, we apply the triangle inequality theorem, which states:

> For any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side.

Let's check all three inequalities:

1. AB + BC > AC

$$
7 + 4 = 11\, \text{cm} > 9\, \text{cm} \quad \checkmark
$$

2. AB + AC > BC

$$
7 + 9 = 16\, \text{cm} > 4\, \text{cm} \quad \checkmark
$$

3. BC + AC > AB

$$
4 + 9 = 13\, \text{cm} > 7\, \text{cm} \quad \checkmark
$$

All inequalities are satisfied, confirming that a triangle with these side lengths can exist.

Step 3: Classifying the Triangle Based on Side Lengths

Now that we know the side lengths are valid, we can classify triangle ABC:

- Equilateral: All three sides are equal.
- Isosceles: Exactly two sides are equal.
- Scalene: All sides are different.

Given the side lengths:

- AB = 7 cm
- BC = 4 cm
- AC = 9 cm

Since all three are different, the triangle is scalene.

Conclusion:
Triangle ABC is a scalene triangle.

Step 4: Additional Analysis — Is Triangle ABC an Isosceles or Equilateral?

To further confirm, check for any equal sides:

- AB ≠ BC (7 cm ≠ 4 cm)
- AB ≠ AC (7 cm ≠ 9 cm)
- BC ≠ AC (4 cm ≠ 9 cm)

Since no sides are equal, the triangle cannot be isosceles or equilateral.

Step 5: Visualizing the Triangle and Additional Properties

While side length calculations help classify the triangle, visualizing its shape can offer additional insights:

- The longest side: AC (9 cm)
- The shortest side: BC (4 cm)
- The middle side: AB (7 cm)

Given the side lengths, triangle ABC appears as a scalene triangle with no special symmetry.

Angles:

- The side opposite the largest angle is the longest side (AC).
- The side opposite the smallest angle is the shortest side (BC).

Using the Law of Cosines, we can find the angles if needed, but for classification, knowing the side lengths suffices.

Step 6: Applying the Law of Cosines (Optional for Further Analysis)

To determine the measure of angles, especially to understand the triangle's shape better, we can apply the Law of Cosines:

\[
\cos C = \frac{a^2 + b^2 - c^2}{2ab}

\]

Where:

- \(a \) and \(b \) are sides adjacent to angle \(C \),
- \(c \) is the side opposite angle \(C \).

Let's find angle \(C \), opposite side \(AC \):

\[
a = AB = 7\, \text{cm}
\]
\[
b = BC = 4\, \text{cm}
\]
\[
c = AC = 9\, \text{cm}
\]

Applying the Law of Cosines:

\[
\cos C = \frac{7^2 + 4^2 - 9^2}{2 \times 7 \times 4} = \frac{49 + 16 - 81}{56} = \frac{-16}{56} = -\frac{2}{7}
\]

Calculate \(C \):

\[
C = \arccos \left(-\frac{2}{7}\right) \approx \arccos(-0.2857) \approx 106.6^\circ
\]

Similarly, we can find other angles, but for the purpose of classification, this confirms that triangle ABC has one angle greater than 90°, indicating an obtuse triangle.

Summary and Final Conclusion

Based on the detailed calculations and geometric principles, we conclude:

- The side lengths of triangle ABC are 7 cm, 4 cm, and 9 cm.
- The sum of any two sides exceeds the third, confirming the validity of the triangle.
- All sides are different, indicating the triangle is scalene.
- The calculated angles further suggest that the triangle is obtuse (with an angle approximately 106.6°).

Therefore, the answer to the question: "By calculation, deduce whether triangle ABC is a" is:

Triangle ABC is a scalene, obtuse triangle with sides measuring 7 cm, 4 cm, and 9 cm.

This comprehensive analysis underscores the importance of applying the triangle inequality theorem, the Law of Cosines, and classification rules to understand the properties of triangles

based on given data. Whether for academic purposes or practical applications, mastering these techniques enhances problem-solving skills in geometry.

Additional Tips for Geometry Students

- Always verify the validity of a triangle with the triangle inequality theorem before proceeding with further calculations.
- Use the Law of Cosines when you know two sides and the included angle or want to find angles based on side lengths.
- Classify triangles based on sides (scalene, isosceles, equilateral) and angles (acute, right, obtuse) for better understanding of their properties.
- Visualize side lengths and angles to develop spatial awareness and better grasp geometric concepts.

If you have more questions about triangle properties or need help with similar problems, feel free to ask!

Frequently Asked Questions

Given triangle ABC has a perimeter of 20 cm, with AB = 7 cm and BC = 4 cm, how can we determine the length of AC?

Subtract the sum of AB and BC from the perimeter: AC = 20 cm - (7 cm + 4 cm) = 9 cm.

What is the length of side AC in triangle ABC?

The length of AC is 9 cm.

How do we verify if triangle ABC is valid using the triangle inequality theorem?

Check if the sum of any two sides is greater than the third: 7 + 4 > 9 (11 > 9), 7 + 9 > 4 (16 > 4), and 4 + 9 > 7 (13 > 7). All are true, so the triangle is valid.

Based on the side lengths, can triangle ABC be classified as an equilateral, isosceles, or scalene triangle?

Since all sides are of different lengths (7 cm, 4 cm, and 9 cm), triangle ABC is scalene.

Is triangle ABC an acute, right, or obtuse triangle?

Calculate the squares of the sides: 7^2 = 49, 4^2 = 16, 9^2 = 81. The largest side squared (81) is greater than the sum of the squares of

the other two (49 + 16 = 65), so the triangle is obtuse.

What calculation confirms whether triangle ABC is a right triangle?

Check if Pythagoras' theorem holds: 7^2 + 4^2 = 49 + 16 = 65, which does not equal 81, so it is not a right triangle.

Is triangle ABC an obtuse triangle based on the side lengths?

Yes, because the square of the longest side (9 cm) is greater than the sum of the squares of the other two sides, indicating an obtuse triangle.

Why is the triangle ABC considered a scalene triangle?

Because all three sides have different lengths: 7 cm, 4 cm, and 9 cm.

Can triangle ABC be classified as a valid triangle based on the given perimeter and side lengths?

Yes, since the side lengths satisfy the triangle inequality theorem, triangle ABC is a valid triangle.

What is the overall conclusion about triangle ABC based on the calculations?

Triangle ABC is a valid, scalene, obtuse triangle with sides 7 cm, 4 cm, and 9 cm.

Additional Resources

Triangle ABC, with its specified perimeter and given side lengths, presents an intriguing geometric scenario that invites analysis to determine its validity as a triangle. This investigation hinges on understanding fundamental properties of triangles, applying the triangle inequality theorem, and performing precise calculations to assess the plausibility of the given dimensions. Through meticulous examination, we aim to clarify whether Triangle ABC, with a perimeter of 20 cm, and side AB = 7 cm, BC = 4 cm, can indeed exist as a legitimate triangle, or if the data suggests otherwise.

Introduction to Triangle Properties and the Significance of Perimeter

Understanding the Basics of Triangle Geometry

Triangles are among the most fundamental shapes in Euclidean geometry, characterized by three

sides and three angles. The properties of a triangle are governed by several theorems and inequalities that determine the feasibility of a given set of side lengths. The perimeter, which is simply the sum of the lengths of all three sides, provides an initial constraint on the possible dimensions of the triangle.

The perimeter of Triangle ABC is given as 20 cm, meaning that the sum of its three sides—AB, BC, and AC—must equal exactly 20 cm:

AB + BC + AC = 20 cm

Given:
- AB = 7 cm
- BC = 4 cm

Therefore, the length of side AC must satisfy:

AC = 20 cm - (AB + BC) = 20 cm - (7 cm + 4 cm) = 20 cm - 11 cm = 9 cm

This calculation reveals that, based on the perimeter, side AC is 9 cm.

Applying the Triangle Inequality Theorem

Fundamentals of the Triangle Inequality

The triangle inequality theorem states that for any triangle, the sum of the lengths of any two

sides must be greater than the length of the remaining side. This theorem is essential for verifying the feasibility of a triangle with given side lengths.

Mathematically, for sides a, b, and c:
- a + b > c
- a + c > b
- b + c > a

Applying this to Triangle ABC with sides AB = 7 cm, BC = 4 cm, and AC = 9 cm, we check:

1. AB + BC = 7 + 4 = 11 > AC = 9 (satisfied)
2. AB + AC = 7 + 9 = 16 > BC = 4 (satisfied)
3. BC + AC = 4 + 9 = 13 > AB = 7 (satisfied)

Since all three inequalities hold true, the set of side lengths (7 cm, 4 cm, and 9 cm) can form a valid triangle.

Analyzing the Triangle's Validity and Its Geometric Properties

Existence of Triangle ABC with the Given Data

The calculations confirm that the side lengths derived from the perimeter and the given sides are consistent with the triangle inequality theorem. This suggests that Triangle ABC, with sides 7 cm, 4 cm, and 9 cm, can indeed exist geometrically.

Furthermore, the sides satisfy the basic conditions for a triangle, indicating no contradictions in the data. It is important to verify if the side lengths adhere to other properties, such as the triangle's shape and potential classification.

Type of Triangle Based on Side Lengths

Side length analysis:
- AB = 7 cm
- BC = 4 cm
- AC = 9 cm

The longest side is AC = 9 cm. To classify the triangle:

- Scalene: All sides are different (7 cm, 4 cm, 9 cm) — Yes, it is scalene.
- Is it acute, right, or obtuse? We can determine this by comparing the square of the longest side with the sum of the squares of the other two sides.

Calculate:
- $AC^2 = 9^2 = 81$
- $AB^2 + BC^2 = 7^2 + 4^2 = 49 + 16 = 65$

Since $AC^2 = 81 > 65$, the triangle is obtuse at angle C (opposite side AC), because the square of the longest side exceeds the sum of the squares of the other two sides.

Confirming the Consistency of the Data

Cross-Verification of Calculations

The initial calculation derived side AC as 9 cm based on the perimeter. The triangle inequality confirms that these side lengths are valid for a triangle, with all inequalities holding true.

This consistency indicates the data is internally coherent, and Triangle ABC can indeed be constructed with the specified dimensions.

Implications of the Triangle's Properties

Given that the triangle is scalene and obtuse, its angles are all distinct, with one angle exceeding 90 degrees. Specifically, the angle opposite the side of length 9 cm (side AC) is obtuse, and the other angles are acute.

This information is significant in understanding the triangle's shape and potential applications, such as in construction or geometric proofs.

Conclusion: Is Triangle ABC A?

The comprehensive analysis demonstrates that

with a perimeter of 20 cm, and the specified sides AB = 7 cm and BC = 4 cm, the remaining side AC must be 9 cm. The calculations satisfy the triangle inequality theorem, confirming the possibility of constructing such a triangle.

Furthermore, the classification as a scalene, obtuse triangle adds depth to our understanding of its geometric properties. All evidence suggests that Triangle ABC, with the given data, can indeed exist as a valid triangle.

In summary, based on the calculations and geometric principles applied, Triangle ABC meets all necessary conditions to be a real, constructible triangle. Therefore, the conclusion is affirmative: Triangle ABC, with the specified dimensions and perimeter, is a valid triangle.

Final thoughts and implications:
This analysis underscores the importance of the triangle inequality theorem as a fundamental tool in geometric problem-solving. It also highlights how perimeter and side length data can be cross-verified to ensure consistency and feasibility. Such exercises are not only theoretical but have practical applications in fields ranging from engineering to architecture, where precise measurements and validation are crucial.

By understanding and applying these principles,

learners and professionals alike can develop sharper insights into the properties and constraints governing geometric figures, ensuring accuracy and integrity in their work.

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