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Triangle ABC Has Vertices At A(3, 3), B(0, 7), And C(3, 0). Determine The Coordinates Of The Vertices

Understanding the precise locations of the vertices of a triangle is fundamental in coordinate geometry, a branch of mathematics that deals with points, lines, and shapes in the coordinate plane. In this article, we will explore how to determine the coordinates of the vertices of triangle ABC, given some of its points. Specifically, we will analyze the points A(3, 3), B(0, 7), and C(3, 0). Our goal is to understand the geometric relationships between these points, verify their positions, and explore how their coordinates define the shape and properties of the triangle.

This comprehensive guide will help students, educators, and enthusiasts enhance their understanding of coordinate geometry, with a focus on practical problem-solving strategies, graphing techniques, and key concepts such as distance, midpoint, and slope calculations. Whether you're preparing for exams, teaching a class, or simply interested in the mathematical beauty of triangles, this article provides detailed insights into how to work with coordinate points and determine the vertices of a triangle.

Understanding the Coordinates of Triangle ABC

What Are Coordinates in Geometry?

Coordinates are numerical values that specify the position of a point in a two-dimensional space. The most common coordinate system is the Cartesian plane, where each point is represented as an ordered pair (x, y). The x-coordinate indicates the horizontal position, while the y-coordinate indicates the vertical position.

For example, the point A(3, 3) is located 3 units along the x-axis and 3 units along the y-axis from the origin (0, 0). Similarly, B(0, 7) is directly above the origin on the y-axis, and C(3, 0) lies on the x-axis, 3 units to the right of the origin.

The Given Points of Triangle ABC

In this problem, the vertices of triangle ABC are given as:
- Point A at (3, 3)
- Point B at (0, 7)
- Point C at (3, 0)

These points form the vertices of the triangle on the coordinate plane. Our task is to analyze these points to understand the shape and properties of the triangle they form, as well as to verify their positions and relationships.

Plotting and Visualizing the Triangle

Graphing Points A, B, and C

To better understand the triangle's shape, start by plotting the points on the Cartesian plane:

1. Point A (3, 3): Located 3 units right of the origin and 3 units up.
2. Point B (0, 7): Located on the y-axis, 7 units above the origin.
3. Point C (3, 0): Located 3 units right of the origin on the x-axis.

When these points are plotted, the triangle's vertices are visually clear, and the shape becomes apparent.

Sketching the Triangle

Connecting the points A, B, and C with straight lines reveals the triangle's outline:

- Line AB connects points A(3, 3) and B(0, 7).
- Line BC connects points B(0, 7) and C(3, 0).
- Line CA connects points C(3, 0) and A(3, 3).

The shape formed is a triangle with specific properties based on the positions of its vertices.

Calculating the Coordinates and Properties of Triangle ABC

Verifying the Coordinates

The first step is to confirm the given points are correctly positioned:

- Point A: (3, 3) – located 3 units right, 3 units up.
- Point B: (0, 7) – located on the y-axis, 7 units above the origin.
- Point C: (3, 0) – located 3 units right, on the x-axis.

These positions are consistent with the provided data, confirming the vertices are accurately represented.

Calculating Lengths of Sides

Using the distance formula, we can find the lengths of the sides of triangle ABC:

$$
\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Applying this formula:

1. AB:

$$
AB = \sqrt{(0 - 3)^2 + (7 - 3)^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

2. BC:

$$
BC = \sqrt{(3 - 0)^2 + (0 - 7)^2} = \sqrt{3^2 + (-7)^2} = \sqrt{9 + 49} = \sqrt{58} \approx 7.62
$$

3. CA:

$$
CA = \sqrt{(3 - 3)^2 + (0 - 3)^2} = \sqrt{0 + (-3)^2} = \sqrt{9} = 3
$$

The side lengths are approximately:

- AB = 5 units
- BC ≈ 7.62 units
- CA = 3 units

This information provides insight into the triangle's proportions.

Calculating Midpoints

Midpoints help identify the center points of each side:

- Midpoint of AB:

$$
M_{AB} = \left(\frac{3 + 0}{2}, \frac{3 + 7}{2}\right) = (1.5, 5)
$$

- Midpoint of BC:

$$
M_{BC} = \left(\frac{0 + 3}{2}, \frac{7 + 0}{2}\right) = (1.5, 3.5)
$$

- Midpoint of CA:

$$
M_{CA} = \left(\frac{3 + 3}{2}, \frac{0 + 3}{2}\right) = (3, 1.5)
$$

Midpoints are useful in various geometric constructions and proofs.

Slope Calculations for Side Analysis

The slope indicates the steepness of each side:

- Slope of AB:

$$
m_{AB} = \frac{7 - 3}{0 - 3} = \frac{4}{-3} = -\frac{4}{3}
$$

- Slope of BC:

$$
m_{BC} = \frac{0 - 7}{3 - 0} = \frac{-7}{3}
$$

- Slope of CA:

$$
m_{CA} = \frac{0 - 3}{3 - 3} = \frac{-3}{0}
$$

Since the denominator is zero, side CA is vertical, indicating a perpendicular relationship with the x-axis.

Understanding the Geometric Properties of Triangle ABC

Type of Triangle Based on Side Lengths

From the side lengths calculated:

- AB ≈ 5 units
- BC ≈ 7.62 units
- CA = 3 units

Since all three sides have different lengths, triangle ABC is scalene (a triangle with all sides of different lengths).

Determining if the Triangle is Right-Angled

To check if the triangle has a right angle, apply the Pythagorean theorem:

- If the square of the longest side equals the sum of the squares of the other two sides, the triangle is right-angled.

The longest side is BC (≈7.62). Let's verify:

$$
(AB)^2 + (CA)^2 = 5^2 + 3^2 = 25 + 9 = 34
$$

$$
(BC)^2 \approx (7.62)^2 \approx 58.1
$$

Since 34 ≠ 58.1, the triangle is not right-angled.

Area of Triangle ABC

Using the coordinate geometry formula for the area:

$$
\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|
$$

Substituting the points:

$$
\text{Area} = \frac{1}{2} |3(7 - 0) + 0(0 - 3) + 3(3 - 7)|
$$

$$
= \frac{1}{2} |3 \times 7 + 0 \times (-3) + 3 \times (-4)|
$$

$$
= \frac{1}{2} |21 + 0 - 12| = \frac{1}{2} |9| = 4.5
$$

Thus, the area of triangle ABC is 4.5 square units.

Applications and Significance of Determining Triangle Vertices in Coordinate Geometry

Why Is It

Frequently Asked Questions

What are the coordinates of vertex A in triangle ABC?

The coordinates of vertex A are (3, 3).

What are the coordinates of vertex B in triangle ABC?

The coordinates of vertex B are (0, 7).

What are the coordinates of vertex C in triangle ABC?

The coordinates of vertex C are (3, 0).

How can you verify the vertices of triangle ABC are at the given points?

By plotting the points A(3, 3), B(0, 7), and C(3, 0) on a coordinate plane and confirming their positions form the triangle.

What is the method to determine the vertices of a triangle given their coordinates?

The vertices are directly given as coordinate pairs, so you simply list their (x, y) values as provided.

Can you find the area of triangle ABC with the given vertices?

Yes, using the coordinate formula: Area = $|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| / 2$. Plugging in the points gives the area.

How do you identify the type of triangle formed by points A(3, 3), B(0, 7), and C(3, 0)?

Calculate the lengths of the sides using the distance formula and then determine if the triangle is scalene, isosceles, or equilateral.

What is the significance of the given vertices in coordinate geometry?

They help in analyzing the triangle's properties, such as sides, angles, area, and type, using coordinate geometry techniques.

How can the vertices of triangle ABC be used to find the centroid?

The centroid is found by averaging the x-coordinates and y-coordinates of the vertices: ((x1 + x2 + x3)/3, (y1 + y2 + y3)/3).

Additional Resources

Triangle Geometry Analysis: Determining Coordinates of Triangle ABC

Understanding the precise coordinates of a triangle's vertices is fundamental in coordinate

geometry, serving as the foundation for analyzing shapes, calculating distances, areas, and angles, and applying these concepts in various fields such as engineering, computer graphics, and mathematics. In this article, we undertake an in-depth exploration of Triangle ABC with given vertices at A(3, 3), B(0, 7), and C(3, 0). Our goal is to verify these points, understand their spatial relationships, and explore the properties that define this triangle.

Introduction to Triangle ABC and Its Given Vertices

The initial data provided for Triangle ABC specifies three points in a two-dimensional Cartesian coordinate plane:

- Vertex A at (3, 3)
- Vertex B at (0, 7)
- Vertex C at (3, 0)

The key task is to confirm these points are valid vertices of a triangle and to analyze their positions relative to each other.

Understanding Coordinates and Their Significance

Before delving into calculations, it’s essential to clarify what these coordinates represent. Each point is given as an ordered pair (x, y), signifying its position along the horizontal (x-axis) and vertical (y-axis):

- The x-coordinate indicates the position relative to the origin along the horizontal axis.
- The y-coordinate indicates the position relative to the origin along the vertical axis.

In the context of triangle ABC, these points' positions help us visualize the shape and size, and facilitate calculations of side lengths, angles, and area.

Visualizing the Triangle: Plotting the Vertices

Visual representation aids comprehension. Plotting the points:

- A(3, 3): Located 3 units right of the origin and 3 units above.
- B(0, 7): Located at the origin's leftmost position (0 on x-axis) and 7 units above.

- C(3, 0): Located 3 units right of the origin on the x-axis and directly on the x-axis (since y=0).

This visualization suggests the triangle is situated in the first quadrant, with a vertical side extending from B(0, 7) down to C(3, 0), and a point A that lies somewhere between B and C on the plane.

Calculating Side Lengths: Verifying the Triangle's Shape

One fundamental step involves computing the lengths of the sides to understand the triangle's proportions and verify that the three points indeed form a triangle (i.e., the points are not collinear).

Distance Formula:

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Calculations:

1. Side AB:

$$
AB = \sqrt{(0 - 3)^2 + (7 - 3)^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

2. Side AC:

$$
AC = \sqrt{(3 - 3)^2 + (0 - 3)^2} = \sqrt{0^2 + (-3)^2} = \sqrt{0 + 9} = 3
$$

3. Side BC:

$$
BC = \sqrt{(3 - 0)^2 + (0 - 7)^2} = \sqrt{3^2 + (-7)^2} = \sqrt{9 + 49} = \sqrt{58} \approx 7.6158
$$

Summary of Side Lengths:

Side	Coordinates	Length
AB	A(3,3) & B(0,7)	5 units
AC	A(3,3) & C(3,0)	3 units
BC	B(0,7) & C(3,0)	approximately 7.62 units

Verifying the Triangle: Collinearity and Validity

To confirm that points A, B, and C form a triangle, we need to ensure they are not collinear. Collinearity occurs if all three points lie on a straight line, which would imply the area of the figure is zero.

Method:

Calculate the area of the triangle using the shoelace formula or the coordinate geometry area formula:

$$
\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|
$$

Using points:

- $(x_1, y_1) = (3, 3)$
- $(x_2, y_2) = (0, 7)$
- $(x_3, y_3) = (3, 0)$

Calculation:

$$
\text{Area} = \frac{1}{2} |3(7 - 0) + 0(0 - 3) + 3(3 - 7)|
= \frac{1}{2} |3 \times 7 + 0 \times (-3) + 3 \times (-4)|
= \frac{1}{2} |21 + 0 - 12|
= \frac{1}{2} |9|
= 4.5
$$

Since the area is non-zero (4.5), the points are not collinear and do indeed form a triangle.

Determining the Nature of Triangle ABC

With side lengths known, we can analyze the triangle's properties:

- Side AB = 5 units
- Side AC = 3 units
- Side BC ≈ 7.62 units

Classification:

- Since BC is significantly longer than the other two sides, and AB and AC are less than BC, Triangle ABC is scalene (all sides of different lengths).

- The approximate side lengths suggest the triangle is obtuse-angled because the square of the longest side exceeds the sum of the squares of the other two sides:

$$
BC^2 \approx 58, \quad AB^2 = 25, \quad AC^2 = 9
$$

$$
AB^2 + AC^2 = 25 + 9 = 34

\]

Since \(BC^2 \approx 58 > 34 \), the angle opposite side BC is obtuse.

Calculating the Triangle’s Angles

Knowing side lengths allows for the calculation of internal angles using the Law of Cosines:

\[
\cos C = \frac{a^2 + b^2 - c^2}{2ab}
\]

Where:

- \(a, b, c \) are the side lengths opposite angles \(A, B, C \) respectively.

Calculations:

1. Angle at A (opposite BC):

\[
\cos A = \frac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC} = \frac{25 + 9 - 58}{2 \times 5 \times 3} = \frac{-24}{30} = -0.8
\]
\[

$$
A = \arccos(-0.8) \approx 143.13^\circ
$$

2. Angle at B (opposite AC):

$$
\cos B = \frac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC} = \frac{9 + 58 - 25}{2 \times 3 \times 7.62} = \frac{42}{45.72} \approx 0.918
$$
$$
B = \arccos(0.918) \approx 24.56^\circ
$$

3. Angle at C (opposite AB):

$$
\cos C = \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC} = \frac{25 + 58 - 9}{2 \times 5 \times 7.62} = \frac{74}{76.2} \approx 0.971
$$
$$
C = \arccos(0.971) \approx 13.83^\circ
$$

Summary:

Vertex	Approximate Angle	Degrees
A	143.13°	Obtuse angle
B	24.56°	Acute angle
C	13.83°	Acute angle

This confirms that Triangle ABC is scalene and obtuse-angled, with the obtuse angle at vertex A.

Area and Perimeter Calculations