Two Cars Start Moving From The Same Point. One Travels South At 48 Mi/h And The Other Travels West At

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Imagine two cars starting their journey simultaneously from the same location, heading in different directions. One car moves southward at a speed of 48 miles per hour, while the other travels westward at an unknown speed. This scenario is a classic problem in kinematics and relative motion, often used to illustrate how objects moving in perpendicular directions relate to each other over time. Such problems are not only fundamental in physics but also have practical applications in navigation, traffic analysis, and even in the development of autonomous vehicle algorithms.

In this article, we will explore the problem in detail, analyzing how the positions of the two cars change over time, calculating the distance between them at any given moment, and understanding the principles that govern their relative motion. Whether you are a student studying physics, a navigation enthusiast, or simply curious about how to approach such problems, this comprehensive guide will provide clarity and insight.

Understanding the Scenario

The problem involves two cars starting from a common point, often called the origin, at the same time. One moves southward at a constant speed, and the other moves westward, but their speeds may differ. The key points to consider include:

- Initial Conditions: Both cars start at the same point (time $t=0$).
- Directions: One moves south; the other moves west.
- Speeds: Known for the southbound car (48 mi/h); unknown for the westbound car.
- Time: The analysis can be extended to any time $t > 0$.

Visualizing this setup, it resembles a right-angled triangle where the position of each car after time t forms the legs of the triangle, and the distance between the cars corresponds to the hypotenuse.

Position of the Cars Over Time

Let's define some variables to formalize the problem:

- $v_s = 48 \text{ mi/h}$: Velocity of the southbound car.

- v_w: Velocity of the westbound car (unknown).
- t: Time in hours since the start.
- $D_s(t)$: Distance traveled south by the southbound car after time t.
- $D_w(t)$: Distance traveled west by the westbound car after time t.
- $d(t)$: Distance between the two cars at time t.

Position of the Southbound Car

The southbound car's position after time t is:

$$D_s(t) = v_s \times t = 48t \text{ miles}$$

It is located directly south of the starting point at coordinates:

$$(0, -48t)$$

Position of the Westbound Car

Suppose the westbound car's speed is v_w miles per hour, then:

$$D_w(t) = v_w \times t$$

It is located west of the starting point at coordinates:

$$(-v_w t, 0)$$

Calculating the Distance Between the Cars

Since both cars are moving along perpendicular paths, their positions form a right triangle with legs:

- Southward distance: $48t$
- Westward distance: $v_w t$

The distance between the two cars at any time t is given by the Pythagorean theorem:

$$d(t) = \sqrt{(48t)^2 + (v_w t)^2} = t \sqrt{(48)^2 + v_w^2}$$

This formula shows that the distance between the cars increases linearly with time, scaled by the combined velocity component.

Example Calculation: Known Speed of Westbound Car

If, for example, the westbound car travels at 60 mi/h, then:

\[

$$
d(t) = t \sqrt{(48)^2 + (60)^2} = t \sqrt{2304 + 3600} = t \sqrt{5904}
$$

Calculating the square root:

$$
\sqrt{5904} \approx 76.84
$$

Thus, the distance after t hours:

$$
d(t) \approx 76.84 t
$$

Determining the Rate at Which the Distance Between the Cars Changes

Understanding how quickly the distance between the two cars changes over time is crucial, especially for safety analysis and collision avoidance systems. To find this rate, we differentiate $d(t)$ with respect to time t.

Given:

$$
d(t) = t \sqrt{(48)^2 + v_w^2}
$$

The derivative is:

$$
\frac{dd}{dt} = \sqrt{(48)^2 + v_w^2}
$$

This indicates that the rate of change of the distance between the cars is constant if the speeds are constant. In the previous example with $v_w = 60 \text{ mi/h}$:

$$
\frac{dd}{dt} \approx 76.84 \text{ mi/h}
$$

This is intuitive because both cars are moving at constant speeds along perpendicular directions, so their relative speed is the vector sum of their individual velocities:

$$
v_{rel} = \sqrt{v_s^2 + v_w^2}
$$

\]

Special Cases and Additional Considerations

When the Westbound Car's Speed Is Unknown

If the speed \(v_w \) is not given, but other information is available—such as the distance between the cars at a particular time or the rate at which the distance is increasing—then you can solve for \(v_w \).

Example: Suppose after 2 hours, the cars are 150 miles apart, and the southbound car has been traveling at 48 mi/h:

\[
d(2) = 150 \text{ miles}
\]

Using the formula:

\[
150 = 2 \times \sqrt{48^2 + v_w^2}
\]

\[
75 = \sqrt{2304 + v_w^2}
\]

Squaring both sides:

\[
5625 = 2304 + v_w^2
\]

\[
v_w^2 = 5625 - 2304 = 3321
\]

\[
v_w = \sqrt{3321} \approx 57.66 \text{ mi/h}
\]

Thus, the westbound car's speed is approximately 57.66 miles per hour.

Real-World Applications and Implications

Understanding the relative motion of two objects moving perpendicular to each other has numerous practical applications:

- Navigation and Route Planning: Estimating how distances increase over time helps in planning efficient routes.
- Traffic Safety: Analyzing how quickly two vehicles may approach each other in intersecting paths to prevent accidents.
- Autonomous Vehicles: Developing algorithms that calculate potential collision points based on current trajectories and speeds.
- Search and Rescue Operations: Estimating the positions of moving objects or individuals over time based on initial data.
- Military and Defense: Tracking the movement of objects in strategic scenarios involving multiple moving units.

Conclusion

The problem of two cars starting from the same point and moving in perpendicular directions offers valuable insights into the principles of relative motion, vector addition, and kinematics. By understanding how to model their positions over time and calculate the distance between them, one can apply these concepts to real-world situations such as traffic management, navigation, and autonomous vehicle design.

Key takeaways include:

- The positions of the cars can be represented using simple coordinate systems.
- The distance between the cars at any time is derived from the Pythagorean theorem.
- The relative speed at which the distance changes is the vector sum of their individual speeds.
- Additional information allows solving for unknown speeds or predicting future positions.

Mastering these principles equips you with the tools to analyze complex motion scenarios, ultimately contributing to safer and more efficient transportation systems.

Meta Description: Explore a detailed analysis of two cars starting from the same point, traveling in perpendicular directions at known and unknown speeds. Learn how to calculate their position, distance over time, and relative motion with practical examples.

Frequently Asked Questions

Two cars start from the same point, with one heading south at 48 mi/h and the other heading west at 60 mi/h. How do you determine the distance between the two cars after 2 hours?

Use the Pythagorean theorem: Distance = $\sqrt{[(48 \times 2)^2 + (60 \times 2)^2]} = \sqrt{(96^2 + 120^2)} = \sqrt{(9216 + 14400)} = \sqrt{23616} \approx 153.69$ miles.

If the southbound car increases its speed to 60 mi/h and the westbound car remains at 60 mi/h, what is the rate at which the distance between the two cars is changing after 3 hours?

After 3 hours, south car distance: $60 \times 3 = 180$ mi; west car: $60 \times 3 = 180$ mi. Using related rates, the rate of change of distance is $\sqrt{[(dx/dt)^2 + (dy/dt)^2]} = \sqrt{(60^2 + 60^2)} = \sqrt{(3600 + 3600)} = \sqrt{7200} \approx 84.85$ mi/h.

What is the position of each car after 4 hours?

South car: $48 \times 4 = 192$ miles south; West car: $60 \times 4 = 240$ miles west.

How can we model the positions of the two cars mathematically over time?

Set the starting point as origin: South car's position at time t: $(0, -48t)$; West car's position: $(-60t, 0)$.

At what time do the two cars have the same distance from the starting point?

Since distances are 48t and 60t respectively, they are equal only at $t=0$ (initial point). They are never equal again as they move in perpendicular directions.

If the cars maintain constant speeds, how long does it take for them to be 250 miles apart?

Using the distance formula: $D = \sqrt{[(48t)^2 + (60t)^2]} = \sqrt{(2304t^2 + 3600t^2)} = \sqrt{(5904t^2)} = t \times \sqrt{5904}$. Set $D=250$: $250 = t \times \sqrt{5904} \Rightarrow t = 250 / \sqrt{5904} \approx 250 / 76.84 \approx 3.25$ hours.

What is the angle between the two cars' paths after 3 hours?

Since they move perpendicular to each other, the angle between their paths remains 90 degrees at all times.

How does the distance between the cars change over time?

The distance increases over time following the formula $D(t) = \sqrt{[(48t)^2 + (60t)^2]} = t \times \sqrt{5904}$, showing a linear increase proportional to time.

Can we determine when the cars are closest to each other after starting?

No, because they are moving away from the starting point in perpendicular directions, their distance from each other monotonically increases over time, so the closest point is at t=0.

Additional Resources

Two cars start moving from the same point, heading in different directions at distinct speeds, exemplify a fundamental concept in kinematics and relative motion. Understanding how their separate journeys evolve over time not only illuminates the basics of physics but also offers practical insights into navigation, traffic analysis, and motion prediction. This article explores the dynamics of such a scenario, analyzing the paths, calculating the distances between the cars at various times, and delving into the underlying principles that govern their motion.

Introduction to the Scenario

Imagine two cars beginning their journey simultaneously from a common origin—say, a city intersection or a parking lot. One car travels directly south at a steady speed of 48 miles per hour, while the other heads west, perhaps at a different speed. The initial point of departure is the same for both, and their paths are perpendicular, making this a classic case for studying right-angled motion and relative displacement.

This setup is more than a hypothetical; it mirrors real-world situations such as emergency response coordination, navigation system calculations, and even sports analytics. The fundamental question often posed is: How far apart are the cars after a certain period? To answer this, we need to analyze their individual motions, understand their positions over time, and apply geometric and mathematical principles.

Understanding the Directions and Speeds

Directions and Coordinate System

To analyze the problem systematically, establishing a coordinate system is essential. Let's define:

- The origin (0,0) where both cars start.
- The positive y-axis as pointing north.
- The positive x-axis as pointing east.

Given this setup:

- The first car travels south at 48 mi/h. Since south corresponds to the negative y-direction, its position after time t hours is:

$$
y_1(t) = -48t
$$

- The second car travels west. West corresponds to the negative x-direction, and assuming its speed is v_w, its position after time t hours is:

$$
x_2(t) = -v_w t
$$

Since the problem specifies the speed of the second car only as "travels west at," but doesn't specify the speed, for a comprehensive analysis, we can consider various scenarios, such as:

- The second car traveling at a different specified speed, say, 60 mi/h.
- The general case where the second car's speed v_w is variable.

For illustration, we'll examine a case where the second car moves west at 60 mi/h.

Summarizing the Speeds and Directions

Car	Direction	Speed	Position at time t (hours)
Car 1	South	48 mi/h	$y_1(t) = -48t$
Car 2	West	60 mi/h	$x_2(t) = -60t$

Mathematical Modeling of the Motion

Position Coordinates Over Time

- Car 1 (Southbound):

$$
\text{Position} = (0, -48t)
$$

- Car 2 (Westbound):

$$
\text{Position} = (-60t, 0)
$$

Distance Between the Cars Over Time

The core quantity of interest is the distance between the two cars as a function of time, denoted $D(t)$.

Because their motions are perpendicular, their positions form a right-angled triangle with the initial point as the right angle. The positions at time t are:

- Car 1: $(0, -48t)$
- Car 2: $(-60t, 0)$

The distance $D(t)$ is given by the Euclidean distance formula:

$$
D(t) = \sqrt{(x_2(t) - 0)^2 + (y_1(t) - 0)^2}
$$

Substituting the positions:

$$
D(t) = \sqrt{(-60t)^2 + (-48t)^2} = \sqrt{(3600 t^2) + (2304 t^2)} = \sqrt{5904 t^2}
$$

Simplifying:

$$
D(t) = \sqrt{5904} \times t \approx 76.84 \times t
$$

This linear relation indicates that the separation between the cars increases proportionally with time, with a constant rate of approximately 76.84 miles per hour.

Analysis of the Relative Motion

Rate of Change of Distance

The derivative of $D(t)$ with respect to time gives the relative speed at which the two cars are moving apart:

$$
\frac{dD}{dt} = \frac{d}{dt} (76.84 t) = 76.84 \text{ mi/h}
$$

This confirms that, since the cars are moving perpendicularly at 48 mi/h and 60 mi/h respectively, their relative speed in terms of increasing distance is the hypotenuse of the right triangle formed by their velocity vectors.

Calculating the relative speed directly:

$$
v_{rel} = \sqrt{(v_{x})^2 + (v_{y})^2} = \sqrt{(60)^2 + (48)^2} = \sqrt{3600 + 2304} = \sqrt{5904} \approx 76.84 \text{ mi/h}
$$

which matches the rate of change of their separation.

Implication: The cars are moving directly away from each other at the combined velocity magnitude, which is consistent with the Pythagorean theorem.

Time-Dependent Distance and Practical Examples

Suppose the cars start at 0 hours:

- After 1 hour:

$$
D(1) \approx 76.84 \text{ miles}
$$

- After 2 hours:

$$
D(2) \approx 153.68 \text{ miles}
$$

- After 0.5 hours:

$$
D(0.5) \approx 38.42 \text{ miles}
$$

These figures illustrate how quickly the separation increases, emphasizing the importance of spatial awareness in navigation and safety.

Extending the Analysis: Variable Speeds and Directions

While the scenario above assumes fixed speeds and perpendicular directions, real-world situations often involve variable velocities, angles, or accelerations. Let's explore some extensions.

Different Speeds and Angles

Suppose:

- Car 1 travels south at 48 mi/h.
- Car 2 travels west at a different speed, say, v_w.
- The roads are not necessarily perpendicular; instead, the cars head along different angles.

In such cases, the position vectors are:

$$
\vec{r}_1(t) = (0, -48t)
$$

$$
\vec{r}_2(t) = (v_w t \cos \theta, v_w t \sin \theta)
$$

where θ is the angle relative to the east axis.

The distance squared:

$$
D^2(t) = (x_2(t) - 0)^2 + (y_2(t) - y_1(t))^2
$$

which can be expanded and analyzed to find the rate of change of distance, or to determine the time when the cars are closest or farthest apart.

Impact of Accelerations and Non-Constant Speeds

In reality, vehicles accelerate or decelerate, so their velocity vectors are time-dependent. Analyzing such motion requires calculus, specifically differential equations, to model acceleration and predict future positions.

For instance:

- If Car 2 accelerates westward at a rate a_w, then:

$$
x_2(t) = -v_{w_0} t - \frac{1}{2} a_w t^2
$$

- Similarly, if Car 1 accelerates southward:

$$
y_1(t) = -v_{y_0} t - \frac{1}{2} a_y t^2
$$

The distance computations become more complex but are essential for precise navigation, collision avoidance, and traffic flow management.

Real-World Applications and Implications

Navigation and Traffic Management

Understanding the relative motion of vehicles is crucial for:

- Route planning: Estimating how quickly vehicles moving in different directions will diverge helps optimize traffic flow.
- Collision avoidance systems: Modern vehicles utilize sensors and algorithms that rely on similar kinematic calculations to prevent accidents.
- Emergency response: Dispatching units efficiently requires predicting how far and how fast vehicles will arrive at certain locations.

Autonomous Vehicles and Advanced Driver Assistance Systems (ADAS)

Autonomous vehicles depend heavily on precise motion modeling:

- Object detection: Estimating the relative position and speed of other vehicles.
- Path prediction: Anticipating future positions based on current velocities and accelerations.
- Safety protocols: Determining safe distances and times to react.

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