Two Jupiter-size Planets Are Released From Rest 1.001011 M Apart. What Are Their Speeds As They Crash

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Understanding the dynamics of celestial bodies colliding in space offers fascinating insights into astrophysics and gravitational physics. In this article, we explore the scenario where two Jupiter-sized planets are initially at rest, separated by a distance of approximately 1.001011 million meters, and analyze their eventual velocities as they accelerate toward each other under mutual gravitational attraction. This investigation combines principles of classical mechanics, specifically gravitational potential energy, kinetic energy, and conservation laws, to determine their collision speeds.

Introduction to the Problem

The scenario involves two planets, each roughly the size of Jupiter, which is approximately 1.898 × 10^27 kilograms in mass. They are initially stationary relative to each other, separated by a very small distance of about 1.001011 million meters (roughly 1,001,011 meters). Given the immense mass of each planet and the proximity, the gravitational pull will cause them to accelerate toward each other, reaching significant velocities by the time they collide.

The key questions we aim to answer are:

- What are the velocities of each planet just before impact?
- How does the initial separation influence their final speeds?
- What physical principles govern this motion?

To answer these, we'll employ conservation of energy principles, assuming no external forces or dissipative effects like friction or interplanetary medium resistance, which are negligible in the vacuum of space.

Fundamentals of Gravitational Motion

Before delving into calculations, understanding the foundational concepts is crucial.

Newton's Law of Universal Gravitation

Newton's law states that the force (F) between two masses (m_1 and m_2) separated by a distance (r) is:

$$F = G \frac{m_1 m_2}{r^2}$$

where G is the gravitational constant ($6.67430 \times 10^{-11} \, \mathrm{m^3\,kg^{-1}\,s^{-2}}$).

Conservation of Mechanical Energy

In a two-body system with no external forces, total mechanical energy remains constant:

$$E_{total} = KE + PE = \text{constant}$$

Where:

- KE is the kinetic energy.
- PE is the gravitational potential energy.

Initially, both planets are at rest, so their kinetic energy is zero, and their potential energy depends on their initial separation.

Modeling the Scenario

Given the initial separation and masses, we analyze the system step by step.

Assumptions

- Both planets are perfect spheres with equal mass m.
- They start from rest at a distance $r_0 = 1.001011 \times 10^6\, \mathrm{m}$.
- No external forces influence their motion.
- The collision occurs when the distance between their centers equals twice the planet radius ($2 R_J \approx 2.898 \times 10^7\, \mathrm{m}$).

Note: Since the initial separation is only about 1 million meters, which is much less than the diameter of Jupiter (~1.4 × 10^7 meters), this indicates the initial distance is close to their physical size, suggesting they are nearly touching or overlapping in a hypothetical scenario. However, for the purpose of physics calculations, we treat the initial separation as the distance between their centers.

Calculations of Final Velocities

To find the velocities at collision, we use conservation of energy.

Step 1: Initial Total Mechanical Energy

Since both planets are initially at rest:

$$
E_{initial} = PE_{initial} = -G \frac{m^2}{r_0}
$$

The negative sign indicates gravitational potential energy is negative, reflecting a bound system.

Step 2: Final Total Mechanical Energy

At the moment just before collision, the planets are at a distance $r_f = 2 R_J$, and their kinetic energies are:

$$
KE_{final} = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2
$$

Since the system is symmetric, and no external forces act, the velocities will be equal in magnitude but opposite in direction:

$$
v_1 = v, \quad v_2 = v
$$

Total kinetic energy:

$$
KE_{final} = m v^2
$$

The final potential energy:

$$
PE_{final} = -G \frac{m^2}{r_f}
$$

Applying conservation of energy:

\[

PE_{initial} = KE_{final} + PE_{final}
\]

\[
- G \frac{m^2}{r_0} = m v^2 - G \frac{m^2}{r_f}
\]

Rearranged:

\[
m v^2 = G m^2 \left(\frac{1}{r_0} - \frac{1}{r_f}\right)
\]

Divide through by \(m\):

\[
v^2 = G m \left(\frac{1}{r_0} - \frac{1}{r_f}\right)
\]

The collision speed of each planet is then:

\[
v = \sqrt{G m \left(\frac{1}{r_0} - \frac{1}{r_f}\right)}
\]

Step 3: Numerical Calculation

Using known values:

- \(m = 1.898 \times 10^{27}\, \mathrm{kg}\)
- \(r_0 = 1.001011 \times 10^{6}\, \mathrm{m}\)
- \(r_f = 2 R_J \approx 1.898 \times 10^{7}\, \mathrm{m}\)

Calculate:

\[
v = \sqrt{6.67430 \times 10^{-11} \times 1.898 \times 10^{27}
\left(\frac{1}{1.001011 \times 10^{6}} - \frac{1}{1.898 \times
10^{7}}\right)}
\]

First, compute the difference in reciprocals:

\[
\frac{1}{1.001011 \times 10^{6}} \approx 9.98989 \times 10^{-7}\,
\mathrm{m^{-1}}
\]
\[
\frac{1}{1.898 \times 10^{7}} \approx 5.268 \times 10^{-8}\, \mathrm{m^{-1}}

\]

Difference:

\[
9.98989 \times 10^{-7} - 5.268 \times 10^{-8} \approx 9.463 \times 10^{-7}
\]

Now, multiply:

\[
6.67430 \times 10^{-11} \times 1.898 \times 10^{27} \approx 1.267 \times 10^{17}
\]

Finally, multiply by the difference:

\[
1.267 \times 10^{17} \times 9.463 \times 10^{-7} \approx 1.2 \times 10^{11}
\]

Square root:

\[
v \approx \sqrt{1.2 \times 10^{11}} \approx 1.095 \times 10^{6}\, \mathrm{m/s}
\]

Result: The planets collide at approximately 1,095 km/s.

Note: This is an immense speed, roughly 3,600 times the speed of sound in air, illustrating the enormous energies involved.

Implications and Physical Insights

The calculated collision speed highlights several critical points:

- High Energy Release: The kinetic energy involved in such a collision would be astronomical, capable of causing massive shock waves, heating, and possibly vaporization of material.
- Astrophysical Relevance: While such direct collisions between Jupiter-sized planets are rare, understanding their dynamics is essential for modeling planetary formation, collisions in dense star clusters, or planetary system evolution.
- Limitations of the Model: Realistically, factors like tidal forces, material deformation, and relativistic effects at such velocities could alter the outcome. Our calculations assume idealized point masses and neglect such complexities.

Additional Factors and Considerations

While the above calculations provide a good estimate, several factors could influence the actual collision speed:

- Planetary Structure: Actual planets are not point masses; their internal composition might influence the collision dynamics.
- Angular Momentum: The assumption of symmetric motion neglects possible angular momentum, which could introduce tangential velocities.
- External Gravitational Fields: Nearby celestial bodies or star influence might modify the trajectories.
- Relativistic Effects: At velocities approaching \(10^6\, \mathrm{m/s}\), relativistic effects are negligible but might be considered in extreme cases.

Conclusion

In summary, when two Jupiter-sized planets are released from rest at a separation of approximately 1.001011 million meters, they accelerate toward each other under mutual gravity, reaching a collision speed of roughly 1,095 km/s just before impact. This calculation, grounded in classical mechanics and conservation of energy,

Frequently Asked Questions

What initial conditions are given for the two Jupiter-sized planets in the problem?

The two planets are initially at rest, separated by a distance of 1.001011 million miles (or units), and are assumed to attract each other gravitationally.

How do we determine the speed of each planet as they collide?

By applying conservation of energy, considering their initial gravitational potential energy and converting it into kinetic energy as they accelerate toward each other.

What is the significance of the initial separation distance in calculating their final speeds?

The initial separation affects the total gravitational potential energy available to convert into kinetic energy, thus influencing the final impact speeds of the planets.

Why are the planets considered to be released from rest in this problem?

They are considered to be released from rest to simplify the initial conditions, meaning their initial kinetic energy is zero, allowing for straightforward application of energy conservation.

What are the typical impact speeds of Jupiter-sized planets colliding under these conditions?

The impact speeds depend on the initial separation and mass but generally can reach thousands of meters per second, often in the range of several kilometers per second, indicating extremely energetic collisions.

Additional Resources

Two Jupiter-Size Planets Are Released From Rest 1.001011 M Apart. What Are Their Speeds As They Crash?

Understanding the dynamic interactions between celestial bodies, especially massive planets like Jupiter, offers profound insights into gravitational physics, orbital mechanics, and astrophysical phenomena. The scenario of two Jupiter-sized planets initially at rest and separated by a specific distance, subsequently accelerating toward each other due to mutual gravitational attraction, serves as a compelling case study. This comprehensive review delves into the physics governing their motion, the calculations involved in determining their velocities upon collision, and the broader implications of such interactions.

Overview of the Scenario

The problem involves two identical planets, each roughly the size and mass of Jupiter, initially at rest relative to each other, separated by a distance of 1.001011 times a certain baseline measure—likely the sum of their radii or perhaps a specific initial separation distance. Their initial state is static, with no initial velocity, and they are then released to move solely under their mutual gravitational attraction.

Key parameters:

- Mass of each planet (M): Approximately 1.898×10^{27} kg (mass of Jupiter)
- Initial separation distance (R): $R_0 = 1.001011 \times D$, where D is a reference distance, possibly the sum of their radii or an initial

orbital radius.
- Initial velocities: Zero (rest)
- Final state: The planets collide; we are interested in their speeds at impact.

Fundamental Principles Involved

The analysis relies on several core physics principles:

Newton’s Law of Universal Gravitation

$$
F = G \frac{M_1 M_2}{r^2}
$$

- G: Gravitational constant ($6.67430 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$)
- M_1, M_2: masses of the planets (equal in this scenario)
- r: instantaneous separation between the planets

Conservation of Energy

Since the planets start from rest and only interact gravitationally, their total mechanical energy remains conserved:

$$
E_{total} = \text{Potential Energy} + \text{Kinetic Energy}
$$

Initially, kinetic energy is zero, and potential energy is:

$$
U_i = - G \frac{M^2}{R_0}
$$

At the point of collision, the separation is effectively zero (assuming point masses for simplicity), and the kinetic energy is at maximum.

Conservation of Momentum

Given the symmetry (equal masses, initial conditions), the planets will move directly toward each other with equal and opposite velocities at impact.

Step-by-Step Calculation of Final Speeds

To determine the velocities of the planets at collision, we apply conservation laws, primarily energy conservation, since the initial kinetic energy is zero.

1. Establish the Initial Conditions

- Initial separation: R_0
- Initial kinetic energy: $KE_i = 0$
- Initial potential energy:

$$
U_i = - G \frac{M^2}{R_0}
$$

2. Final Conditions at Collision

- Separation: effectively zero ($r \to 0$)
- Final kinetic energy:

$$
KE_f = 2 \times \frac{1}{2} M v^2 = M v^2
$$

- Final potential energy: tends to negative infinity if point masses are assumed. To avoid singularities, in real scenarios, the finite size of planets must be considered. For simplicity, we assume they collide at a minimal separation equal to the sum of their radii or a specified collision radius R_c.

Assuming a collision at minimal separation R_c, the potential energy at collision:

$$
U_f = - G \frac{M^2}{R_c}
$$

Note: For a realistic calculation, the finite size of planets dictates R_c (roughly $2 R_{Jupiter}$). For the purpose of this calculation, we can approximate the collision radius as the sum of their radii.

3. Applying Conservation of Energy

$$
U_i + KE_i = U_f + KE_f
$$

$$
- G \frac{M^2}{R_0} + 0 = - G \frac{M^2}{R_c} + M v^2
$$

Rearranged for v:

$$
M v^2 = G M^2 \left(\frac{1}{R_0} - \frac{1}{R_c} \right)
$$

$$
v = \sqrt{ G M \left(\frac{1}{R_0} - \frac{1}{R_c} \right) }
$$

This expression gives the impact velocity of each planet relative to the center of mass frame.

Numerical Evaluation

Suppose specific values for the initial separation and collision radius:

- Mass of Jupiter: $M = 1.898 \times 10^{27}$ kg
- Radius of Jupiter: $R_{J} \approx 7.1492 \times 10^{7}$ m
- Collision radius: $R_c \approx 2 R_{J} \approx 1.42984 \times 10^{8}$ m
- Initial separation: $R_0 = 1.001011 \times R_c \approx 1.001011 \times 1.42984 \times 10^{8} \approx 1.43152 \times 10^{8}$ m

Calculating:

$$
v = \sqrt{ (6.67430 \times 10^{-11}) \times (1.898 \times 10^{27}) \times \left(\frac{1}{1.43152 \times 10^{8}} - \frac{1}{1.42984 \times 10^{8}} \right) }
$$

Compute the difference inside parentheses:

$$
\frac{1}{1.43152 \times 10^{8}} \approx 6.987 \times 10^{-9}
$$

$$
\frac{1}{1.42984 \times 10^{8}} \approx 6.996 \times 10^{-9}
$$

$$
\Delta = 6.987 \times 10^{-9} - 6.996 \times 10^{-9} = - 9 \times 10^{-12}
$$

Note: The negative sign indicates that the potential energy becomes more negative as they approach, so the actual kinetic energy increase corresponds to the magnitude of this difference.

Putting the absolute value:

$$
v = \sqrt{ (6.67430 \times 10^{-11}) \times (1.898 \times 10^{27}) \times 9 \times 10^{-12} }
$$

Calculating:

$$
(6.67430 \times 10^{-11}) \times (1.898 \times 10^{27}) \approx 1.267 \times 10^{17}
$$

Then:

$$
v = \sqrt{ 1.267 \times 10^{17} \times 9 \times 10^{-12} } = \sqrt{ 1.140 \times 10^{6} } \approx 1068\, \text{m/s}
$$

Result: Each planet would collide at approximately 1.07 km/s relative to the other.

Implications of the Calculations

The derived impact velocity (~1 km/s) is significant but much less than typical planetary escape velocities (~60 km/s for Jupiter). This indicates:

- The initial separation was close; a slightly larger initial separation would reduce the impact velocity.
- In real astrophysical environments, other forces (e.g., tidal interactions, orbital angular momentum, external influences) could alter this simple scenario.
- The assumption of point masses simplifies calculations but neglects internal structure and finite size effects, which would influence the

collision dynamics.

Broader Context and Real-World Relevance

Understanding such gravitational interactions is fundamental in astrophysics, especially concerning:

- Planetary formation: Accretion of gas giants and planetary collisions
- Binary planet systems: Potential formation and stability considerations
- Collision outcomes: Impact velocity influences collision aftermath, such as merging, fragmentation, or atmospheric stripping
- Exoplanet studies: Observing high-velocity impacts as indicators of dynamic planetary systems

Additionally, these calculations are essential in modeling:

- N-body simulations
- Gravitational wave emission during massive collisions
- Planetary system evolution over astronomical timescales

Limitations and Further Considerations

While the calculations provide a foundational understanding, several factors could refine or complicate the scenario:

- Finite sizes and shapes: Actual planets are not point masses; their internal structures influence collision dynamics.
- Angular momentum: Any initial tangential velocity or orbital motion would alter impact velocities.
- Relativistic effects: For extremely high velocities or close encounters, relativistic corrections might be necessary.
- Energy dissipation

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