Use This Rational Function And Complete A Full Analysis According To The Listed Instructions:1. Write

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Understanding rational functions is a fundamental aspect of algebra and calculus, as they serve as essential tools for modeling real-world phenomena, analyzing asymptotic behaviors, and solving various mathematical problems. This comprehensive guide aims to provide a detailed analysis of a specific rational function, guiding you through each step of the process, from identifying key features to graphing and interpreting the function's behavior.

In this article, we will walk through the complete analysis of a rational function, demonstrating how to apply various mathematical techniques and concepts systematically. Whether you're a student preparing for exams or a professional seeking to deepen your understanding, this guide offers valuable insights into the intricacies of rational functions.

Understanding Rational Functions

A rational function is any function that can be expressed as the ratio of two polynomials:

$$
f(x) = \frac{P(x)}{Q(x)}
$$

where $P(x)$ and $Q(x)$ are polynomials, and $Q(x) \neq 0$.

Key features of rational functions include:

- Domain restrictions due to the denominator $Q(x)$, where the function is undefined.
- Vertical asymptotes occurring at zeros of the denominator.
- Horizontal or oblique asymptotes describing end behavior.
- Holes in the graph where factors cancel out.
- Intercepts where the graph crosses axes.

Step-by-Step Analysis of the Rational Function

Suppose we are given the rational function:

$$
f(x) = \frac{2x^2 - 3x + 1}{x^2 - 4}
$$

Our goal is to analyze this function thoroughly following the instructions: find domain, intercepts, asymptotes, analyze end behavior, determine increasing/decreasing intervals, concavity, and sketch the graph.

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1. Domain of the Function

The domain includes all real numbers except where the denominator equals zero.

$$
x^2 - 4 = 0 \implies x^2 = 4 \implies x = \pm 2
$$

Therefore:

$$
\boxed{\text{Domain}: \quad (-\infty, -2) \cup (-2, 2) \cup (2, \infty)}
$$

---

2. Intercepts

a. y-intercept:

Set $x = 0$:

$$
f(0) = \frac{2(0)^2 - 3(0) + 1}{(0)^2 - 4} = \frac{1}{-4} = -\frac{1}{4}
$$

Y-intercept:

$$
\boxed{(0, -\frac{1}{4})}
$$

b. x-intercepts:

Set numerator equal to zero:

$$
2x^2 - 3x + 1 = 0
$$

Solve quadratic:

$$
x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(1)}}{2 \times 2} = \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{3 \pm 1}{4}
$$

So:

$$
x = \frac{3 + 1}{4} = 1 \quad \text{and} \quad x = \frac{3 - 1}{4} = \frac{1}{2}
$$

x-intercepts:

$$
\boxed{(1, 0)} \quad \text{and} \quad \left(\frac{1}{2}, 0\right)
$$

---

3. Vertical Asymptotes

Vertical asymptotes occur where the denominator is zero and the numerator is not zero simultaneously.

Since denominator zero at $x = \pm 2$:

- At $x = 2$, numerator:

$$
2(2)^2 - 3(2) + 1 = 8 - 6 + 1 = 3 \neq 0
$$

- At $x = -2$:

$$
2(-2)^2 - 3(-2) + 1 = 8 + 6 + 1 = 15 \neq 0
$$

Thus, vertical asymptotes:

$$
x = 2, \quad x = -2
$$

---

4. Horizontal or Oblique Asymptotes

Since degrees of numerator and denominator are equal (2), the end behavior is determined by the ratio of leading coefficients.

$$
\lim_{x \to \pm \infty} f(x) = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}} = \frac{2}{1} = 2
$$

Horizontal asymptote:

$$
\boxed{y = 2}
$$

---

5. Holes in the Graph

Holes occur where a factor cancels out in numerator and denominator.

Factor numerator and denominator:

$$
f(x) = \frac{(2x - 1)(x - 1)}{(x - 2)(x + 2)}
$$

Since no common factors exist, there are no holes in this function.

---

6. Sign Analysis and Intervals of Increase/Decrease

Create a sign chart to determine where the function is increasing or decreasing.

Critical points:

- $x = -2, 2$ (vertical asymptotes)

- $x = \frac{1}{2}$ and $x = 1$ (x-intercepts where numerator zero)

Test points in each interval:

Interval	Test point	Sign of numerator	Sign of denominator	f(x) sign	Behavior
$(-\infty, -2)$	-3	negative	positive	negative	decreasing
$(-2, \frac{1}{2})$	0	positive	negative	negative	decreasing
$(\frac{1}{2}, 1)$	0.75	positive	negative	negative	decreasing
$(1, 2)$	1.5	positive	negative	negative	decreasing
$(2, \infty)$	3	positive	positive	positive	increasing

Conclusion:

- $f(x)$ decreases on $(-\infty, 2)$
- $f(x)$ increases on $(2, \infty)$

---

7. Concavity and Inflection Points

To analyze concavity, compute the second derivative $f''(x)$ or analyze the first derivative $f'(x)$.

First derivative:

Using the quotient rule:

$$
f'(x) = \frac{(P'(x)Q(x) - P(x)Q'(x))}{[Q(x)]^2}
$$

where

$$
P(x) = 2x^2 - 3x + 1,\quad Q(x) = x^2 - 4
$$

Compute derivatives:

$$
P'(x) = 4x - 3,\quad Q'(x) = 2x
$$

Thus,

$$
f'(x) = \frac{(4x - 3)(x^2 - 4) - (2x^2 - 3x + 1)(2x)}{(x^2 - 4)^2}
$$

Simplify numerator step-by-step:

$$
N(x) = (4x - 3)(x^2 - 4) - 2x(2x^2 - 3x + 1)
$$

Expand:

$$
(4x)(x^2 - 4) - 3(x^2 - 4) - 2x(2x^2 - 3x + 1)
$$

$$
= 4x^3 - 16x - 3x^2 + 12 - 4x^3 + 6x^2 - 2x
$$

Combine like terms:

$$
(4x^3 - 4x^3) + (-3x^2 + 6x^2) + (-16x - 2x) + 12 = 0 + 3x^2 - 18x + 12
$$

Therefore,

$$
f'(x) = \frac{3x^2 - 18x + 12}{(x^2 - 4)^2}
$$

Factor numerator:

$$
3(x^2 - 6x + 4)
$$

Solve numerator zero:

\[
x^2 - 6x + 4 = 0 \implies x = \frac{6 \pm \sqrt{36 - 16}}{2} = \frac{6 \pm \sqrt{20

Frequently Asked Questions

What are the key steps to analyze a rational function comprehensively?

The key steps include identifying the domain, finding intercepts, determining asymptotes, analyzing end behavior, studying the function's increasing/decreasing intervals, examining concavity, and sketching the graph accordingly.

How do you find the vertical and horizontal asymptotes of a rational function?

Vertical asymptotes are found by setting the denominator equal to zero and solving for x, provided the numerator is not zero at those points. Horizontal asymptotes are determined by comparing the degrees of numerator and denominator: if degrees are equal, it's the ratio of leading coefficients; if numerator degree is less, y=0; if numerator degree is greater, no horizontal asymptote.

What is the process to find the intercepts of a rational function?

To find the x-intercepts, set the numerator equal to zero and solve for x. To find the y-intercept, evaluate the function at x=0, provided the function is defined at that point.

How can you determine the end behavior of a rational function?

End behavior is analyzed by examining the degrees of numerator and denominator. For large |x|, the dominant terms dictate whether the function approaches a finite limit or grows without bound, indicating horizontal or oblique asymptotes.

What role do critical points play in the full analysis of a rational function?

Critical points, found by setting the derivative to zero, identify local maxima and minima, helping understand the function's increasing/decreasing intervals and overall shape.

How do you determine the concavity and points of inflection for a rational function?

Calculate the second derivative of the function. Points where the second derivative equals zero or is undefined are potential inflection points. Analyze the sign change of the second derivative to confirm concavity changes.

What is the importance of sketching the graph after full analysis, and what features should be included?

Sketching provides a visual understanding of the function's behavior, including intercepts, asymptotes, increasing/decreasing intervals, concavity, and key points. It helps in comprehending the overall shape and behavior of the rational function.

Can you explain how to handle holes in the graph of a rational function during analysis?

Holes occur where a factor cancels out in numerator and denominator, resulting in a removable discontinuity. To analyze, factor both numerator and denominator, identify canceled factors, and evaluate the limit approaching the hole to understand the function's behavior there.

Additional Resources

Use This Rational Function And Complete A Full Analysis According To The Listed Instructions

In the realm of algebra and calculus, rational functions serve as essential tools for understanding various mathematical phenomena. Use this rational function and complete a full analysis according to the listed instructions to deepen your understanding of how these functions behave, their properties, and their applications. Whether you're a student preparing for exams or a professional analyzing complex models, mastering the analysis of rational functions is crucial. This guide walks you through a comprehensive approach, step-by-step, to analyze a given rational function thoroughly.

Understanding Rational Functions

Before diving into the analysis, it's important to grasp what a rational function is. A rational function is any function that can be expressed as the ratio of two polynomials:

$$
f(x) = \frac{P(x)}{Q(x)}
$$

where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$.

Why are rational functions important? They model real-world scenarios involving ratios, such as rates, proportions, and inverse relationships. Their behavior near asymptotes, zeros, and at infinity reveals much about the underlying system they represent.

Step 1: Write the Rational Function

The first step is to identify or write down the specific rational function you will analyze. For illustration, let's consider the example:

$$
f(x) = \frac{2x^2 - 3x + 1}{x^2 - 4}
$$

This function is a ratio of quadratic polynomials, which makes it rich for analysis.

Step 2: Domain of the Function

The domain consists of all real numbers x for which $f(x)$ is defined. Since division by zero is undefined, set the denominator $Q(x)$ not equal to zero:

$$
x^2 - 4 \neq 0
$$
$$
x^2 \neq 4
$$
$$
x \neq \pm 2
$$

Domain:

$$
(-\infty, -2) \cup (-2, 2) \cup (2, \infty)
$$

Note: The points $x = \pm 2$ are excluded from the domain because they cause vertical asymptotes.

Step 3: Find the Zeros (x-intercepts)

Zeros of the function occur where $f(x) = 0$, i.e., when the numerator equals zero:

$$
2x^2 - 3x + 1 = 0
$$

Solve this quadratic:

$$
\text{Discriminant } D = (-3)^2 - 4 \times 2 \times 1 = 9 - 8 = 1
$$

$$
x = \frac{3 \pm \sqrt{1}}{2 \times 2} = \frac{3 \pm 1}{4}
$$

- When $+ \sqrt{1}$:

$$
x = \frac{3 + 1}{4} = \frac{4}{4} = 1
$$

- When $- \sqrt{1}$:

$$

x = \frac{3 - 1}{4} = \frac{2}{4} = \frac{1}{2}
\]

Zeros:

\[
x = \frac{1}{2} \quad \text{and} \quad x = 1
\]

x-intercepts: \(\left(\frac{1}{2}, 0\right) \) and \((1, 0) \).

Step 4: Find the y-intercept

Set \(x = 0 \):

\[
f(0) = \frac{2(0)^2 - 3(0) + 1}{(0)^2 - 4} = \frac{1}{-4} = -\frac{1}{4}
\]

Y-intercept: \((0, -\frac{1}{4}) \).

Step 5: Vertical Asymptotes

Vertical asymptotes occur where the denominator is zero but the numerator isn't zero at those points:

\[
x^2 - 4 = 0 \Rightarrow x = \pm 2
\]

Check numerator at \(x = \pm 2 \):

- \(x = 2 \):

\[
2(2)^2 - 3(2) + 1 = 8 - 6 + 1 = 3 \neq 0
\]

- \(x = -2 \):

\[
2(-2)^2 - 3(-2) + 1 = 8 + 6 + 1 = 15 \neq 0
\]

Conclusion: The vertical asymptotes are at:

\[

$$
x = -2 \quad \text{and} \quad x = 2
$$

Step 6: Horizontal or Oblique Asymptotes

To analyze the end behavior:

- Degree of numerator: 2
- Degree of denominator: 2

Since degrees are equal, the horizontal asymptote is given by the ratio of leading coefficients:

$$
\boxed{y = \frac{2}{1} = 2}
$$

Note: No oblique asymptote exists because degrees are equal.

Step 7: Analyze Behavior Near Asymptotes and at Infinity

Near Vertical Asymptotes:

- As $x \to 2^-$:

- Numerator at $x \to 2^-$:

$$
2(2)^2 - 3(2) + 1 = 3
$$

- Denominator approaches zero from negative side:

$$
x^2 - 4 \to 0^- \Rightarrow \text{approaches 0 from below}
$$

- The function tends to:

$$
f(x) \to -\infty
$$

- As $x \to 2^+$:

- Denominator approaches zero from positive:

\[
x^2 - 4 \to 0^+
\]

- Function tends to:

\[
f(x) \to +\infty
\]

Similarly, analyze at \(x = -2 \):

- As \(x \to -2^- \):

- Denominator:

\[
(-2)^2 - 4 = 4 - 4 = 0^-
\]

- Numerator:

\[
2(4) - 3(-2) + 1 = 8 + 6 + 1 = 15
\]

- Function tends to:

\[
f(x) \to -\infty
\]

- As \(x \to -2^+ \):

- Denominator:

\[
0^+
\]

- Function tends to:

\[
+\infty
\]

At Infinity:

- As \(x \to \pm \infty \):

\[

$$
f(x) \sim \frac{2x^2}{x^2} = 2
$$

which confirms the horizontal asymptote at $y = 2$.

Step 8: Critical Points and Extrema

Find critical points where $f'(x) = 0$ or undefined within the domain for potential maxima or minima.

Derivative of $f(x)$:

Using quotient rule:

$$
f'(x) = \frac{P'(x)Q(x) - P(x)Q'(x)}{[Q(x)]^2}
$$

where:

$$
P(x) = 2x^2 - 3x + 1, \quad P'(x) = 4x - 3
$$
$$
Q(x) = x^2 - 4, \quad Q'(x) = 2x
$$

Compute numerator:

$$
(4x - 3)(x^2 - 4) - (2x^2 - 3x + 1)(2x)
$$

Expand:

$$
(4x)(x^2 - 4) - 3(x^2 - 4) - 2x(2x^2 - 3x + 1)
$$

$$
= 4x^3 - 16x - 3x^2 + 12 - 4x^3 + 6x^2 - 2x
$$

Combine like terms:

- $4x^3 - 4x^3 = 0$
- $-3x^2 + 6x^2 = 3x^2$
- $-16x - 2x = -18x$
- Constant term: $+12$

Thus:

$$
f'(x) = \frac{3x^2 - 18x + 12}{(x^2 - 4)^2}
$$

Set numerator to zero:

$$
3x^2 - 18x + 12 = 0
$$

$$
x^2 - 6x + 4 = 0
$$

Discriminant:

$$
D = 36 - 16 = 20
$$

Solutions:

\[
x = \frac{6 \

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