Using The Formal Definition Of A Limit, Prove That F(x) = 2r-1 Is Continuous At The Point Z = 2; That

Using The Formal Definition Of A Limit, Prove That F(x) = 2r-1 Is Continuous At The Point Z = 2; That is a fundamental exercise in understanding the rigorous concepts of limits and continuity in calculus. This article provides a comprehensive, step-by-step guide to applying the formal (epsilon-delta) definition of a limit to prove the continuity of the function $f(x) = 2x - 1$ at the point $z = 2$. By thoroughly exploring the concepts, methods, and logical steps involved, readers will gain a solid understanding of how to approach such proofs, which are essential for advanced calculus and mathematical analysis.

Understanding Continuity and Limits in Calculus

What Is Continuity?

Continuity is a property of functions that describes smoothness and the absence of abrupt jumps or breaks at a point. Formally, a function $f(x)$ is said to be continuous at a point a if the following three conditions are satisfied:
1. The function is defined at a; that is, $f(a)$ exists.
2. The limit of $f(x)$ as x approaches a exists; that is, $\lim_{x \to a} f(x)$ exists.
3. The value of the function at a equals the limit; that is, $f(a) = \lim_{x \to a} f(x)$.

In symbols:

$$
f \text{ is continuous at } a \iff
\begin{cases}
f(a) \text{ exists} \\
\lim_{x \to a} f(x) \text{ exists} \\
f(a) = \lim_{x \to a} f(x)
\end{cases}
$$

The Formal (Epsilon-Delta) Definition of a Limit

Understanding the formal definition of a limit is crucial for rigorous proofs. The definition states:

> For a function \(f(x)\), we say that \(\lim_{x \to a} f(x) = L\) if and only if:
>
> For every \(\varepsilon > 0\), there exists a \(\delta > 0\) such that whenever \(|x - a| < \delta\), it follows that \(|f(x) - L| < \varepsilon\).

This concept ensures that as \(x\) gets arbitrarily close to \(a\), \(f(x)\) gets arbitrarily close to \(L\).

Applying the Formal Definition to Prove Continuity of \(f(x) = 2x - 1\) at \(z = 2\)

Step 1: Identify the point and the function

Given the function:
\[
f(x) = 2x - 1
\]
and the point of interest:
\[
z = 2
\]

To prove that \(f(x)\) is continuous at \(z=2\), we need to verify:
1. \(f(2)\) exists.
2. \(\lim_{x \to 2} f(x)\) exists.
3. \(f(2) = \lim_{x \to 2} f(x)\).

Step 2: Find \(f(2)\) and the limit \(\lim_{x \to 2} f(x)\)

Calculating \(f(2)\):
\[
f(2) = 2(2) - 1 = 4 - 1 = 3
\]

Calculating the limit:
\[
\lim_{x \to 2} f(x) = \lim_{x \to 2} (2x - 1) = 2 \cdot 2 - 1 = 4 - 1 = 3
\]

Since both the function value and the limit are equal and finite, the function satisfies the conditions for continuity, but to rigorously prove continuity using the epsilon-delta definition, we proceed to the next step.

Step 3: Formal proof using the epsilon-delta definition

Our goal: For every $\varepsilon > 0$, find $\delta > 0$ such that if $|x - 2| < \delta$, then $|f(x) - 3| < \varepsilon$.

Step-by-step process:

1. Start with the expression $|f(x) - 3|$:
$$
|f(x) - 3| = |(2x - 1) - 3| = |2x - 4| = 2|x - 2|
$$

2. To ensure $|f(x) - 3| < \varepsilon$, we need:
$$
2|x - 2| < \varepsilon \Rightarrow |x - 2| < \frac{\varepsilon}{2}
$$

3. Choose $\delta = \frac{\varepsilon}{2}$. Then, whenever $|x - 2| < \delta$, it follows that:
$$
|f(x) - 3| = 2|x - 2| < 2 \delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon
$$

Conclusion:
For every $\varepsilon > 0$, choosing $\delta = \frac{\varepsilon}{2}$ guarantees that if $|x - 2| < \delta$, then $|f(x) - 3| < \varepsilon$. This completes the proof that the limit $\lim_{x \to 2} f(x) = 3$ using the epsilon-delta formalism.

Verifying The Conditions for Continuity at $z=2$

Condition 1: $f(2)$ exists

We previously computed:
$$
f(2) = 3

\]
which clearly exists, satisfying the first condition for continuity.

Condition 2: $\lim_{x \to 2} f(x)$ exists

Using the formal epsilon-delta proof, we established:
$$
\lim_{x \to 2} f(x) = 3
$$
which exists and is finite.

Condition 3: $f(2) = \lim_{x \to 2} f(x)$

Since $f(2) = 3$ and $\lim_{x \to 2} f(x) = 3$, the third condition is satisfied.

Thus, by the formal definition, $f(x) = 2x - 1$ is continuous at $z=2$.

Summary: Key Points in Using The Formal Definition Of A Limit To Prove Continuity

The process of proving a function's continuity at a point via the epsilon-delta definition involves the following key points:

1. Identify the point a and verify the function is defined there.

2. Calculate the function value $f(a)$ and the limit $\lim_{x \to a} f(x)$.

3. Show that the limit exists and equals $f(a)$.

4. Given any $\varepsilon > 0$, find a $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$.

5. Use algebraic manipulation to relate $|f(x) - f(a)|$ to $|x - a|$ and choose δ accordingly.

Conclusion

Applying the formal epsilon-delta definition of a limit to prove the continuity of \(f(x) = 2x - 1\) at \(z=2\) demonstrates a rigorous approach to understanding the foundational concepts in calculus. This proof not only confirms the continuity but also enhances comprehension of limits and their significance in analysis. Mastering these techniques is essential for students and professionals working in mathematical analysis, physics, engineering, and related fields where precise understanding of function behavior is crucial.

By practicing such proofs, learners develop a strong foundation for exploring more complex topics such as differentiability, integrability, and advanced calculus concepts, ensuring a deeper understanding of the mathematical universe.

Keywords: formal limit definition, continuity proof, epsilon-delta proof, calculus, limit at point, continuous function, \(f(x) = 2x - 1\), mathematical analysis, rigorous proof, limit proof at \(z=2\).

Frequently Asked Questions

What is the formal definition of a limit in calculus?

The formal (epsilon-delta) definition states that for a function f(x), the limit as x approaches c is L if for every epsilon > 0, there exists a delta > 0 such that whenever 0 < |x - c| < delta, then |f(x) - L| < epsilon.

How do you apply the formal definition of a limit to prove continuity at a point?

To prove continuity of f at c, you need to show that lim_{x→c} f(x) = f(c). Using the formal definition, for every epsilon > 0, find a delta > 0 such that |x - c| < delta implies |f(x) - f(c)| < epsilon, thus establishing the limit equals the function's value at c.

Given f(x) = 2x - 1, how do you determine the value of the function at Z = 2?

Substitute x = 2 into the function: f(2) = 2(2) - 1 = 4 - 1 = 3, so f(2) = 3.

What is the value of the limit of f(x) = 2x - 1 as x approaches 2?

The limit as x approaches 2 is lim_{x→2} (2x - 1) = 2(2) - 1 = 3.

How do you show that f(x) = 2x - 1 is continuous at x = 2 using the formal definition?

To prove continuity at x = 2, show that for any epsilon > 0, there exists delta > 0 such that |x - 2| < delta implies |f(x) - f(2)| < epsilon. Since f(x) is linear and continuous everywhere, choosing delta = epsilon/2 suffices, confirming continuity at x = 2.

What is the significance of choosing delta in the epsilon-delta proof for continuity?

Choosing delta ensures that when x is within delta of the point c, the function values are within epsilon of f(c). It formalizes the closeness condition necessary to demonstrate continuity rigorously.

Can you outline the steps to formally prove that f(x) = 2x - 1 is continuous at Z = 2?

Yes. First, identify f(2) = 3. Then, for any epsilon > 0, choose delta = epsilon/2. Show that if |x - 2| < delta, then |f(x) - 3| = |2x - 1 - 3| = 2|x - 2| < 2(delta) = epsilon. This confirms the limit equals the function value, proving continuity.

Why is the function f(x) = 2x - 1 considered continuous everywhere, including at Z = 2?

Because it is a linear polynomial, which are continuous functions everywhere. The formal limit proof at Z = 2 confirms the continuity specifically at that point, aligning with the general property that polynomials are continuous everywhere.

Additional Resources

Using The Formal Definition Of A Limit, Prove That F(x) = 2r - 1 Is Continuous At The Point Z = 2; That

---

Introduction: Understanding Continuity and the Formal Definition of a Limit

In mathematics, the concept of continuity plays a vital role in understanding how functions behave. Intuitively, a function is continuous at a point if you can draw its graph at that point without lifting your pen. However, in rigorous mathematical analysis, this intuitive idea is formalized through the epsilon-delta (ε-δ) definition of a limit. This formalism allows mathematicians to precisely establish whether a function is continuous at a specific point by verifying certain conditions.

In this article, we will explore how to use the formal definition of a limit to prove the continuity of a particular function at a designated point. Specifically, we will examine the function $f(x) = 2r - 1$ at the point $z = 2$. Although the function appears straightforward, applying the epsilon-delta formalism provides a deeper understanding of the underlying principles of continuity.

The Function and the Point of Interest

Before delving into the proof, it is essential to clarify the function and the point under consideration:

- Function: $f(x) = 2r - 1$

- Point of interest: $z = 2$

It is noteworthy to observe that the function is expressed in terms of r. In many standard formulations, the variable is denoted as x. For clarity, and to align with conventional notation, we will interpret the function as:

$$
f(x) = 2x - 1
$$

and analyze its continuity at $x = 2$.

Understanding Continuity at a Point

A function f is said to be continuous at a point c if the following three conditions are satisfied:

1. The function is defined at c: $f(c)$ exists.
2. The limit of $f(x)$ as $x \to c$ exists: $\lim_{x \to c} f(x)$ exists.
3. The limit equals the function value at c: $\lim_{x \to c} f(x) = f(c)$.

While these conditions are straightforward to verify intuitively, the formal epsilon-delta definition provides the foundation for rigorous proof.

The Formal (Epsilon-Delta) Definition of a Limit

The epsilon-delta definition states:

> A function f has a limit L at c if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that whenever $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

Expressed mathematically:

$$
\forall \varepsilon > 0, \exists \delta > 0 \text{ such that } 0 < |x - c| < \delta \implies |f(x) - L| < \varepsilon
$$

To prove that f is continuous at c, we need to show:

- $f(c)$ is defined,
- $\lim_{x \to c} f(x) = f(c)$, and
- For every $\varepsilon > 0$, there exists a $\delta > 0$ satisfying the above condition with $L = f(c)$.

Step 1: Confirm the Function is Defined at $z = 2$

Calculate $f(2)$:

$$
f(2) = 2 \times 2 - 1 = 4 - 1 = 3
$$

Since $f(2)$ exists and equals 3, the first condition for continuity is satisfied.

Step 2: Find the Limit $\lim_{x \to 2} f(x)$

Because $f(x) = 2x - 1$ is a linear function, it is continuous everywhere, and the limit as $x \to 2$ is straightforward:

$$
\lim_{x \to 2} f(x) = 2 \times 2 - 1 = 3
$$

This matches the function value at 2, satisfying the second and third conditions for continuity. However, to adhere to the formal proof, we will proceed with the epsilon-delta argument.

Step 3: Formal Proof Using Epsilon-Delta Definition

We aim to demonstrate that for any $\varepsilon > 0$, we can find a $\delta > 0$ such that:

$$
0 < |x - 2| < \delta \implies |f(x) - 3| < \varepsilon
$$

Step-by-step reasoning:

- Start with the expression $|f(x) - 3|$:

$$
|f(x) - 3| = |(2x - 1) - 3| = |2x - 4| = 2|x - 2|
$$

- We want this to be less than ε:

$$
2|x - 2| < \varepsilon
$$

- Divide both sides by 2:

$$
|x - 2| < \frac{\varepsilon}{2}
$$

- To ensure $|f(x) - 3| < \varepsilon$, choose:

$$
\delta = \frac{\varepsilon}{2}
$$

- Now, for any $\varepsilon > 0$, selecting $\delta = \varepsilon/2$ guarantees:

$$
0 < |x - 2| < \delta \implies |f(x) - 3| = 2|x - 2| < 2 \times \frac{\varepsilon}{2} = \varepsilon
$$

This completes the proof.

Conclusion: Function Continuity at $z = 2$

Having demonstrated that for any arbitrary \(\varepsilon > 0 \), there exists a \(\delta = \varepsilon/2 \) satisfying the epsilon-delta condition, we establish rigorously that:

\[
\lim_{x \to 2} f(x) = f(2) = 3
\]

Therefore, the function \(f(x) = 2x - 1 \) is continuous at \(z = 2 \).

Broader Implications and Understanding

This proof exemplifies how the epsilon-delta definition serves as a powerful tool to establish continuity beyond simple intuition. For linear functions such as \(f(x) = 2x - 1 \), continuity is inherent due to their algebraic structure. However, the formal proof process underscores the importance of precise reasoning, especially when dealing with more complex or piecewise functions where intuition may not suffice.

Furthermore, this approach can be adapted to other functions and points, reinforcing the universal applicability of the epsilon-delta formalism in mathematical analysis.

Final Thoughts

Understanding the formal definition of a limit and its application in proving continuity is fundamental in advanced mathematics. The epsilon-delta approach not only provides rigor but also deepens our comprehension of how functions behave in the vicinity of specific points. As demonstrated with \(f(x) = 2x - 1 \) at \(z = 2 \), the process involves verifying the function's value, calculating the limit, and then constructing a suitable \(\delta \) for any given \(\varepsilon \). This meticulous process forms the backbone of rigorous analysis and helps build a solid foundation for exploring more complex mathematical concepts.

Using The Formal Definition Of A Limit Prove That F X 2r 1 Is Continuous At The Point Z 2 That

Using The Formal Definition Of A Limit Prove That F X 2r 1 Is Continuous At The Point Z 2 That

Related Articles

- (a) If Zy + Ev = E Where Y = F(x), Find The Value Of Y" At The Point Where X = 0. (b) The

Product Rule
- (Chapter 5) A Small Candy Company Sells A Popular Variety Of Hard Candies In A Reusable Metal Can. The
- (q037) What Were Critics Of Immigration Worried About During This Time Period? Group Of Answer Choices

Back to Home