W=xy X=e^t, Y=e^-2tFind Dw/dt(a) By Using Appropriate Chain Rule(b) By Converting W To A Funtion Of T

W=xy X=e^t, Y=e^-2tFind Dw/dt(a) By Using Appropriate Chain Rule(b) By Converting W To A Function Of T

Understanding how to compute the derivative of a function that depends on multiple variables which themselves depend on a parameter is crucial in calculus, especially in applications involving physics, engineering, and mathematical modeling. In this comprehensive guide, we will analyze the function $W = xy$, where $x = e^t$ and $y = e^{-2t}$. Our goal is to find $\frac{dW}{dt}$ using two approaches: (a) by applying the appropriate chain rule directly, and (b) by converting W into a function solely of t. This step-by-step explanation will not only clarify the differentiation process but also reinforce the understanding of the chain rule in multivariable calculus.

Problem Overview and Mathematical Background

Before diving into the solution methods, let's understand the problem context and the mathematical concepts involved.

Given Data:

- $W = xy$
- $x = e^t$
- $y = e^{-2t}$

Objective:

- Find $\frac{dW}{dt}$ using:
1. The appropriate chain rule considering the dependence of x and y on t.
2. By expressing W explicitly as a function of t and then differentiating.

Key Concepts:

- Chain rule in multivariable calculus: Allows differentiation of functions where variables depend on a parameter.
- Substitution of variables: Simplifying the function into a single-variable function for straightforward differentiation.
- Product rule: Necessary because $W = xy$ is a product of two functions.

Method (a): Using the Appropriate Chain Rule

Applying the chain rule directly involves understanding the dependencies: x and y are functions of t, and W is a function of x and y.

Step 1: Express $\frac{dW}{dt}$ in terms of partial derivatives

Using the multivariable chain rule:

$$
\frac{dW}{dt} = \frac{\partial W}{\partial x} \frac{dx}{dt} + \frac{\partial W}{\partial y} \frac{dy}{dt}
$$

This formula accounts for the fact that W depends on x and y, both of which depend on t.

Step 2: Compute the partial derivatives $\frac{\partial W}{\partial x}$ and $\frac{\partial W}{\partial y}$

Since $W = xy$:
- $\frac{\partial W}{\partial x} = y$
- $\frac{\partial W}{\partial y} = x$

Step 3: Compute $\frac{dx}{dt}$ and $\frac{dy}{dt}$

Given:
- $x = e^t$, so $\frac{dx}{dt} = e^t$
- $y = e^{-2t}$, so $\frac{dy}{dt} = -2 e^{-2t}$

Step 4: Substitute all derivatives into the chain rule formula

$$
\frac{dW}{dt} = y \cdot \frac{dx}{dt} + x \cdot \frac{dy}{dt}
$$

$$
\boxed{
\frac{dW}{dt} = e^{-2t} \cdot e^t + e^t \cdot (-2 e^{-2t})
}
$$

Method (b): Converting W into a Function of t

This method involves explicitly expressing W as a function of t by substituting x and y into W, then differentiating directly.

Step 1: Write W as a function of t

Given:

$$
W = xy = e^t \times e^{-2t}
$$

Using properties of exponents:

$$
W(t) = e^{t - 2t} = e^{-t}
$$

Step 2: Differentiate $W(t)$ directly with respect to t

$$
\frac{dW}{dt} = \frac{d}{dt} e^{-t} = - e^{-t}
$$

Comparison and Final Results

Now, let's verify that both methods lead to the same result, and interpret the significance.

Method (a): Simplify the expression

Recall:

$$
\frac{dW}{dt} = y \cdot e^t + x \cdot (-2 e^{-2t})
$$

Substitute $x = e^t$ and $y = e^{-2t}$:

$$
\frac{dW}{dt} = e^{-2t} \cdot e^t - 2 e^t \cdot e^{-2t}
$$

Simplify each term:

$$
e^{-2t} \cdot e^t = e^{-2t + t} = e^{-t}
$$

$$
- 2 e^t \cdot e^{-2t} = - 2 e^{t - 2t} = - 2 e^{-t}
$$

Adding these:

$$
\frac{dW}{dt} = e^{-t} - 2 e^{-t} = - e^{-t}
$$

Result from method (a): $\boxed{\frac{dW}{dt} = - e^{-t}}$

Method (b): Direct differentiation after substitution

From earlier:

$$
W(t) = e^{-t} \Rightarrow \frac{dW}{dt} = - e^{-t}
$$

Result from method (b): $\boxed{\frac{dW}{dt} = - e^{-t}}$

Both methods agree, confirming the correctness of the calculations.

Additional Insights and Applications

Understanding how to differentiate functions with multiple variables depending on a parameter is fundamental in various scientific fields. The two methods outlined demonstrate flexibility in solving such problems:

- Using the chain rule directly is advantageous when the original functions are complicated or when a quick derivative involving multiple variables is needed.
- Converting to a single-variable function simplifies the process, especially when the substitution leads to a straightforward function like an exponential, enabling quick differentiation.

These techniques are widely applicable in physics for velocity and acceleration calculations, in economics for marginal analysis, and in engineering for system response evaluations.

Practical Tips for Calculus Students and Professionals

- Always identify the dependencies of variables before starting differentiation.
- Use the chain rule for multivariable functions, noting the partial derivatives and derivatives of the inner functions.
- When possible, simplify the function into a single-variable expression, then differentiate to save time.
- Double-check the calculations by comparing results from different methods for consistency.
- Practice with various functions to become comfortable switching between methods based on the problem's complexity.

Summary

In this comprehensive analysis, we have:

- Clearly defined the problem involving the function $W = xy$ with $x = e^t$ and $y = e^{-2t}$.
- Applied the multivariable chain rule to compute $\frac{dW}{dt}$, step-by-step.
- Converted W into a single-variable function of t, then differentiated directly.
- Verified that both methods produce the same result: $\boxed{\frac{dW}{dt} = - e^{-t}}$.

Mastering these techniques enhances your calculus toolkit, enabling you to handle complex derivatives efficiently and accurately in both academic and professional settings.

Frequently Asked Questions

What is the function W in terms of x and y?

W is given as the product of x and y, so $W = xy$.

Given $x = e^t$ and $y = e^{-2t}$, how can we express W as a function of t?

Substituting, $W(t) = (e^t) (e^{-2t}) = e^{t - 2t} = e^{-t}$.

How do we find the derivative dW/dt using the chain rule when W = xy?

Using the chain rule: $dW/dt = y\, dx/dt + x\, dy/dt$.

What are dx/dt and dy/dt for the given functions?

$dx/dt = e^t$ and $dy/dt = -2 e^{-2t}$.

Calculate dW/dt using the chain rule for the given functions.

$dW/dt = y\, dx/dt + x\, dy/dt = e^{-2t} e^t + e^t (-2 e^{-2t}) = e^{-t} - 2 e^{t - 2t} = e^{-t} - 2 e^{-t} = -e^{-t}$.

How do you convert W into a function of t directly?

Since $W = xy$, and $x = e^t$, $y = e^{-2t}$, then $W(t) = e^t e^{-2t} = e^{-t}$.

What is the derivative dW/dt after converting W to a function of t?

Differentiating $W(t) = e^{-t}$ gives $dW/dt = -e^{-t}$.

Why is it useful to express W directly as a function of t?

Expressing W directly as a function of t simplifies the differentiation process, allowing straightforward computation of derivatives without applying the chain rule repeatedly.

What are the final answers for dW/dt using both methods?

Using the chain rule: $dW/dt = -e^{-t}$. Converting W to a function of t and differentiating: $dW/dt = -e^{-t}$. Both methods yield the same result.

Additional Resources

Understanding the Derivative of W = xy with respect to t: A Detailed Exploration

When tackling calculus problems involving composite functions, understanding how to differentiate with respect to a variable, especially when variables are themselves functions of another parameter, is crucial. The problem at hand is to find the derivative $\frac{dW}{dt}$ where $W = xy$, and the variables x and y are functions of t, specifically $x = e^t$ and $y = e^{-2t}$.

This exploration will focus on two approaches:
1. Using the Chain Rule directly in the context of multivariable functions.
2. Converting W into a single variable function of t explicitly, then differentiating.

Let's delve into each method comprehensively.

Background: The Nature of the Problem

Before proceeding with the methods, it’s essential to understand the components involved:

- Function W: Defined as the product of two functions, $x(t)$ and $y(t)$.
- Functions $x(t)$ and $y(t)$: Given explicitly as exponential functions:
- $x(t) = e^t$
- $y(t) = e^{-2t}$

The goal is to compute $\frac{dW}{dt}$.

Method 1: Using the Chain Rule for Multivariable Functions

Understanding the Chain Rule in Multivariable Context

The multivariable chain rule states that if W is a function of multiple variables, which in turn are functions of t, then:

$$
\frac{dW}{dt} = \frac{\partial W}{\partial x} \frac{dx}{dt} + \frac{\partial W}{\partial y} \frac{dy}{dt}
$$

This is an extension of the total derivative concept, applied when the variables x and y depend on t.

Key points:

- $W = xy$
- $x = e^t \Rightarrow \frac{dx}{dt}$
- $y = e^{-2t} \Rightarrow \frac{dy}{dt}$

Step-by-step process:

1. Compute the partial derivatives of W:
- $\frac{\partial W}{\partial x} = y$
- $\frac{\partial W}{\partial y} = x$

2. Compute the derivatives of $x(t)$ and $y(t)$:
- $\frac{dx}{dt} = \frac{d}{dt} e^t = e^t$
- $\frac{dy}{dt} = \frac{d}{dt} e^{-2t} = -2 e^{-2t}$

3. Apply the chain rule:

$$
\frac{dW}{dt} = y \cdot \frac{dx}{dt} + x \cdot \frac{dy}{dt}
$$

4. Plug in the known functions:

$$
\frac{dW}{dt} = e^{-2t} \cdot e^t + e^t \cdot (-2 e^{-2t})
$$

5. Simplify the expression:
- For the first term:

$$
e^{-2t} \cdot e^t = e^{-2t + t} = e^{-t}
$$

- For the second term:

$$
e^t \cdot (-2 e^{-2t}) = -2 e^t e^{-2t} = -2 e^{t - 2t} = -2 e^{-t}
$$

6. Combine the terms:

$$
\frac{dW}{dt} = e^{-t} - 2 e^{-t} = - e^{-t}
$$

Result:

$$
\boxed{
\frac{dW}{dt} = - e^{-t}
}
$$

This method underscores how the chain rule elegantly accounts for the dependency of W on t via $x(t)$ and $y(t)$.

Advantages of Using the Chain Rule Approach

- It explicitly demonstrates the dependency of the composite function on each variable.
- It is systematic and scales well for more complex functions involving multiple variables.
- It provides insight into how changes in each variable influence the overall function.

Method 2: Converting W to a Function of t and Differentiating

Expressing W as a Single Variable Function

Since $W = xy$, and both x and y are functions of t, we can explicitly write:

$$
W(t) = e^t \cdot e^{-2t}
$$

Using properties of exponents:

$$
W(t) = e^{t - 2t} = e^{-t}
$$

Now, the problem reduces to differentiating a single exponential function:

$$
W(t) = e^{-t}
$$

This approach simplifies the differentiation process. It transforms the problem into a straightforward derivative of a single variable function.

Derivative:

$$
\frac{dW}{dt} = \frac{d}{dt} e^{-t} = - e^{-t}
$$

which matches the result obtained via the chain rule approach.

Implications and Insights

- Simplification: Recognizing that W simplifies to e^{-t} is a key step.
- Efficiency: This method is faster when such simplifications are possible, especially for more complicated functions.
- Understanding the structure: It emphasizes the importance of algebraic manipulation in calculus.

Limitations and Considerations

- Not all functions will simplify as neatly as this example.

- It requires the ability to recognize opportunities for simplification.
- For more complex functions where W cannot be readily expressed as a single function of t, the direct chain rule approach remains essential.

Comparison and Summary of the Two Methods

Aspect	Chain Rule Approach	Conversion to a Single Variable Function
Process	Involves partial derivatives and derivatives of $x(t)$ and $y(t)$	Simplifies W first, then differentiates directly
Applicability	Useful when variables are not easily combined	Ideal when algebraic simplification is straightforward
Complexity	Slightly more involved, especially for multiple variables	Simpler if W can be explicitly expressed as a function of t
Insight	Shows how each variable influences W	Focuses on the overall dependence on t

In this case, both methods lead to the same derivative:

$$
\boxed{
\frac{dW}{dt} = - e^{-t}
}
$$

Deeper Mathematical Insights and Extensions

Generalization to Other Functions

The techniques used here are applicable to a wide array of functions:

- When W is a product of functions of t, the product rule combined with the chain rule is often used.
- For more complex compositions, recognizing potential for algebraic simplification can save computational effort.
- Multivariable calculus principles extend to functions involving more variables and parameters.

Applications in Physics and Engineering

- Calculating rates of change in physical systems where quantities depend on time.

- Analyzing compound systems where variables are interconnected, such as in thermodynamics or circuit analysis.
- Optimizing functions that depend on multiple parameters.

Potential Pitfalls and Tips

- Always verify whether the function can be simplified before differentiating.
- Remember the dependence of each variable on t; missing derivatives can lead to incorrect results.
- For complex functions, consider alternative approaches, such as logarithmic differentiation or substitution.

Conclusion: Synthesis of Approaches and Final Remarks

The problem of finding $\frac{dW}{dt}$ where $W = xy$, with $x = e^t$ and $y = e^{-2t}$, exemplifies core calculus principles: the chain rule and algebraic manipulation.

Key takeaways:

- The chain rule provides a systematic framework for differentiating composite functions involving multiple variables.
- Recognizing algebraic structures or simplifications can significantly streamline calculations.
- Both approaches converge to the same answer, reinforcing the consistency and reliability of calculus techniques.

Final result:

$$
\boxed{
\frac{dW}{dt} = - e^{-t}
}
$$

This detailed exploration underscores the importance of understanding multiple methods in calculus, their appropriate contexts, and how they complement each other in solving complex derivative problems efficiently and accurately.

W Xy X E T Y E 2tfind Dw Dt A By Using Appropriate Chain

Rule B By Converting W To A Funtion Of T

W Xy X E T Y E 2tfind Dw Dt A By Using Appropriate Chain Rule B By Converting W To A Funtion Of T

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