What Is An Equation Of The Line That Passes Through The Point (2,-3) And Is Perpendicular To The Line

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Understanding how to find the equation of a line that passes through a specific point and is perpendicular to another line is a fundamental concept in coordinate geometry. This topic often appears in algebra and calculus, and mastering it enhances your ability to analyze geometric relationships, solve problems involving slopes, and interpret the spatial orientation of lines. In this article, we will explore in detail what an equation of such a line entails, the mathematical principles behind it, and step-by-step methods to determine it.

Introduction to Lines and Their Equations

Before diving into perpendicular lines and their equations, it’s essential to understand the basics of lines in the coordinate plane.

The Slope-Intercept Form

The most common way to express the equation of a line in a Cartesian plane is the slope-intercept form:

$$
y = mx + b
$$

where:
- m is the slope of the line,
- b is the y-intercept, the point where the line crosses the y-axis.

This form provides a straightforward way to graph and analyze lines.

Understanding the Slope

The slope m describes the steepness and direction of the line:
- If $m > 0$, the line rises from left to right.
- If $m < 0$, the line falls from left to right.
- If $m = 0$, the line is horizontal.

- If the slope is undefined (vertical line), the equation is of the form $x = c$.

Perpendicular Lines and Their Slopes

Perpendicular lines are lines that intersect at a right angle (90 degrees). The relationship between their slopes is a key to finding the equation of a perpendicular line.

Relationship Between Slopes of Perpendicular Lines

If two lines are perpendicular, their slopes m_1 and m_2 satisfy:

$$
m_1 \times m_2 = -1
$$

or equivalently,

$$
m_2 = -\frac{1}{m_1}
$$

This means:
- The slope of the line perpendicular to a given line with slope m_1 is the negative reciprocal of m_1.

Implications for Finding the Perpendicular Line

Given a line with a known slope m_1, the slope of a line perpendicular to it is:

$$
m_\perp = -\frac{1}{m_1}
$$

Using this, once you determine the original line’s slope, you can find the perpendicular slope and then derive the equation of the new line passing through a specific point.

Step-by-Step Process to Find the Equation of

the Perpendicular Line

Given the point $(2, -3)$ and the original line, the process involves several key steps:

1. Find the Slope of the Original Line

To determine the perpendicular line, you need the slope of the original line. Depending on the problem:

- If the original line’s equation is known, extract its slope m_1.
- If only the line’s equation is given, convert it into slope-intercept form to identify m_1.

Example: Suppose the original line is $y = 4x + 1$. Here, $m_1 = 4$.

2. Calculate the Slope of the Perpendicular Line

Using the relationship:

$$
m_\perp = -\frac{1}{m_1}
$$

Continuing the example:

$$
m_\perp = -\frac{1}{4}
$$

This is the slope of the line passing through $(2, -3)$ and perpendicular to the original line.

3. Use the Point-Slope Form to Write the Equation

The point-slope form of a line is:

$$
y - y_1 = m(x - x_1)
$$

where (x_1, y_1) is a point on the line, and m is the slope.

Applying to our point $(2, -3)$:

$$
y - (-3) = -\frac{1}{4}(x - 2)
$$

which simplifies to:

$$
y + 3 = -\frac{1}{4}x + \frac{1}{2}
$$

4. Convert to Slope-Intercept Form (if needed)

Subtract 3 from both sides:

$$
y = -\frac{1}{4}x + \frac{1}{2} - 3
$$

$$
y = -\frac{1}{4}x - \frac{5}{2}
$$

This is the equation of the line passing through $(2, -3)$ and perpendicular to the original line $y = 4x + 1$.

Special Cases and Additional Considerations

While the process above covers typical scenarios, some special cases require additional attention.

Vertical and Horizontal Lines

- Original line is horizontal: $y = c$

The perpendicular line is vertical, which has an equation of the form $x = c'$. To find this line passing through $(2, -3)$, simply write:

$$
x = 2
$$

- Original line is vertical: $x = c$

The perpendicular line is horizontal with an equation:

$$
y = -3
$$

passing through $(2, -3)$.

Line with Zero or Undefined Slope

- Zero slope (horizontal line): slope $m = 0$. Its perpendicular line is vertical.
- Undefined slope (vertical line): slope is undefined; the perpendicular is horizontal.

Real-World Applications of Perpendicular Lines

Understanding the equations of perpendicular lines has practical applications across various fields:

- **Architecture and Engineering:** Designing structures with perpendicular walls and supports.
- **Navigation:** Calculating bearings and directions that are at right angles.
- **Computer Graphics:** Rendering scenes with perpendicular axes and components.
- **Physics:** Analyzing forces acting at right angles.
- **Mathematics Education:** Teaching concepts of orthogonality and geometric relationships.

Practice Problems to Reinforce Learning

To deepen understanding, try solving these problems:

1. Find the equation of the line passing through $(2, -3)$ and perpendicular to $y = 2x + 5$.
2. Given the line $y = -\frac{1}{3}x + 4$, determine the equation of the line passing through $(2, -3)$ and perpendicular to it.

3. Write the equation of a line passing through $(2, -3)$ that is perpendicular to the line $x = 5$. (Hint: Recognize the type of line and its perpendicular counterpart.)

Solutions:

1. Original slope $m_1 = 2$, perpendicular slope $m_\perp = -\frac{1}{2}$:

$$
y + 3 = -\frac{1}{2}(x - 2)
$$

Simplify:

$$
y + 3 = -\frac{1}{2}x + 1
$$

$$
y = -\frac{1}{2}x - 2
$$

2. Original slope $m_1 = -\frac{1}{3}$, perpendicular slope:

$$
m_\perp = -\frac{1}{-\frac{1}{3}} = 3
$$

Equation:

$$
y + 3 = 3(x - 2)
$$

$$
y + 3 = 3x - 6
$$

$$
y = 3x - 9
$$

3. The original line $x = 5$ is vertical; its perpendicular is horizontal:

$$
y = -3
$$

Conclusion

Finding the equation of a line passing through a specific point and perpendicular to another line involves understanding the relationship between slopes of perpendicular lines, converting equations into appropriate forms, and applying the point-slope formula. Whether dealing with standard lines, vertical/horizontal lines, or more complex scenarios, the core principles remain consistent.

Mastering these techniques provides a strong foundation in coordinate geometry, enabling you to analyze and solve a wide range of mathematical and real-world problems with confidence. Remember to carefully identify slopes, consider special cases, and verify your solutions for accuracy. With practice, determining perpendicular lines and their equations will become an intuitive and valuable skill in your mathematical toolkit.

Frequently Asked Questions

How do I find the equation of a line passing through the point (2, -3) perpendicular to a given line?

First, determine the slope of the given line, then find the negative reciprocal to get the perpendicular slope. Use the point-slope form with point (2, -3) and this slope to write the equation.

What is the formula to find the equation of a line perpendicular to another line passing through a specific point?

Use the point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point (2, -3) and m is the negative reciprocal of the original line's slope.

If the original line has an equation y = 4x + 1, what is the equation of the line passing through (2, -3) and perpendicular to it?

The slope of the original line is 4, so the perpendicular slope is -1/4. Plugging into point-slope form: $y + 3 = -1/4(x - 2)$. Simplifying, the equation is $y = -1/4\ x - 5/2$.

Can I find the equation of the perpendicular line without knowing the original line's equation?

No, you need the original line's slope to determine the perpendicular slope.

Without it, you cannot uniquely define the perpendicular line passing through the point.

What are the steps to write the equation of a line passing through (2, -3) and perpendicular to a given line?

1. Find the slope of the given line. 2. Calculate the negative reciprocal of that slope. 3. Use the point-slope form with point (2, -3) and this slope. 4. Simplify to slope-intercept or other desired form.

Additional Resources

What Is An Equation Of The Line That Passes Through The Point (2,-3) And Is Perpendicular To The Line

Understanding the equations of lines is fundamental in geometry, algebra, and various applied sciences. When working with lines, one common problem involves finding the equation of a line that passes through a specific point and is perpendicular to another given line. This task combines concepts of slope, perpendicularity, and point-slope form, offering a window into the interconnected nature of algebraic and geometric principles. In this article, we will explore what it means to find such a line, how to approach the problem systematically, and the mathematical principles behind it—all in a clear, informative manner suitable for students, educators, and enthusiasts alike.

Foundations of Line Equations and Slope

Before delving into the specific problem, it’s essential to review some fundamental concepts related to lines in coordinate geometry.

Understanding the Equation of a Line

A line in the coordinate plane can be represented mathematically in various forms, with the most common being:

- Slope-Intercept Form: $y = mx + b$
where m is the slope of the line, and b is the y-intercept.

- Point-Slope Form: $y - y_1 = m(x - x_1)$
where (x_1, y_1) is a point on the line, and m is the slope.

- Standard Form: $Ax + By = C$

These forms are interchangeable, and the choice depends on the problem at hand.

The Concept of Slope

The slope (m) of a line measures its steepness and is calculated as the ratio of the change in y over the change in x between two points on the line:

$m = (y_2 - y_1) / (x_2 - x_1)$

The slope determines the direction of the line:

- Positive slope: line rises from left to right.
- Negative slope: line falls from left to right.
- Zero slope: horizontal line.
- Undefined slope: vertical line.

Perpendicular Lines and Their Slopes

Perpendicular lines are lines that intersect at a right angle (90 degrees). The relationship between their slopes is a key aspect of coordinate geometry.

Relation Between Slopes of Perpendicular Lines

If two lines are perpendicular, and their slopes are m_1 and m_2, then:

$m_1 m_2 = -1$

This means:

- If one line has slope m, the line perpendicular to it will have slope -1/m.
- A vertical line (undefined slope) is perpendicular to a horizontal line (zero slope), and vice versa.

Implications for Finding a Perpendicular Line

Given the slope of one line, its perpendicular counterpart will have a slope that is the negative reciprocal:

- For example, if the original line has a slope of 2, the perpendicular line will have a slope of -1/2.

This reciprocal relationship is crucial when constructing the equation of the perpendicular line passing through a specific point.

Step-by-Step Approach to Find the Equation of the Perpendicular Line

Let's now focus on the specific problem: finding the equation of a line passing through point (2, -3) that is perpendicular to a given line. Since the original line is not explicitly provided, we will assume it's given or derivable from context. The general approach remains consistent:

1. Identify the slope of the original line.
2. Determine the slope of the perpendicular line.
3. Use the point-slope form with the point (2, -3) and the perpendicular slope.
4. Simplify to the desired form (e.g., slope-intercept or standard form).

This systematic process ensures clarity and correctness in solving such problems.

Step 1: Find the Slope of the Original Line

Depending on the given original line, its equation could be provided directly or derived from points or other information. For example, consider the original line has the equation:

$y = 3x + 4$

Here, the slope m_1 is 3.

If the line is given in standard form or other formats, convert it to slope-intercept form to identify the slope.

Step 2: Calculate the Slope of the Perpendicular Line

Using the relation between slopes of perpendicular lines:

$m_2 = -1 / m_1$

For the example with $m_1 = 3$:

$m_2 = -1/3$

This is the slope of the line we seek, which passes through (2, -3).

Step 3: Use the Point-Slope Formula

Apply the point-slope form:

$y - y_1 = m(x - x_1)$

Plugging in the point (2, -3) and slope $m_2 = -1/3$:

$y - (-3) = -1/3 (x - 2)$

Simplify:

$y + 3 = -1/3 (x - 2)$

Step 4: Simplify and Write the Equation

Distribute the slope:

$y + 3 = -1/3 x + 2/3$

Subtract 3 from both sides to write in slope-intercept form:

$y = -1/3 x + 2/3 - 3$

Express -3 as -9/3 to combine:

$y = -1/3 x + 2/3 - 9/3$

$y = -1/3 x - 7/3$

This is the equation of the line passing through (2, -3) and perpendicular to the original line $y = 3x + 4$.

Note: If the original line is different, the process remains the same; only the initial slope calculation changes.

Special Cases and Additional Considerations

While the above steps cover most scenarios, some special cases merit attention:

Original Line is Vertical or Horizontal

- If the original line is vertical (x = a), its slope is undefined.
- The perpendicular line must then be horizontal (slope 0).
- The perpendicular line passing through (2, -3) would be y = -3.

- Conversely, if the original line is horizontal (y = b), then the perpendicular line is vertical (x = 2).

Multiple Lines and Variations

In real-world problems, multiple lines and intersecting scenarios can exist. Always clarify:

- The exact original line's equation.
- The point through which the new line must pass.
- The orientation of the lines involved.

Practical Applications and Significance

Understanding how to find the equation of a perpendicular line through a specific point has numerous applications:

- Engineering: Designing perpendicular components.
- Navigation: Calculating perpendicular paths.
- Computer Graphics: Rendering lines at right angles.
- Mathematical Modeling: Solving geometric constraints.

In educational contexts, mastering this concept strengthens foundational skills in algebra and geometry, enabling students to approach more complex problems with confidence.

Conclusion

Finding the equation of a line that passes through a given point and is perpendicular to another line combines multiple core principles of coordinate geometry. It involves understanding slopes, the relationships between perpendicular lines, and the application of point-slope form. Whether the original line's equation is known or needs to be derived, the systematic approach outlined ensures clarity and accuracy.

Mastering these techniques enhances mathematical literacy, supports problem-solving in various fields, and deepens appreciation for the elegant interconnectedness of geometric concepts. As you practice with different line equations and points, you'll develop a more intuitive grasp of how lines interact in the plane—an essential skill for anyone delving into mathematics or related disciplines.

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