What Is An Equation Of The Line That Passes Through The Point (8,-2) And Is Perpendicular To The Line

What Is An Equation Of The Line That Passes Through The Point (8,-2) And Is Perpendicular To The Line is a fundamental question in coordinate geometry that often arises in algebra and calculus. Understanding how to determine this equation involves grasping concepts such as slopes, perpendicular lines, and point-slope forms. Whether you're a student working on homework, a teacher preparing lesson plans, or a math enthusiast exploring analytic geometry, mastering this topic is essential for solving various problems involving lines and their relationships. This article provides a comprehensive guide to understanding, deriving, and applying the equation of a line passing through a specific point and perpendicular to another line, optimized for clarity and SEO.

Understanding the Basics: Lines, Slopes, and Perpendicularity

Before diving into the specific problem, it’s important to review some fundamental concepts in coordinate geometry.

What Is a Line Equation?

A line in a two-dimensional plane can be represented by an equation, typically in the form:
- Slope-Intercept Form: $y = mx + b$
- Where:
- m is the slope of the line
- b is the y-intercept

The Concept of Slope

The slope of a line indicates its steepness and direction. It is calculated as:
- $m = (\text{change in } y) / (\text{change in } x) = \Delta y / \Delta x$

For example, if a line passes through points (x_1, y_1) and (x_2, y_2), its slope is:
- $m = (y_2 - y_1) / (x_2 - x_1)$

Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at a right angle (90 degrees). A key property is:
- The slopes of two perpendicular lines are negative reciprocals of each other.
- If one line has slope m, the perpendicular line has slope $-1/m$ (provided $m \neq 0$).

This means:
- If a line has slope 2, a line perpendicular to it has slope -1/2.
- If a line is vertical (slope undefined), the perpendicular line is horizontal (slope zero).

Step-by-Step Guide to Find the Equation of the Perpendicular Line

Given a point and a line, the main task is to find the equation of the line passing through the point that is perpendicular to the given line. Here's a structured approach.

1. Identify the Slope of the Original Line

- Obtain the equation of the original line (if provided).
- Rewrite it in slope-intercept form $y = mx + b$ or identify the slope directly.

2. Determine the Slope of the Perpendicular Line

- Use the property that perpendicular slopes are negative reciprocals:
- If original slope m, then perpendicular slope $m_p = -1/m$.

3. Use the Point-Slope Form

- The point-slope form of a line's equation:
- $y - y_1 = m(x - x_1)$
- Where (x_1, y_1) is the given point (8, -2), and m is the perpendicular slope.

4. Write the Equation of the Perpendicular Line

- Substitute the point and the perpendicular slope into the point-slope form.
- Simplify to slope-intercept or standard form as needed.

Example: Finding the Equation of the Line Perpendicular to a Given Line and Passing Through (8, -2)

Suppose the original line is given by the equation:
- $3x + 4y = 7$

Let's walk through the process.

Step 1: Find the slope of the original line

- Rewrite in slope-intercept form:
- $3x + 4y = 7$
- $4y = -3x + 7$
- $y = (-3/4)x + 7/4$
- The slope m of the original line is -3/4.

Step 2: Find the slope of the perpendicular line

- Perpendicular slope $m_p = -1 / (-3/4) = 4/3$

Step 3: Use the point (8, -2) and the perpendicular slope in point-slope form

- $y - (-2) = (4/3)(x - 8)$
- Simplify:
- $y + 2 = (4/3)(x - 8)$

Step 4: Convert to slope-intercept form

- $y + 2 = (4/3)x - (4/3)8$
- $y + 2 = (4/3)x - 32/3$
- $y = (4/3)x - 32/3 - 2$
- $y = (4/3)x - 32/3 - 6/3$
- $y = (4/3)x - 38/3$

Final Equation:

$y = (4/3)x - 38/3$

This is the equation of the line passing through (8, -2) and perpendicular to the line $3x + 4y = 7$.

Special Cases and Additional Tips

When working with perpendicular lines, certain scenarios require special attention.

Horizontal and Vertical Lines

- If the original line is horizontal (slope = 0), the perpendicular line is vertical.
- Vertical line passing through (8, -2): $x = 8$.
- If the original line is vertical (slope undefined), the perpendicular line is horizontal.
- Horizontal line passing through (8, -2): $y = -2$.

Lines with Undefined or Zero Slopes

- Always check the form of the original line.
- Convert to slope-intercept form when possible.

Using the Standard Form of a Line

- Sometimes, lines are given in standard form ($Ax + By = C$).
- To find the slope: $m = -A/B$ (if $B \neq 0$).
- Follow the same steps as above.

Practical Applications of Finding Perpendicular Lines

Understanding how to find the equation of a perpendicular line has numerous practical applications:

- **Geometry and Design:** Ensuring right angles in construction or graphic design.
- **Physics:** Calculating trajectories and force components at right angles.
- **Navigation:** Determining paths perpendicular to existing routes.
- **Advanced Mathematics:** Solving systems of equations where perpendicularity is a constraint.
- **Computer Graphics:** Rendering perpendicular lines and shapes accurately.

Conclusion: Mastering the Equation of Perpendicular Lines

Finding the equation of a line that passes through a specific point and is perpendicular to another line is a crucial skill in coordinate geometry. The key steps involve understanding the original line's slope, calculating the negative reciprocal for the perpendicular slope, and applying the point-slope formula. By practicing with different types of lines—standard, vertical, or horizontal—you can develop a robust understanding of line relationships and their equations.

Whether you're tackling algebra homework, preparing for exams, or applying these concepts in real-world scenarios, mastering this process enhances your analytical skills and deepens your comprehension of geometric relationships. Remember, the core principle is that perpendicular lines have slopes that are negative reciprocals, a simple yet powerful concept that unlocks many geometric and algebraic problems.

Keywords for SEO Optimization:
- Equation of a line passing through a point
- Perpendicular line equation
- Slope of a line
- Negative reciprocal slopes
- Point-slope form
- Coordinate geometry
- Line passing through (8, -2)
- Find perpendicular line equation
- How to find line equations
- Geometry and algebra tutorial

Frequently Asked Questions

How do I find the equation of a line passing through the point (8, -2) that is perpendicular to a given line?

First, determine the slope of the given line, then find the negative reciprocal of that slope to get the perpendicular line's slope. Next, use the point-slope form with the point (8, -2) and this perpendicular slope to find the equation.

What is the significance of the negative reciprocal when finding a perpendicular line?

The negative reciprocal of a slope is used because perpendicular lines have slopes that multiply to -1, meaning their slopes are negative reciprocals of each other.

If the original line has the equation y = 3x + 4, what is the equation of the line passing through (8, -2) and perpendicular to it?

The slope of the original line is 3, so the perpendicular slope is -1/3. Using point-slope form: y + 2 = -1/3(x - 8). Simplifying, the equation is y = -1/3x + 2/3 - 2, which simplifies to y = -1/3x - 4/3.

Can I use the slope-intercept form directly to write the perpendicular line's equation?

Yes, after finding the perpendicular slope, you can substitute the point (8, -2) into the slope-intercept form y = mx + b to solve for b, thus obtaining the complete equation.

Why is it important to verify that the line is perpendicular to the original line?

Verifying ensures that the calculated line truly has a slope that is the negative reciprocal of the original line's slope, confirming the lines are perpendicular as required.

Additional Resources

Equation of the Line That Passes Through the Point (8, -2) And Is Perpendicular To The Line is a fundamental concept in coordinate geometry that involves understanding the relationships between lines, their slopes, and points through which they pass. This topic not only helps students and professionals solve geometric problems but also enhances their analytical thinking and algebraic skills. In this comprehensive review, we will explore what such an equation entails, how to derive it step-by-step, the underlying mathematical principles, and practical applications. Whether you are a student preparing for exams or a teacher explaining the concept, this article aims to clarify the process and deepen your understanding of perpendicular lines and their equations.

Understanding the Basics of Lines in Coordinate Geometry

Before diving into the specifics of the problem, it’s important to review some foundational concepts related to lines on the Cartesian plane.

Slope of a Line

The slope of a line, often denoted as m, measures its steepness and is calculated as the ratio of the change in y-coordinates to the change in x-coordinates between two points:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

This value indicates whether the line rises or falls as it moves from left to right:
- Positive slope: line rises.
- Negative slope: line falls.
- Zero slope: horizontal line.
- Undefined slope: vertical line.

Equation of a Line in Various Forms

The most common forms include:
- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$

- Standard form: $Ax + By = C$

Perpendicular Lines and Their Slopes

Two lines are perpendicular if their slopes are negative reciprocals:
$$
m_1 \times m_2 = -1
$$
For example, if one line has a slope of 2, then a line perpendicular to it has a slope of $-\frac{1}{2}$.

Determining the Equation of a Perpendicular Line Through a Given Point

The core of this problem involves several steps:
1. Find the slope of the original line.
2. Determine the slope of the perpendicular line.
3. Use the point-slope form to find the equation of the perpendicular line passing through the given point.

Let’s analyze each step carefully.

Step 1: Identify the Slope of the Original Line

The problem states: "perpendicular to the line," but does not specify which line. Usually, in such problems, the original line is given explicitly, or the context specifies it. For illustrative purposes, assume the original line is given by an equation, say:
$$
L: y = mx + c
$$
or, more generally, we know its slope m.

If the original line’s equation is known, extract its slope directly.

If the original line is not specified, the problem might be to find the equation of a line perpendicular to a given line with a known slope.

Example: Suppose the original line is $y = 3x + 4$. Its slope is $m = 3$.

Step 2: Find the Slope of the Perpendicular Line

Since perpendicular lines have slopes that are negative reciprocals:

$$
m_{perp} = -\frac{1}{m}
$$

Using our example:

$$
m_{perp} = -\frac{1}{3}
$$

Note: If the original line is vertical (slope undefined), then the perpendicular line will be horizontal (slope 0), and vice versa.

Step 3: Write the Equation of the Perpendicular Line Through (8, -2)

Now, using the point-slope form:

$$
y - y_1 = m_{perp} (x - x_1)
$$

Substitute $(x_1, y_1) = (8, -2)$ and $m_{perp} = -\frac{1}{3}$:

$$
y - (-2) = -\frac{1}{3} (x - 8)
$$

Simplify:

$$
y + 2 = -\frac{1}{3} (x - 8)
$$

Distribute:

$$
y + 2 = -\frac{1}{3} x + \frac{8}{3}
$$

Subtract 2 from both sides:

$$
y = -\frac{1}{3} x + \frac{8}{3} - 2
$$

Express 2 as $\frac{6}{3}$:

$$
y = -\frac{1}{3} x + \frac{8}{3} - \frac{6}{3} = -\frac{1}{3} x + \frac{2}{3}
$$

Final equation:

$$
\boxed{ y = -\frac{1}{3} x + \frac{2}{3} }
$$

This is the equation of the line passing through (8, -2) and perpendicular to the original line

$y = 3x + 4$.

General Approach for Any Given Line

If the original line is not explicitly provided, but the problem involves finding a line perpendicular to a given line (say, with slope m), and passing through a point (x_0, y_0), follow these steps:

1. Identify the slope of the original line, m.
2. Calculate the negative reciprocal, $m_{perp} = -1/m$.
3. Use the point-slope form with the point (x_0, y_0) and m_{perp}:
$$
y - y_0 = m_{perp}(x - x_0)
$$
4. Simplify to slope-intercept or standard form as needed.

Examples and Applications

Understanding this concept is vital in various practical contexts:

- Engineering and Design: Designing perpendicular components or pathways.
- Navigation and Mapping: Calculating routes perpendicular to existing roads.
- Physics: Analyzing forces and directions where perpendicularity is key.
- Computer Graphics: Rendering perpendicular lines for shading or modeling.

Example 1: Find the equation of the line passing through (8, -2) and perpendicular to $y = -2x + 5$.

Solution:
- Slope of original line: $m = -2$.
- Slope of perpendicular line: $m_{perp} = -1/(-2) = 1/2$.
- Equation through (8, -2):

$$
y - (-2) = \frac{1}{2}(x - 8)
$$
$$
y + 2 = \frac{1}{2}x - 4
$$
$$
y = \frac{1}{2}x - 4 - 2
$$

$$
y = \frac{1}{2}x - 6
$$

Final Equation: $y = \frac{1}{2}x - 6$.

Example 2: Find the line perpendicular to a vertical line $x = 4$ passing through (8, -2).

Solution:
- A vertical line has an undefined slope.
- The perpendicular line will be horizontal with slope 0.
- Equation passing through (8, -2):

$$
y = -2
$$

Final Equation: $y = -2$.

Features and Limitations

Features:
- Simplifies complex geometric problems into algebraic calculations.
- Uses basic slope properties, making it accessible.
- Applicable to various geometrical and real-world problems.
- Extends to three-dimensional geometry with similar principles.

Limitations:
- Requires knowledge of the original line or its slope.
- Assumes lines are in the Euclidean plane; different geometries may complicate the concept.
- Cannot determine the original line if only the perpendicular line and point are given without additional information.

Conclusion

The process of deriving the equation of a line passing through a specific point and perpendicular to another line hinges on understanding slopes and their relationships. By identifying the slope of the original line, calculating its negative reciprocal, and applying the point-slope form, you can efficiently find the required line's equation. This skill is fundamental in coordinate geometry and has widespread applications across science, engineering, and mathematics.

Mastering this concept enhances problem-solving abilities and provides a solid foundation for exploring more advanced topics such as transformations, conic sections, and vector geometry. Whether working with explicit equations or deducing unknown lines, the principles outlined here serve as a reliable guide for tackling perpendicular line problems effectively.

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