What Is An Equation Of The Line That Passes Through The Point (8,-7)(8,7) And Is Parallel To The Line

What Is An Equation Of The Line That Passes Through The Point (8,-7)(8,7) And Is Parallel To The Line

Understanding how to find the equation of a line that passes through specific points and is parallel to another line is a fundamental concept in coordinate geometry. This process involves calculating the slope of the original line, ensuring the new line maintains the same slope (since parallel lines have equal slopes), and then using point-slope or slope-intercept forms to derive the equation. In particular, when given points such as (8, -7) and (8, 7), and a line to which the new line must be parallel, a step-by-step approach helps clarify the process. This article provides comprehensive guidance on how to determine the equation of such a line, including definitions, formulas, and example calculations, all structured for clarity and SEO optimization.

Understanding the Basics: Line Equations and Parallel Lines

What Is a Line Equation?

A line equation describes a straight line in a coordinate plane. The most common forms are:

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By = C$

where:
- m is the slope of the line,
- (x_1, y_1) is a point on the line,
- A, B, C are constants.

Understanding these forms enables us to manipulate and derive equations for various lines based on given points and slopes.

What Does It Mean for Lines to Be Parallel?

Two lines are parallel if they lie in the same plane and do not intersect, no matter how far they extend. A key property of parallel lines is that they have identical slopes. Therefore, if you know the slope of one line, the parallel line must have the same slope, but possibly a different y-intercept.

Analyzing the Given Points: (8, -7) and (8, 7)

Identifying the Type of Line Formed by the Points

The points (8, -7) and (8, 7) share the same x-coordinate, $x = 8$, but have different y-coordinates. This indicates that:

- The line passing through these points is vertical.
- Its equation is simply $x = 8$.

Calculating the Slope of the Line Through (8, -7) and (8, 7)

The slope formula is:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

Plugging in our points:

$$ m = \frac{7 - (-7)}{8 - 8} = \frac{14}{0} $$

Since division by zero is undefined, the slope of the line through these points is undefined, confirming that the line is vertical.

Implication: Any line parallel to a vertical line must also be vertical, i.e., of the form x = constant.

Finding the Equation of the Parallel Line

Properties of Parallel Lines to a Vertical Line

- Since the original line $x = 8$ is vertical, any line parallel to it must also be vertical.
- The equation of such a line will be of the form $x = k$, where k is a constant.

Determining the Equation When Passing Through a Specific Point

Suppose we are asked to find the equation of a line passing through a particular point and parallel to the original line. For example, if the problem states:

> Find the equation of the line passing through point (x_0, y_0) and parallel to the line passing through (8, -7) and (8, 7).

Given the original line is x = 8, the parallel line must also be x = 8.

But what if the question involves a different point?

General Approach to Find the Equation of a Line Parallel to a Given Line

Case 1: Original Line is Vertical

- If the original line is vertical (x = a), then the parallel line is also vertical.
- The equation will be $x = x_0$, where x_0 is the x-coordinate of the point through which the parallel line passes.

Case 2: Original Line is Non-Vertical

- Find the slope m of the original line.
- Use the point-slope form with the point through which the new line passes and the slope m:

$$ y - y_1 = m(x - x_1) $$

- Simplify to slope-intercept form if necessary.

Applying the Concepts: Example Problem

Suppose the problem is:

> Find the equation of the line passing through (8, -7) and parallel to the line y = 2x + 3.

Step 1: Identify the slope of the original line

- The original line's slope: m = 2.

Step 2: Use the point-slope form with the given point (8, -7)

$$ y - (-7) = 2(x - 8) $$

$$ y + 7 = 2(x - 8) $$

Step 3: Simplify to slope-intercept form

$$y + 7 = 2x - 16$$

$$y = 2x - 23$$

Result: The equation of the line passing through (8, -7) and parallel to the original line $y = 2x + 3$ is:

$$y = 2x - 23$$

Special Considerations in Parallel Line Problems

Vertical and Horizontal Lines

- Vertical lines: x = constant.
- Horizontal lines: y = constant.

When given points with the same x-coordinate or y-coordinate:

- For vertical lines, the equation is x = the shared x-value.
- For horizontal lines, the equation is y = the shared y-value.

Finding Parallel Lines When the Original Line Is Not in Slope-Intercept Form

- Convert the given line to slope-intercept form or identify the slope directly.
- Use the identified slope to find the parallel line's equation through the given point.

Summary of Steps to Find the Equation of a Parallel Line

Here's a quick step-by-step checklist:

1. Identify the original line's slope (m).
- If the line is vertical, note that the slope is undefined.
- If the line is non-vertical, determine the slope from the equation.

2. Determine the form of the parallel line:
- Vertical line: x = constant.
- Non-vertical line: $y = mx + b$.

3. Use the given point(s):
- For non-vertical lines, use the point-slope form:

$$ y - y_1 = m(x - x_1) $$

- For vertical lines, the x-coordinate of the point gives the line's equation: $x = x_1$.

4. Solve for the desired form (if necessary):
- Convert to slope-intercept form or standard form for clarity.

5. Verify the line's properties:
- Confirm the slope matches the original line's slope.
- Check that the line passes through the specified point.

Additional Examples and Practice Problems

Example 1:
Find the equation of a line passing through (5, 2) and parallel to $y = -3x + 4$.

Solution:
- Slope of original line: $m = -3$
- Use point-slope form:
$y - 2 = -3(x - 5)$
- Simplify:
$y - 2 = -3x + 15$
$y = -3x + 17$

Answer: $y = -3x + 17$

Example 2:
Find the line passing through (0, 5) and parallel to the vertical line $x = -2$.

Solution:
- The original line is vertical with $x = -2$.
- Parallel line must also be vertical: $x = -2$.
- Since the point (0, 5) does not lie on $x = -2$, the line passing through this point parallel to $x = -2$ is simply $x = 0$.

Answer: $x = 0$

Conclusion

Determining the equation of a line passing through specific points and parallel to a given line involves understanding the properties of slopes and line equations. When the given points form a vertical line, the parallel line is also vertical, simplifying the process to identifying the x-coordinate. For non-vertical lines, calculating the slope and applying point-slope or slope-intercept forms is essential. Recognizing these principles ensures accurate and efficient solutions to various problems involving parallel lines in coordinate geometry.

FAQs About Parallel Line Equations

Q1: How do I find the equation of a line parallel to a given line that passes through a specific

Frequently Asked Questions

What is the general form of the equation of a line parallel to a given line passing through the point (8, -7)?

The equation of a line parallel to a given line will have the same slope. To find it, first determine the slope of the original line, then use point-slope form with the point (8, -7).

How do I find the equation of a line passing through (8, -7) and parallel to the line passing through (8, 7)?

Since both points share the same x-coordinate, the original line is vertical. The parallel line will also be vertical, with the equation x = 8.

What is the significance of the points (8, -7) and (8, 7) in determining the line's equation?

Both points have the same x-coordinate, indicating the original line passing through them is vertical at x=8. The parallel line passing through (8, -7) will also be vertical at x=8.

If a line passes through (8, -7) and is parallel to a vertical line through (8, 7), what is its equation?

The line is vertical at x=8, so its equation is x = 8.

Are the points (8, -7) and (8, 7) on the same vertical line, and what does that imply about the equation of a parallel line?

Yes, both points lie on the vertical line x=8, which means any line parallel to this passing through (8, -7) is also x=8.

Additional Resources

What Is An Equation Of The Line That Passes Through The Point (8, -7) and (8, 7), and Is Parallel To The Line

Understanding the equations of lines is fundamental in the study of coordinate geometry, serving as a cornerstone for more complex concepts in mathematics and its applications. When given specific points and conditions—such as parallelism—the task becomes an intriguing problem that combines algebraic skills with geometric insight. This article delves into the question: What is an equation of the line that passes through the points (8, -7) and (8, 7), and is parallel to a given line? We will explore the underlying principles, methodically derive the equations, and clarify the geometric relationships involved.

Introduction to Lines and Their Equations

Before addressing the specific problem, it is essential to revisit the fundamental concepts related to lines in the Cartesian plane.

Linear Equations and Their Forms

A line in the plane can be represented by a linear equation, commonly expressed in various forms:

- Slope-Intercept Form: $y = mx + b$
where m is the slope, and b is the y-intercept.

- Point-Slope Form: $y - y_1 = m(x - x_1)$
used when a point (x_1, y_1) on the line and its slope m are known.

- Standard Form: $Ax + By + C = 0$

The choice of form depends on the information available and the context of the problem.

Understanding Slope and Parallelism

The slope (m) of a line indicates its steepness and direction. Two lines are parallel if and only if they have the same slope and are distinct lines (not coincident).

Formally:

> If line 1 has slope m_1 and line 2 has slope m_2, then the lines are parallel if $m_1 = m_2$, provided the lines are not identical.

The Given Points and Conditions

The problem involves two points: (8, -7) and (8, 7).

Analyzing the Points

- Points:
$P_1 = (8, -7)$
$P_2 = (8, 7)$

- Observation: Both points share the same x-coordinate (x=8), but different y-coordinates.

Implication of the Points' Coordinates

The points are vertically aligned:

- The line passing through (8, -7) and (8, 7) is a vertical line at x = 8.

- The slope of this line is undefined (since the change in x is zero).

This leads to an important conclusion:

> The line passing through these two points is a vertical line at x=8.

Equation of the Line Passing Through (8, -7) and (8, 7)

Given the points, the line passing through them is straightforward:

Equation of the Vertical Line

- Since x is constant at 8, the line's equation is:

x = 8

This vertical line has an undefined slope and is unique in its direction.

Understanding Parallel Lines to the Vertical Line

The core of the problem involves identifying lines parallel to the given line.

What Does Parallelism Mean for Vertical Lines?

- Vertical lines are all lines with an equation of the form x = constant.

- Parallel vertical lines are also vertical lines, each with a different x-coordinate.

Therefore:

> Any line parallel to x=8 is also vertical, with an equation x = k, where k is a constant different from 8.

Finding the Equation of the Parallel Line Passing Through (8, -7)

The problem specifies that the line passes through the point (8, -7). Since the line is parallel to the vertical line x=8, it must also be vertical.

Equation of the Parallel Line

- Because it passes through (8, -7), and is parallel to x=8, its equation is:

x = 8

This vertical line passes through (8, -7), as expected.

Key Observation

- The line passing through (8, -7) and (8, 7) is the line x=8.

- Any line parallel to it that passes through (8, -7) is identical to x=8.

Alternative Scenario: If the Given Line Is Not Vertical

The problem mentions "And Is Parallel To The Line"—possibly implying an existing line that is not necessarily vertical.

Suppose the original line to which the line is parallel is given by an explicit equation, such as:

- $y = m_0x + c_0$

In that case, the problem would involve:

1. Determining the slope m_0 of the original line.

2. Constructing a line passing through (8, -7) with the same slope m_0.

3. Writing the equation in the preferred form.

Since the current scenario involves the points (8, -7) and (8, 7), which define a vertical line, the slope is undefined, and the only lines parallel to it are also vertical lines.

Summary of Key Points

- The line passing through (8, -7) and (8, 7) is x=8.

- Vertical lines are characterized by their x-coordinate.

- Any line parallel to x=8 is also vertical, with an equation x=k, where $k \neq 8$.

- The line passing through (8, -7) and parallel to x=8 is therefore x=8 itself.

Conclusion: Final Equation and Its Significance

Given the points and the nature of the line:

- The equation of the line passing through (8, -7) and (8, 7) is:

$x = 8$

- The equation of the line passing through (8, -7) and parallel to this line is:

$x = 8$

This outcome underscores the unique characteristics of vertical lines in coordinate geometry. It also illustrates that when dealing with vertical lines, the concept of slope is undefined, but the parallelism can be directly understood through their x-coordinates.

Implications and Broader Context

Understanding the behavior of lines with undefined slopes and their parallels is crucial in many mathematical applications:

- Geometric constructions: Vertical lines often serve as reference axes or boundaries.

- Analytical geometry: Recognizing the nature of line equations aids in solving systems of equations, optimization problems, and in graphing.

- Real-world applications: Vertical and horizontal lines are common in architecture, engineering, and computer graphics.

Final Remarks

This exploration highlights the importance of carefully analyzing given points and conditions. In the specific case of points with identical x-coordinates, the line is vertical, and its parallels are similarly vertical. The clarity of this geometric property simplifies the problem and leads to a straightforward solution.

Understanding these principles enhances one's ability to interpret and solve a wide variety of problems related to lines, slopes, and parallelism in coordinate geometry, providing a solid foundation for more advanced mathematical endeavors.

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