What Is The Coefficient Of X^3 Y^4 In (-3x + 4y)^7? What Is The Coefficient Of X^2 Y^7 In (5x - Y)^9?

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Understanding how to find specific coefficients in binomial expansions is a fundamental skill in algebra and combinatorics. This article provides a comprehensive guide to calculating the coefficients of particular terms in binomial expressions, focusing on two examples: determining the coefficient of $x^3 y^4$ in $(-3x + 4y)^7$, and finding the coefficient of $x^2 y^7$ in $(5x - y)^9$. By exploring binomial theorem principles, step-by-step calculations, and practical tips, this guide aims to enhance your understanding and confidence in handling similar problems.

Understanding the Binomial Theorem

What Is the Binomial Theorem?

The binomial theorem is a key algebraic principle that provides a formula for expanding expressions raised to a power, such as $(a + b)^n$. It states that:

$$
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^{k}
$$

where:

- n is a non-negative integer.
- $\binom{n}{k}$ is the binomial coefficient, read as "n choose k," calculated as:

$$
\binom{n}{k} = \frac{n!}{k! (n - k)!}
$$

This expansion involves summing all terms of the form $\binom{n}{k} a^{n - k} b^{k}$.

Applying the Binomial Theorem to Find Specific Terms

When looking for the coefficient of a specific term like $x^a y^b$ in an expansion:

- Identify the powers of a and b in the term.
- Match these powers to the general term in the binomial expansion.
- Calculate the corresponding binomial coefficient and the powers of the individual terms.

Calculating the Coefficient of $x^3 y^4$ in $(-3x + 4y)^7$

Step 1: Recognize the Binomial Expression

Given the expression:

$$
(-3x + 4y)^7
$$

- The overall exponent is 7.
- The terms inside the parentheses are $-3x$ and $4y$.

Step 2: Write the General Term in the Expansion

Using the binomial theorem, the general term T_{k+1} for $k=0, 1, ..., 7$:

$$
T_{k+1} = \binom{7}{k} (-3x)^{7 - k} (4y)^k
$$

This simplifies to:

$$
T_{k+1} = \binom{7}{k} (-3)^{7 - k} x^{7 - k} \times 4^{k} y^{k}
$$

Step 3: Find the Relevant Term for $x^3 y^4$

- The power of x in the term is $7 - k$. Set this equal to 3:

$$
7 - k = 3 \Rightarrow k = 4
$$

- The power of y in the term is k. Check if $k=4$ matches the desired power:

$$
k=4 \quad \Rightarrow y^{4}
$$

Yes, this aligns with the target term $x^3 y^4$.

Step 4: Calculate the Coefficient

Plug $k=4$ into the general term:

$$
T_{5} = \binom{7}{4} (-3)^{7 - 4} x^{3} \times 4^{4} y^{4}
$$

Compute each component:

- $\binom{7}{4} = \frac{7!}{4! \times 3!} = \frac{5040}{24 \times 6} = 35$

- $(-3)^{3} = -27$ (since $7-4=3$)

- $4^{4} = 256$

Putting it all together:

$$
\text{Coefficient} = 35 \times (-27) \times 256
$$

Calculate step-by-step:

1. $35 \times -27 = -945$
2. $-945 \times 256 = -241,920$

Answer: The coefficient of $x^3 y^4$ in $(-3x + 4y)^7$ is $-241,920$.

Finding the Coefficient of $x^2 y^7$ in $(5x - y)^9$

Step 1: Recognize the Binomial Expression

Given:

$$
(5x - y)^9
$$

- The overall exponent is 9.
- The terms are $5x$ and $- y$.

Step 2: Write the General Term in the Expansion

Using the binomial theorem:

$$
T_{k+1} = \binom{9}{k} (5x)^{9 - k} (- y)^k
$$

Simplify:

$$
T_{k+1} = \binom{9}{k} 5^{9 - k} x^{9 - k} \times (-1)^k y^{k}
$$

Step 3: Find the Relevant Term for $x^2 y^7$

- The power of x is $9 - k$. Set equal to 2:

$$
9 - k = 2 \Rightarrow k = 7
$$

- The power of y is k. Confirm it matches the desired:

$$
k=7 \quad \Rightarrow y^{7}
$$

which aligns with the y^7 term.

Step 4: Calculate the Coefficient

Plug $k=7$ into the general term:

$$
T_{8} = \binom{9}{7} 5^{9 - 7} x^{2} (-1)^{7} y^{7}
$$

Calculate each component:

- $\binom{9}{7} = \binom{9}{2} = \frac{9!}{2! \times 7!} = \frac{362,880}{2 \times 5040} = 36$

- $5^{2} = 25$

- $(-1)^7 = -1$

Putting it together:

$$
\text{Coefficient} = 36 \times 25 \times (-1) = -900
$$

Answer: The coefficient of $x^2 y^7$ in $(5x - y)^9$ is -900.

Summary of Key Steps in Binomial Coefficients Calculation

To efficiently find specific coefficients in binomial expansions, follow these steps:

1. Identify the powers: Determine the required powers of the variables in the term.
2. Match the powers to the general term: Use the binomial theorem formula to find which k corresponds to the desired powers.

3. Calculate the binomial coefficient: Use factorial formulas or Pascal's triangle.
4. Compute the powers of each term: Raise the constants to the appropriate powers.
5. Multiply all components: Combine the binomial coefficient and the powers to find the coefficient.

Practical Tips for Solving Binomial Coefficient Problems

- Always start by writing the general term for the binomial expansion.
- Pay attention to the signs, especially when terms involve negative signs.
- Use factorials or Pascal's triangle for quick binomial coefficient calculation.
- Confirm the powers of variables match the target term before proceeding.
- Practice with different binomial expressions to become comfortable with pattern recognition.

Conclusion

Understanding how to find specific coefficients in binomial expansions is essential for mastering algebraic manipulations and problem-solving in mathematics. By carefully applying the binomial theorem, identifying the correct term based on variable powers, and performing precise calculations, you can accurately determine coefficients like those in $(-3x + 4y)^7$ and $(5x - y)^9$. This skill not only enhances your algebraic proficiency but also provides a foundation for more advanced topics in combinatorics, probability, and calculus. Keep practicing with diverse binomial problems to strengthen your understanding and efficiency in coefficient calculations.

Frequently Asked Questions

How do you find the coefficient of $x^3 y^4$ in the expansion of $(-3x + 4y)^7$?

Use the Binomial Theorem: the coefficient of $x^3 y^4$ is given by the binomial coefficient $C(7, 3)$ multiplied by $(-3)^3$ and 4^4. Calculate $C(7, 3)=35$, then multiply: $35 (-3)^3 4^4 = 35 (-27) 256 = -35\ 27\ 256 = -35\ 6912 = -241,920$.

What is the coefficient of $x^2 y^7$ in the expansion of $(5x - y)^9$?

Apply the Binomial Theorem: the coefficient is $C(9, 2) (5)^2 (-1)^7$. Calculate $C(9, 2)=36$, then multiply: $36\ 25 (-1)^7 = 36\ 25 (-1) = -900$.

In the expansion of $(-3x + 4y)^7$, what term contains $x^3 y^4$ and what is its coefficient?

The term containing $x^3 y^4$ is given by $C(7, 3) (-3x)^3 (4y)^4$, with coefficient $-241,920$ as calculated earlier.

How can I generalize finding the coefficient of $x^m y^n$ in $(ax + by)^k$?

The coefficient is $C(k, m) a^m b^n$, where $m + n = k$. Identify the relevant binomial coefficient and multiply by the corresponding powers of a and b.

What is the binomial coefficient C(7, 3) and how is it used in this context?

C(7, 3) = 35. It indicates the number of ways to choose 3 x's (or y's) in the expansion, and is used to determine the coefficient of the specific term $x^3 y^4$.

Why do we use the binomial theorem for finding specific coefficients in binomial expansions?

Because the binomial theorem provides a systematic way to expand expressions like $(a + b)^n$ and identify the coefficients of individual terms based on binomial coefficients and powers.

In the expression $(5x - y)^9$, how does the negative sign affect the coefficient calculation?

The negative sign contributes a factor of $(-1)^n$ to the term's coefficient, depending on the power of y, which impacts the sign of the coefficient.

Can these methods be applied to any binomial expression to find specific term coefficients?

Yes, as long as the powers are non-negative integers, the binomial theorem provides a direct way to find any specific term's coefficient.

What are the key steps to find the coefficient of a specific term in a binomial expansion?

Identify the powers of each variable in the term, determine the corresponding binomial coefficient C(n, m), calculate the powers of each term's coefficient, and multiply everything together.

Additional Resources

What Is The Coefficient Of $X^3 Y^4$ In $(-3x + 4y)^7$? What Is The Coefficient Of $X^2 Y^7$ In $(5x - Y)^9$?

Understanding the coefficients in polynomial expansions is a fundamental aspect of algebra that bridges the gap between abstract mathematical theory and practical problem-solving. These coefficients not only reveal the weight of specific terms within expanded expressions but also embody combinatorial principles that underpin many areas of mathematics, from algebra to calculus and beyond. Today, we will explore the coefficients associated with particular terms in binomial expansions: specifically, the coefficient of X^3Y^4 in $(-3x + 4y)^7$, and the coefficient of X^2Y^7 in $(5x - y)^9$. These calculations, while seemingly straightforward, involve a deep understanding of binomial theorem and combinatorics—a blend of algebraic manipulation and counting principles that form the backbone of polynomial expansion.

The Significance of Binomial Coefficients in Polynomial Expansion

Before diving into the specific problems, it is essential to grasp the importance of binomial coefficients in polynomial expansions. The binomial theorem states that for any positive integer n:

$$
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^{k}
$$

\]

Here, \(\binom{n}{k}\) is the binomial coefficient, representing the number of ways to choose \(k\) elements from a set of \(n\). It is calculated as:

\[
\binom{n}{k} = \frac{n!}{k!(n - k)!}
\]

In the context of polynomial expansion, each term's coefficient involves these binomial coefficients, which directly determine the numerical weight of each term after expansion.

Analyzing the Coefficient of \(x^3y^4\) in \((-3x + 4y)^7\)

Step 1: Recognize the Binomial Structure

The expression \((-3x + 4y)^7\) is a binomial raised to the 7th power. Its expansion can be written as:

\[
(-3x + 4y)^7 = \sum_{k=0}^{7} \binom{7}{k} (-3x)^{7 - k} (4y)^k
\]

Each term in the sum corresponds to a specific value of \(k\), ranging from 0 to 7.

Step 2: Identify the Target Term

We are interested in the term containing \(x^3y^4\). Let's analyze the general term:

\[
\binom{7}{k} (-3x)^{7 - k} (4y)^k
\]

Note that:

- \((-3x)^{7 - k} = (-3)^{7 - k} x^{7 - k}\)
- \((4y)^k = 4^k y^k\)

Since the powers of \(x\) and \(y\) are \(7 - k\) and \(k\), respectively, to match the powers \(x^3y^4\), we need:

\[
7 - k = 3 \quad \Rightarrow \quad k = 4
\]

and

\[
k = 4
\]

which aligns perfectly.

Step 3: Compute the Coefficient for \(k=4\)

Using the binomial coefficient:

\[

\binom{7}{4} = \frac{7!}{4! \times 3!} = \frac{5040}{24 \times 6} = \frac{5040}{144} = 35
\]

Calculating each component:

- \((-3)^{7 - 4} = (-3)^3 = -27\)
- \(4^{4} = 256\)

Putting it together:

\[
\text{Coefficient} = \binom{7}{4} \times (-3)^{3} \times 4^{4} = 35 \times (-27) \times 256
\]

Calculating step-by-step:

- \(35 \times (-27) = -945\)
- \(-945 \times 256\)

Break down \(256\):

\[
-945 \times 256 = -945 \times (200 + 50 + 6) = -945 \times 200 - 945 \times 50 - 945 \times 6
\]

Calculate each:

- \(-945 \times 200 = -189,000\)
- \(-945 \times 50 = -47,250\)
- \(-945 \times 6 = -5,670\)

Sum:

\[
-189,000 - 47,250 - 5,670 = -241,920
\]

Answer: The coefficient of \(X^3Y^4\) in \((-3x + 4y)^7\) is \(-241,920\).

Analyzing the Coefficient of \(X^2Y^7\) in \((5x - y)^9\)

Step 1: Recognize the Binomial Structure

The expression \((5x - y)^9\) expands as:

\[
(5x - y)^9 = \sum_{k=0}^9 \binom{9}{k} (5x)^{9 - k} (-y)^k
\]

Each term corresponds to a particular \(k\).

Step 2: Identify the Target Term

We want the term containing \(X^2Y^7\). The general term is:

\[
\binom{9}{k} (5x)^{9 - k} (-y)^k

which simplifies to:

$$
\binom{9}{k} 5^{9 - k} x^{9 - k} \times (-1)^k y^{k}
$$

Matching the exponents:

$$
9 - k = 2 \quad \Rightarrow \quad k = 7
$$

and

$$
k = 7
$$

which matches with the power of Y.

Step 3: Compute the Coefficient for $k=7$

Calculate the binomial coefficient:

$$
\binom{9}{7} = \binom{9}{2} = \frac{9!}{2! \times 7!} = \frac{362,880}{2 \times 5040} = \frac{362,880}{10,080} = 36
$$

Calculate the numerical components:

- $5^{9 - 7} = 5^2 = 25$
- $(-1)^7 = -1$

Putting everything together:

$$
\text{Coefficient} = \binom{9}{7} \times 5^{2} \times (-1)^7 = 36 \times 25 \times (-1)
$$

Compute:

- $36 \times 25 = 900$

Therefore:

$$
900 \times (-1) = -900
$$

Answer: The coefficient of X^2Y^7 in $(5x - y)^9$ is -900.

Broader Implications and Applications

Calculating coefficients in binomial expansions is more than an academic exercise; it has practical applications across fields such as computer science, engineering, and physics. For example, in probability theory, binomial coefficients represent the number of ways events can occur, which underpins calculations in binomial distributions. In combinatorics, they count arrangements and selections, and in algebra, they aid in polynomial factorization and solving equations.

Moreover, understanding these coefficients allows mathematicians and scientists to estimate the behavior of polynomial functions, determine dominant terms in asymptotic analysis, and facilitate polynomial approximation methods like Taylor series expansions. The ability to derive specific coefficients equips students and professionals with tools for modeling complex systems, analyzing algorithms, and solving real-world problems involving algebraic structures.

Summary

In this exploration, we've demonstrated how to compute specific coefficients in binomial expansions through a step-by-step process involving the binomial theorem and combinatorial calculations:

- The coefficient of \(X^3Y^4\) in \((-3x + 4y)^7\) is \(-241,920\).
- The coefficient of \(X^2Y^7\) in \((5x - y)^9\) is \(-900\).

These calculations exemplify the power of binomial coefficients and the importance of systematic approaches in algebra. Mastery of these principles enhances problem-solving skills and deepens understanding of the mathematical structures that underpin much of science and engineering.

Final Thoughts

While the process of calculating specific coefficients can seem intricate at first, it becomes more intuitive with practice. Recognizing patterns, understanding the structure of binomial expansions, and applying combinatorial formulas are essential skills for anyone delving into higher mathematics. As you encounter more complex polynomial expressions, remember that the foundational concepts of binomial coefficients and algebraic manipulation will serve as reliable tools for uncovering the rich information contained within these expressions.

What Is The Coefficient Of X 3 Y 4 In 3x 4y 7 What Is The Coefficient Of X 2 Y 7 In 5x Y 9

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