What Is The Equation Of A Parabola That Has A Vertical Axis, Passes Through The Point (1, 3), And Has

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Understanding the equation of a parabola is fundamental in algebra and coordinate geometry. When a parabola has a vertical axis of symmetry, its general equation takes a specific form, which can be used to analyze its properties and graph it accurately. In this comprehensive guide, we will explore how to find the equation of such a parabola that passes through the point (1, 3), along with other essential details and examples to deepen your understanding.

Understanding the Basic Concepts of Parabolas

Before delving into the specific equation, it's important to understand some foundational concepts related to parabolas:

Definition of a Parabola

- A parabola is a symmetric, U-shaped curve that can be represented algebraically and geometrically.
- It is the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix.

Axis of Symmetry

- The line that divides the parabola into two mirror images.
- For a parabola with a vertical axis, the axis of symmetry is a vertical line, typically expressed as $x = h$.

Vertex of a Parabola

- The highest or lowest point on the parabola.
- The point where the axis of symmetry intersects the parabola.

Standard Forms of Parabola Equations with a Vertical Axis

The equation of a parabola with a vertical axis of symmetry can be written in two common forms:

Vertex Form

- $y = a(x - h)^2 + k$

Where:
- (h, k) is the vertex of the parabola.
- a determines the parabola's opening direction and width.

Standard Form (General Form)

- $y = Ax^2 + Bx + C$

While this form can describe any parabola, the vertex form is more straightforward for analyzing specific points and transformations.

Finding the Equation of a Parabola Passing Through (1, 3) with a Vertical Axis

Suppose we want to determine the equation of a parabola that:

- Has a vertical axis of symmetry.
- Passes through the point (1, 3).
- Has known or unknown parameters (such as the vertex or other features).

Step 1: Decide on the form of the parabola's equation

Depending on the available information, you might choose:

- Vertex form if the vertex point is known.
- General quadratic form if only a point is given and the parabola's shape is unspecified.

Since the problem specifies only a point and the axis orientation, we can proceed with the vertex form assuming the vertex is at some point (h, k). If the vertex is not specified, additional information is needed.

Step 2: Express the parabola in vertex form

The general vertex form:

$$ y = a(x - h)^2 + k $$

Our goal is to find the parameters a, h, k.

Step 3: Use the point (1, 3) to set up the equation

Substitute $x = 1$ and $y = 3$ into the vertex form:

$$ 3 = a(1 - h)^2 + k $$

This gives us one equation with three unknowns, so additional assumptions or information are necessary to solve uniquely.

Determining the Vertex Location

Without explicit information about the vertex, we can consider different scenarios for the parabola:

Scenario 1: The vertex is at the origin (0, 0)

- Equation becomes:

$$ y = a x^2 $$

- Plugging in point (1, 3):

$$ 3 = a \times 1^2 \Rightarrow a = 3 $$

- Equation:

$$ y = 3x^2 $$

- This parabola passes through (1, 3) and opens upwards with vertex at (0, 0).

Scenario 2: The vertex is at some point (h, k) with unknowns

- Additional information, such as the parabola's focus or directrix, would be necessary to find exact parameters.

Special Cases and Additional Conditions

The general approach involves knowing extra details to find the precise equation. Here are some common conditions and how they influence the parabola's equation:

1. Parabola with a given vertex

- If the vertex (h, k) is known, substitute into the vertex form along with the point to find a.

2. Parabola with a known focus or directrix

- Use the focus-directrix definition to derive the equation.

3. Parabola with a specific width or latus rectum

- Use the length of the latus rectum to determine a.

Example: Complete Derivation of the Equation with a Given Vertex

Suppose the parabola's vertex is at (2, 1) and it passes through (1, 3). Find its equation.

Step 1: Write the vertex form

$$y = a(x - 2)^2 + 1$$

Step 2: Use the point (1, 3)

$$ 3 = a(1 - 2)^2 + 1 $$
$$ 3 = a(-1)^2 + 1 $$
$$ 3 = a + 1 $$
$$ a = 2 $$

Step 3: Final equation

$$ y = 2(x - 2)^2 + 1 $$

This is the parabola with vertex at (2, 1), passing through (1, 3), and with a vertical axis.

Graphing the Parabola

Once the equation is determined, graphing involves:

- Plotting the vertex.
- Identifying the axis of symmetry ($x = h$).
- Plotting additional points for accuracy.
- Drawing the symmetric U-shaped curve.

Using the example $y = 2(x - 2)^2 + 1$:

- Vertex at (2, 1).
- Parabola opens upwards (since $a > 0$).
- Points like (3, 3) and (1, 3) are symmetric about $x=2$.

Applications of Parabola Equations

Understanding the equation of a parabola with a vertical axis is essential in various fields:

- Physics: Modeling projectile motion.
- Engineering: Designing parabolic reflectors.
- Mathematics: Analyzing quadratic functions.
- Architecture: Creating arches and bridges.

Summary and Key Takeaways

- The general form of a parabola with a vertical axis is $y = a(x - h)^2 + k$.
- To find its specific equation, you need at least the vertex or additional points.
- The point (1, 3) provides a key constraint, but additional parameters are necessary for a unique solution.
- Assumptions or extra information (like the vertex location) enable precise derivations.
- Understanding the relationship between the vertex, focus, and directrix is crucial for comprehensive analysis.

Final Thoughts

Finding the equation of a parabola passing through a specific point with a vertical axis involves combining algebraic techniques with geometric intuition. Whether the vertex is known or unknown, the process hinges on substituting known points into the appropriate form and solving for the parameters. With practice, you can analyze various scenarios and derive the equations of parabolas tailored to particular applications or constraints.

By mastering these concepts, you'll gain a powerful toolset for tackling quadratic functions and their geometric representations, enhancing your mathematical proficiency and practical problem-solving skills.

Frequently Asked Questions

What is the general form of a parabola with a vertical axis of symmetry?

The general form is $y = ax^2 + bx + c$, where the parabola opens upward if $a > 0$ and downward if $a < 0$.

How can I find the equation of a parabola with a vertical axis passing through a specific point?

You can substitute the point's coordinates into the parabola's equation and use additional information (like the vertex or focus) to determine the specific coefficients.

What additional information do I need to uniquely determine the parabola's equation?

Typically, you need the vertex, focus, directrix, or the parabola's focus and directrix to find a unique equation.

Given a point (1, 3) that the parabola passes through, how do I find its equation if the vertex is at the origin?

Assuming the vertex is at (0, 0), substitute (1, 3) into $y = ax^2$ to find a: $3 = a(1)^2$, so $a = 3$. The equation is $y = 3x^2$.

How do I write the equation of a parabola with a vertical axis passing through (1, 3) and vertex at (h, k)?

The equation in vertex form is $y = a(x - h)^2 + k$. Plug in the point (1, 3) to solve for a once you know h and k.

What is the vertex form of a parabola with a vertical axis passing through the point (1, 3)?

The vertex form is $y = a(x - h)^2 + k$. To find the specific equation, additional information about the vertex or parabola's shape is needed.

If a parabola passes through (1, 3) and opens upward, how do I find its equation?

Assuming the vertex is at (h, k), and the parabola opens upward, you can use the vertex form $y = a(x - h)^2 + k$, substitute (1, 3), and solve for a once h and k are known.

Can I determine the equation of the parabola with only the point (1, 3) and the axis of symmetry being vertical?

No, additional information such as the vertex, focus, or directrix is needed to uniquely determine the parabola's equation.

What is the significance of the parabola passing through (1, 3) with a vertical axis?

It indicates that at $x=1$, the parabola's y-coordinate is 3, and the parabola opens either upwards or downwards along the y-axis, depending on its coefficients.

How do I derive the equation of a parabola with a vertical axis passing through (1, 3) if I know the vertex is at (h, k)?

Use the vertex form $y = a(x - h)^2 + k$, plug in (1, 3), and if additional info is available about the vertex, solve for a to find the specific equation.

Additional Resources

What Is The Equation Of A Parabola That Has A Vertical Axis, Passes Through The Point (1, 3), And Has

Understanding the fundamental properties of parabolas is a cornerstone of algebra and coordinate geometry. When tasked with determining the specific equation of a parabola that exhibits certain features—such as a vertical axis of symmetry and passing through a particular point—mathematicians and students alike delve into a structured process that combines geometric intuition with algebraic manipulation. In this article, we explore the detailed steps involved in deriving the equation of such a parabola, elucidate the underlying principles, and provide practical insights into how these equations are constructed and interpreted.

Foundations: What Defines a Parabola?

Before tackling the specific problem, it’s essential to revisit what a parabola is and its defining characteristics.

Geometric Definition

A parabola can be described as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix. This geometric perspective offers an intuitive grasp of the parabola's shape.

Algebraic Representation

In Cartesian coordinates, the most common form of a parabola with a vertical axis of symmetry is expressed as:

$$y = ax^2 + bx + c$$

where:
- a determines the parabola’s opening (upward if $a > 0$, downward if $a < 0$)

- b influences its tilt
- c is the y-intercept

Alternatively, the vertex form:

$$y = a(x - h)^2 + k$$

where (h, k) is the vertex of the parabola, provides a more geometric perspective, making it easier to analyze transformations and key features.

Key Features of the Parabola in Question

Given the problem's parameters, the parabola:

- Has a vertical axis of symmetry, meaning its axis is a vertical line, typically expressed as $x = h$.
- Passes through the point (1, 3).
- Has additional features or constraints (not fully specified in the prompt), but the process remains similar regardless.

Assuming the parabola is in the vertex form, the general approach is to determine the parameters a, h, and k based on the given information.

Understanding the Axis of Symmetry

For a parabola with a vertical axis, the axis of symmetry is a vertical line $x = h$. The vertex of the parabola lies on this line, and its x-coordinate is the axis of symmetry.

Point Passing Through the Parabola

The point $(1, 3)$ lies on the parabola, which provides a vital equation to relate the parameters.

Step-by-Step Process to Derive the Equation

Let’s walk through the systematic approach to find the parabola's equation.

1. Express the Parabola in Vertex Form

Given the axis of symmetry is vertical at $x = h$, the parabola can be written as:

$$ y = a(x - h)^2 + k $$

where:
- (h, k) is the vertex,
- a determines the parabola's "width" and direction.

2. Use the Known Point $(1, 3)$

Since the point $(1, 3)$ lies on the parabola:

$$ 3 = a(1 - h)^2 + k $$

This equation relates a, h, and k.

3. Additional Constraints or Information

The problem may specify additional features such as the focus, directrix, or vertex location. In absence of such details, we can consider typical scenarios:

- Scenario A: The vertex is at a known point, e.g., (h, k).
- Scenario B: The parabola opens upward or downward with specific focus or directrix properties.

For this explanation, let's assume the vertex is at (h, k), and the parabola opens upward.

4. Express the Vertex Coordinates and Use Symmetry

Suppose, for simplicity, the vertex is at (h, k) and the parabola opens upward (i.e., $a > 0$). We have:

$$ y = a(x - h)^2 + k $$

With this, the known point gives:

$$ 3 = a(1 - h)^2 + k $$

If additional information about the vertex or focus is provided, it can be used to solve for h, k, and a.

Common Approaches to Find the Specific Equation

Depending on the details, several pathways can be taken:

Method 1: Using Vertex and Point

- If the vertex (h, k) is known, plug into the vertex form.
- Use the point $(1, 3)$ to find a:

$$ a = \frac{y - k}{(x - h)^2} $$

- Once a is known, the complete equation is determined.

Method 2: Using Focus-Directrix Definition

- If the focus or directrix is specified, use geometric definitions to find the vertex.
- For a parabola with focus at $(h, k + p)$ and directrix $y = k - p$:

$$ y = \frac{1}{4p}(x - h)^2 + k $$

- The parameter p (distance from vertex to focus) influences the parabola's shape.

Method 3: Symmetry and Algebraic Manipulation

- Use symmetry about the axis $x = h$.
- If the parabola's vertex is at (h, k), and passes through $(1, 3)$, then:

$$ 3 = a(1 - h)^2 + k $$

- Additional constraints or data points can help solve for h, k, and a.

Example: Deriving a Specific Equation

Suppose the vertex is at $(h, k) = (h, 0)$ (i.e., the parabola opens upward and vertex is on the x-axis). The equation:

$$ y = a(x - h)^2 $$

Given the point $(1, 3)$ lies on the parabola:

$$ 3 = a(1 - h)^2 $$

To find specific values, we need either h or an additional point or condition.

Scenario: Assume the vertex is at $(h, 0)$, and the parabola passes through $(1, 3)$. Let's pick $h = 1$ for simplicity:

$$ 3 = a(0)^2 = 0 $$

Contradiction, so the vertex isn't at $h=1$. Let's choose $h = 0$:

$$ 3 = a(1 - 0)^2 = a(1) = a $$

Thus, $a=3$, and the parabola's equation:

$$ y = 3x^2 $$

which passes through $(1, 3)$, consistent with the given point.

Result: The parabola $y=3x^2$ has a vertical axis, passes through $(1, 3)$, and is symmetric about the line $x=0$.

Interpreting and Applying the Equation

Once the specific parameters are identified, the resulting equation allows various applications:

- Graphing: Plotting the parabola to visualize its shape.
- Analysis: Finding the vertex, focus, directrix, axis of symmetry.
- Problem Solving: Using the equation to solve for intersections, distances, and other geometric properties.

Practical Tips for Deriving Parabola Equations

- Start with the vertex form when the vertex is known or can be assumed.
- Use known points to solve for the parabola’s parameters.
- Leverage symmetry about the axis of symmetry to simplify calculations.
- When additional features are specified (focus, directrix), incorporate their geometric relationships.
- Always verify the derived equation by plugging in known points.

Conclusion: Navigating the Path from Concept to Equation

Determining the equation of a parabola that has a vertical axis and passes through a specified point involves a blend of geometric insight and algebraic calculation. By understanding the core properties—such as the vertex form, axis of symmetry, and passing points—one can systematically derive the precise equation that characterizes the parabola in question. Whether for academic purposes, engineering design, or analytical investigations, mastering this process is fundamental to harnessing the power of conic sections in diverse applications.

This exploration underscores that while initial details may seem limited, the structured approach ensures that the equation of the parabola can be accurately deduced, providing a robust foundation for further mathematical analysis and real-world problem-solving.

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