What Is The Length Of A Side Of An Equilateral Triangle Inscribed In A Circle With A Radius Of 10 Cm?

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Understanding the relationship between circles and inscribed polygons is a fundamental aspect of geometry. When dealing with an equilateral triangle inscribed in a circle, a crucial question often arises: What is the length of each side of the triangle when the circle's radius is known? Specifically, if the circle has a radius of 10 centimeters, how long is each side of the inscribed equilateral triangle? In this comprehensive guide, we will explore this question in detail, covering the necessary geometric principles, the derivation of the formula, step-by-step calculations, and practical applications.

Understanding the Basics: Circle and Equilateral Triangle

Before diving into the calculations, it's essential to grasp the fundamental concepts involved:

What Is an Inscribed Polygon?

- An inscribed polygon is a polygon whose vertices all lie on the circumference of a circle.
- The circle itself is called the circumscribed circle or circumcircle of the polygon.

Characteristics of an Equilateral Triangle

- All sides are of equal length.
- Each interior angle measures 60 degrees.
- When inscribed in a circle, the vertices of an equilateral triangle are equally spaced around the circle.

Why Focus on Equilateral Triangles?

- They have symmetrical properties that make calculations straightforward.
- Their inscribed angles and side lengths relate directly to the circle's radius, facilitating formula derivation.

Key Geometric Principles and Formulas

To find the side length of the inscribed equilateral triangle, it's vital to understand the relationship between the circle's radius and the triangle's side length.

Central Angles and Inscribed Angles

- The central angle of a regular polygon with n sides is given by:

$$
\theta = \frac{360^\circ}{n}
$$

- For an equilateral triangle (n=3), the central angle is:

$$
\theta = \frac{360^\circ}{3} = 120^\circ
$$

- Each side of the inscribed triangle subtends a 120-degree arc at the circle's center.

Chord Length Formula

- The side length of a polygon inscribed in a circle can be viewed as a chord of the circle.
- The chord length c for a given central angle θ and radius r is:

$$
c = 2r \sin \left(\frac{\theta}{2} \right)
$$

- This formula is derived from the right triangle formed by the radius and the half-chord.

Applying to Equilateral Triangle

- Since the inscribed equilateral triangle has vertices on the circle, each side corresponds to a chord with a central angle of 120°.
- Therefore, the side length s is:

$$
s = 2r \sin \left(\frac{120^\circ}{2} \right) = 2r \sin(60^\circ)
$$

- Recall that $\sin(60^\circ) = \frac{\sqrt{3}}{2}$.

Calculating the Side Length for a Circle with Radius 10 Cm

Given the above formulas, let’s now perform the calculation step by step.

Step 1: Identify the Known Values

- Radius of the circle $r = 10\, \text{cm}$
- Central angle for each side $\theta = 120^\circ$

Step 2: Compute Half of the Central Angle

- $\frac{\theta}{2} = \frac{120^\circ}{2} = 60^\circ$

Step 3: Calculate the Sine of 60 Degrees

- $\sin(60^\circ) = \frac{\sqrt{3}}{2} \approx 0.8660$

Step 4: Apply the Chord Length Formula

- $s = 2 \times 10\, \text{cm} \times \sin(60^\circ)$
- $s = 20\, \text{cm} \times \frac{\sqrt{3}}{2}$

Step 5: Simplify the Expression

- $s = 20\, \text{cm} \times \frac{\sqrt{3}}{2} = 10\, \text{cm} \times \sqrt{3}$

Step 6: Final Calculation

- $s \approx 10\, \text{cm} \times 1.7321$
- $s \approx 17.321\, \text{cm}$

Therefore, each side of the inscribed equilateral triangle measures approximately 17.32 centimeters when the circle's radius is 10 centimeters.

Additional Insights and Related Concepts

Understanding the side length of an inscribed equilateral triangle opens doors to various related geometric concepts and practical applications.

1. Relationship with the Triangle's Area

- The area A of an equilateral triangle with side length s is:

$$
A = \frac{\sqrt{3}}{4} s^2
$$

- Plugging in $s \approx 17.32\, \text{cm}$, you can compute the precise area.

2. Circumradius and Inradius

- The circumradius (radius of the circumscribed circle) for an equilateral triangle relates to its side as:

$$
R = \frac{s}{\sqrt{3}}
$$

- Confirming that for $s \approx 17.32\, \text{cm}$, $R \approx 10\, \text{cm}$.

- The inradius (radius of inscribed circle) is:

$$
r_{in} = \frac{s}{2\sqrt{3}}
$$

- Useful for understanding inscribed circles within equilateral triangles.

3. Practical Applications

- Engineering: Designing components with specific inscribed polygon properties.
- Architecture: Creating symmetric structures with inscribed polygons.
- Education: Teaching fundamental geometric principles through tangible examples.

Summary and Key Takeaways

- The length of each side of an equilateral triangle inscribed in a circle with radius r is given by:

$$
s = 2r \sin \left(\frac{180^\circ}{n} \right)
$$

where $n = 3$ for an equilateral triangle.

- For a circle with $r = 10\, \text{cm}$:

$$
s = 2 \times 10\, \text{cm} \times \sin(60^\circ) \approx 17.32\, \text{cm}
$$

- This method leverages the properties of chords and central angles to simplify the calculation.

Conclusion

Understanding how to determine the side length of an inscribed equilateral triangle in a circle is a vital skill in geometry, embodying the elegant relationship between angles, chords, and radii. When the circle’s radius is known, such as 10 centimeters, applying the chord length formula with the central angle provides a straightforward yet insightful solution. The side length of approximately 17.32 centimeters reflects the harmonious proportions inherent in equilateral triangles and circles. Mastery of these concepts enhances both theoretical knowledge and practical problem-solving skills in geometry, making it an essential topic for students, educators, and professionals alike.

Frequently Asked Questions

What is the length of a side of an equilateral triangle inscribed in a circle with a radius of 10 cm?

The side length is approximately 17.32 cm, calculated using the formula s = □3 × radius, so s = □3 × 10 cm.

How do you find the side length of an equilateral triangle inscribed in a circle with a given radius?

Use the formula s = □3 × radius; for a radius of 10 cm, multiply 10 by □3 to get the side length.

Why is the side length of an equilateral triangle inscribed in a circle related to the circle's radius?

Because in an inscribed equilateral triangle, each vertex lies on the circle, and the side length relates to the radius through geometric relationships involving equilateral triangles and circles.

Can you provide the formula to calculate the side of an equilateral triangle inscribed in a circle?

Yes, the side length s = □3 × radius of the circle.

What is the approximate measurement of the side length for an inscribed equilateral triangle in a circle with a radius of 10 cm?

Approximately 17.32 cm, since $s \approx \sqrt{3} \times 10$ cm.

Is there a simple way to remember the relation between the radius and side length of an inscribed equilateral triangle?

Yes, remember that the side length equals the square root of 3 times the radius, so $s = \sqrt{3} \times$ radius.

How does the inscribed circle's radius affect the size of an equilateral triangle inside it?

The larger the radius, the longer the side length of the inscribed equilateral triangle, following the formula $s = \sqrt{3} \times$ radius.

What is the step-by-step method to find the side length of an inscribed equilateral triangle in a circle?

Identify the circle's radius, then multiply that radius by $\sqrt{3}$ to get the side length: $s = \sqrt{3} \times$ radius. For a 10 cm radius, $s \approx 17.32$ cm.

Additional Resources

What Is The Length Of A Side Of An Equilateral Triangle Inscribed In A Circle With A Radius Of 10 Cm?

Understanding the relationship between inscribed polygons and their circumscribing circles is fundamental in geometry. When dealing with an equilateral triangle inscribed in a circle with a radius of 10 cm, calculating the side length involves applying core principles of geometry, trigonometry, and circle properties. This question not only tests our grasp of geometric relationships but also demonstrates how different elements of a circle and inscribed shapes interconnect. In this comprehensive guide, we'll explore the step-by-step process of determining the side length of this equilateral triangle, along with relevant concepts and formulas.

Understanding the Geometric Context

Before diving into calculations, it's essential to clarify the geometric setup:

- The circle has a radius, $r = 10$ cm.
- An equilateral triangle is inscribed within this circle, meaning all three vertices of the triangle touch the circle.
- The goal is to find the length of one side of this inscribed equilateral triangle.

Key Concepts and Definitions

1. Inscribed Polygon
An inscribed polygon is one where all vertices lie on the circle's circumference. In this case, the polygon is an equilateral triangle, which has all sides equal and all angles equal to 60°.

2. Circumcircle and Circumradius
The circle in which the polygon is inscribed is called the circumcircle, and the radius of this circle is the circumradius (denoted as R). For our problem, the circumradius $R = 10$ cm.

3. Central and Inscribed Angles
- The central angle subtended at the circle's center between two vertices of the inscribed polygon.
- The inscribed angle subtended at a point on the circle between two points on the circumference.

Understanding the relationship between these angles is crucial for our calculations.

Step-by-Step Calculation Process

Let's now walk through the process of calculating the side length of the inscribed equilateral triangle.

Step 1: Determine the Central Angle of the Equilateral Triangle

Since the triangle is inscribed in the circle:

- The three vertices divide the circle into three equal arcs.
- Each arc subtends an angle of 360° / 3 = 120° at the center of the circle.

Therefore, the central angle between any two vertices of the inscribed equilateral triangle is 120°.

Step 2: Relate the Side Length to the Radius and Central Angle

The side length of the triangle, denoted as s, can be determined using the Law of Cosines or by analyzing the chord length corresponding to the central angle.

Chord length formula:

$$
s = 2 \times R \times \sin\left(\frac{\theta}{2}\right)
$$

where:

- R is the radius of the circle,

- θ is the central angle between the two vertices (in degrees or radians).

In our case:

- $R = 10\, \text{cm}$,
- $\theta = 120^\circ$.

Calculating the Side Length

Using the chord length formula:

$$
s = 2 \times 10 \times \sin\left(\frac{120^\circ}{2}\right)
$$

$$
s = 20 \times \sin(60^\circ)
$$

Recall that:

$$
\sin(60^\circ) = \frac{\sqrt{3}}{2} \approx 0.8660
$$

Substituting:

$$
s = 20 \times \frac{\sqrt{3}}{2} = 10 \times \sqrt{3}

\]

Result:

$$
\boxed{
s = 10 \sqrt{3}\, \text{cm} \approx 10 \times 1.732 = 17.32\, \text{cm}
}
$$

Answer:
The length of a side of the inscribed equilateral triangle is $10\sqrt{3}$ cm, approximately 17.32 cm.

Additional Insights and Related Concepts

1. Relationship Between Side Length and Radius
The formula used demonstrates a direct relationship: as the radius increases, so does the side length of the inscribed equilateral triangle.

2. General Formula for Regular n-Gon Inscribed in a Circle
For a regular polygon with n sides inscribed in a circle of radius R, the side length s is:

$$
s = 2 R \sin\left(\frac{180^\circ}{n}\right)
$$

This formula applies to all regular polygons and highlights the importance of central angles.

3. Application of the Law of Cosines

Alternatively, one could derive the side length using the Law of Cosines, considering the triangle formed by two vertices and the circle's center:

$$
s^2 = R^2 + R^2 - 2 R R \cos(120^\circ)
$$

$$
s^2 = 2 R^2 (1 - \cos 120^\circ)
$$

Since $\cos 120^\circ = -\frac{1}{2}$:

$$
s^2 = 2 R^2 \left(1 - \left(-\frac{1}{2}\right)\right) = 2 R^2 \left(\frac{3}{2}\right) = 3 R^2
$$

$$
s = R \sqrt{3}
$$

which matches the previous result when $R = 10\, \text{cm}$.

Practical Applications and Considerations

- Design and Engineering: Knowing the side lengths of inscribed polygons can aid in designing mechanical parts, decorative patterns, or architectural elements.
- Mathematical Education: This problem offers an excellent example for teaching circle geometry, trigonometry, and the relationships between inscribed polygons and their circumscribing circles.

- Further Exploration: Variations include inscribing other regular polygons, calculating inscribed circle radii for given side lengths, or exploring properties of inscribed polygons in different circle configurations.

Summary and Final Thoughts

In conclusion, the length of a side of an equilateral triangle inscribed in a circle with a radius of 10 cm is $10\sqrt{3}$ cm, approximately 17.32 cm. This calculation hinges on understanding the relationship between the central angle of the inscribed polygon and the chord length, utilizing fundamental geometric formulas involving sine functions. Whether approached through the chord length formula or the Law of Cosines, both methods yield the same elegant result, illustrating the harmony and consistency inherent in geometric principles.

Quick Recap:

- The central angle between vertices of an inscribed equilateral triangle: 120°.
- The side length formula: $s = 2 R \sin(\theta/2)$.
- For R = 10 cm and $\theta = 120°$, $s = 10\sqrt{3}$ cm.

Understanding these relationships enhances not only our problem-solving toolkit but also deepens our appreciation of the interconnectedness within geometry.

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