What Is The Smallest Value Of X When The Function F(x)= 3 2 X 3 2x 2 10x+8, Has A Slope That Equals 0

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Understanding the behavior of functions, particularly in calculus, often involves analyzing their slopes and critical points. When a function's slope equals zero, it indicates potential local maxima, minima, or saddle points. In this article, we will explore the process of determining the smallest value of x for the function $F(x) = 3 \times 2^x + 3 \times 2x^2 + 10x + 8$ (assuming the intended function based on typical notation) when its derivative or slope equals zero. We will break down the steps involved, explain key concepts, and provide a comprehensive guide to solving such problems.

Understanding the Function and Its Components

What is the Function $F(x)$?

The given function appears to be:

$$
F(x) = 3 \times 2^x + 3 \times 2x^2 + 10x + 8
$$

This function combines exponential, polynomial, and linear terms:

- 3×2^x: an exponential component
- $3 \times 2x^2$: a quadratic term scaled by 3
- $10x$: a linear term
- 8: a constant

Understanding each part helps in differentiating and analyzing the function's behavior.

Why Find When the Slope Equals Zero?

In calculus, the slope of a function at a point x is given by its first derivative $F'(x)$. When $F'(x) = 0$, the function has a critical point; it could be a maximum, minimum, or saddle point depending on the second derivative test or the context.

Finding the smallest x where $F'(x) = 0$ involves solving the equation $F'(x) = 0$ for x, and selecting the minimum among those solutions.

Step-by-Step Approach to Find the Smallest x for $F'(x) = 0$

1. Differentiate the Function $F(x)$

To find where the slope is zero, we first compute $F'(x)$.

Given:

$$
F(x) = 3 \times 2^x + 3 \times 2 x^2 + 10x + 8
$$

Differentiate term-by-term:

- Derivative of 3×2^x:

Recall that the derivative of a^x with respect to x is $a^x \ln a$.

$$
\frac{d}{dx} [3 \times 2^x] = 3 \times 2^x \ln 2
$$

- Derivative of $3 \times 2 x^2$:

This is a polynomial term:

$$
\frac{d}{dx} [3 \times 2 x^2] = 3 \times 2 \times 2x = 3 \times 2 \times 2x = 12x
$$

- Derivative of $10x$:

$$
\frac{d}{dx} [10x] = 10
$$

- Derivative of constant 8:

$$
\frac{d}{dx} [8] = 0
$$

2. Write the Derivative Expression

Putting it all together:

$$
F'(x) = 3 \times 2^x \ln 2 + 12x + 10
$$

This derivative function is key to locating critical points.

---

3. Set the Derivative Equal to Zero and Solve for x

To find the critical points:

$$
F'(x) = 0 \Rightarrow 3 \times 2^x \ln 2 + 12x + 10 = 0
$$

Rearranged:

$$
3 \times 2^x \ln 2 = -12x - 10
$$

Or:

$$
2^x = \frac{-12x - 10}{3 \ln 2}
$$

Now, note that 2^x is always positive for all real x. Therefore, the right side must be positive:

$$
\frac{-12x - 10}{3 \ln 2} > 0
$$

Since $\ln 2$ is positive (~0.693), the sign of the denominator is positive, so the numerator must be positive:

$$
-12x - 10 > 0
$$

$$
-12x > 10
$$

$$
x < -\frac{10}{12} = -\frac{5}{6} \approx -0.8333
$$

Thus, critical points only exist for $x < -\frac{5}{6}$.

Numerical and Analytical Solutions

The equation:

$$
2^x = \frac{-12x - 10}{3 \ln 2}
$$

cannot be solved algebraically for x in closed form because of the transcendental nature (combining exponential and linear terms). Therefore, we resort to numerical methods or graphing techniques.

1. Graphical Analysis

Plotting both sides as functions:

- $y_1 = 2^x$
- $y_2 = \frac{-12x - 10}{3 \ln 2}$

The solutions correspond to intersections of these graphs.

Using graphing calculators or software like Desmos, GeoGebra, or WolframAlpha, we can approximate the solutions.

2. Numerical Approximation

Suppose we evaluate 2^x and $\frac{-12x - 10}{3 \ln 2}$ at specific x values:

x	2^x	$\frac{-12x - 10}{3 \ln 2}$
-2	0.25	$\frac{24 - 10}{3 \times 0.693} \approx \frac{14}{2.079} \approx 6.73$
-1.5	0.3536	$\frac{18 - 10}{2.079} \approx \frac{8}{2.079} \approx 3.85$
-1	0.5	$\frac{12 - 10}{2.079} \approx \frac{2}{2.079} \approx 0.96$
-0.8	0.574	$\frac{9.6 - 10}{2.079} \approx \frac{-0.4}{2.079} \approx -0.192$

Observations:

- At $x = -1$, RHS ≈ 0.96, LHS $= 0.5 \rightarrow$ LHS $<$ RHS
- At $x = -0.8$, RHS ≈ -0.192, LHS $\approx 0.574 \rightarrow$ LHS $>$ RHS

Between $x = -1$ and $x = -0.8$, the two functions intersect. The approximate critical point occurs near $x \approx -0.9$.

Similarly, further refinements suggest the critical point is around $x \approx -0.95$.

Determining the Smallest Critical Point

Given the above analysis, the critical points occur for $x < -\frac{5}{6}$ (approximately -0.8333). Numerical approximation indicates the critical point is near $x \approx -0.95$.

Since the derivative equates to zero at approximately $x \approx -0.95$, and considering the behavior of the functions, this critical point is likely the smallest x where the slope of $F(x)$ is zero.

Verifying the Nature of the Critical Point

To confirm whether this critical point corresponds to a minimum or maximum, we examine the second derivative $F''(x)$.

1. Find $F''(x)$

Recall:

$$
F'(x) = 3 \times 2^x \ln 2 + 12x + 10
$$

Differentiate again:

\[
F''(x) = \frac{d}{dx} [3 \times 2^x \ln 2] + 12
\]

\[
F''(x) = 3 \times 2^x (\ln 2)^2 + 12
\]

Since \(2^x > 0 \), \(\ln 2 > 0 \), and the constants are positive, \(F''(x) > 0 \) for all \(x \). Therefore, the critical point at \(x \approx -0.95 \) is a local minimum.

2. Implication

This confirms that the smallest \(x \) where the slope of \(F(x) \) equals zero is approximately at \(x \approx -0.

Frequently Asked Questions

How do you find the smallest value of x when the function f(x) = 3x^2 - 2x - 10x + 8 has a slope of zero?

First, simplify the function, then find its derivative to set the slope to zero, and solve for x to identify the critical points. The smallest x-value among these is the smallest x where the slope is zero.

What is the derivative of the function f(x) = 3x^2 - 2x - 10x + 8, and how does it relate to finding when the slope is zero?

The derivative is f'(x) = 6x - 2 - 10. Setting f'(x) = 0 allows us to find the x-values where the slope of the function is zero.

How do you simplify the function f(x) = 3x^2 - 2x - 10x + 8 before finding its critical points?

Combine like terms: f(x) = 3x^2 - 12x + 8. Then, differentiate and set the derivative equal to zero.

What steps are involved in determining the smallest x-value where the

slope of $f(x) = 3x^2 - 12x + 8$ is zero?

Differentiate to find $f'(x) = 6x - 12$, set this equal to zero, solve for x, which gives $x = 2$. Since it's a quadratic with a positive leading coefficient, this is the minimum point.

Is the critical point at x=2 the smallest x-value where the slope of the function is zero?

Yes, since the derivative is linear and crosses zero at x=2, and the parabola opens upward, x=2 is the smallest x-value where the slope is zero, corresponding to the vertex of the parabola.

What is the significance of finding the point where the derivative equals zero in relation to the function's graph?

Finding where the derivative equals zero identifies the critical points, which correspond to local minima or maxima; in this case, it indicates the lowest point on the parabola where the slope is zero.

Additional Resources

What Is The Smallest Value Of X When The Function F(x)= 3 2 X 3 2x 2 10x+8, Has A Slope That Equals 0

Introduction

Understanding the behavior of functions, especially their critical points where slopes change, is fundamental in calculus. Among these, identifying the smallest value of x where the slope of a function equals zero is a key step in analyzing the function's local minima, maxima, and inflection points. This article delves into the specific function:

$$
F(x) = 3 \times 2^x + 3 \times 2x^2 + 10x + 8
$$

and aims to determine the smallest value of x such that its derivative, representing the slope, equals zero. Through a comprehensive exploration of the function's derivatives, critical points, and their nature, we will answer the central question with clarity and mathematical rigor.

Understanding the Function

Before proceeding, it’s essential to interpret the given function correctly. The notation suggests the function:

$$
F(x) = 3 \times 2^x + 3 \times 2x^2 + 10x + 8
$$

which can be written explicitly as:

$$
F(x) = 3 \cdot 2^x + 6x^2 + 10x + 8
$$

This function combines exponential and polynomial components, making its derivative and critical point analysis more interesting.

Derivative of the Function

To find where the slope of the function is zero, we need to compute the first derivative $F'(x)$:

$$
F'(x) = \frac{d}{dx} \left(3 \cdot 2^x + 6x^2 + 10x + 8 \right)
$$

Let's differentiate each term separately:

- Derivative of $3 \cdot 2^x$:

Using the exponential differentiation rule:

$$
\frac{d}{dx} 2^x = 2^x \ln 2
$$

Thus,

$$
\frac{d}{dx} 3 \cdot 2^x = 3 \cdot 2^x \ln 2
$$

- Derivative of $6x^2$:

$$
\frac{d}{dx} 6x^2 = 12x
$$

- Derivative of $10x$:

$$
\frac{d}{dx} 10x = 10
$$

- Derivative of constant 8:

$$
\frac{d}{dx} 8 = 0
$$

Putting it all together:

$$
F'(x) = 3 \cdot 2^x \ln 2 + 12x + 10
$$

Setting the Derivative Equal to Zero

To find the critical points where the slope is zero:

$$
F'(x) = 0 \Rightarrow 3 \cdot 2^x \ln 2 + 12x + 10 = 0
$$

Rearranged as:

$$
3 \cdot 2^x \ln 2 = - (12x + 10)
$$

or

$$

$$
2^x = - \frac{12x + 10}{3 \ln 2}
$$

This is a transcendental equation involving both exponential and linear terms in x, which generally doesn't lend itself to algebraic solutions. Therefore, numerical methods or graphing techniques are necessary to find solutions.

Analyzing the Critical Points

Nature of the Equation

The key equation:

$$
2^x = - \frac{12x + 10}{3 \ln 2}
$$

has the following characteristics:

- The left side, 2^x, is always positive for all real x.

- The right side, $- (12x + 10) / (3 \ln 2)$, can be positive or negative depending on x.

Since $2^x > 0$, the right side must also be positive, which implies:

$$
- (12x + 10) / (3 \ln 2) > 0
$$

Because $3 \ln 2 > 0$, this inequality becomes:

$$
- (12x + 10) > 0 \Rightarrow 12x + 10 < 0
$$

which simplifies to:

$$
12x < -10 \Rightarrow x < - \frac{10}{12} = - \frac{5}{6} \approx -0.8333
$$

Conclusion:

- Critical points where $F'(x) = 0$ can only occur for $x < -0.8333$.

Numerical Approximation of Critical Points

Given the nature of the equation, applying numerical methods such as the Newton-Raphson method or graphing is effective. For illustrative purposes, we can analyze the function graphically or estimate critical points:

Step 1: Find approximate values where the equation holds.

Let's try some values:

- At $x = -1$:

$$
2^{-1} = 0.5
$$

$$
- (12 \times -1 + 10) / (3 \ln 2) = - (-12 + 10) / (3 \times 0.6931) = - (-2) / 2.079 \approx 0.962
$$

Compare:

$$
LHS: 0.5, \quad RHS: 0.962
$$

LHS < RHS, so the equality isn't met at $x=-1$.

- At $x = -2$:

$$
2^{-2} = 0.25
$$

$$
- (12 \times -2 + 10) / (3 \times 0.6931) = - (-24 + 10) / 2.079 = - (-14) / 2.079 \approx 6.73
$$

LHS: 0.25, RHS: 6.73 → LHS < RHS.

- At $x = -0.9$:

$$
2^{-0.9} \approx 0.535
$$

$$
- (12 \times -0.9 + 10) / 2.079 = - (-10.8 + 10) / 2.079 = - (-0.8) / 2.079 \approx 0.385
$$

Compare:

$$
0.535 \text{ vs } 0.385
$$

LHS > RHS; so the solution is between -1 and -0.9.

- At $x = -0.95$:

$$
2^{-0.95} \approx 0.52
$$

$$
- (12 \times -0.95 + 10) / 2.079 = - (-11.4 + 10) / 2.079 = - (-1.4) / 2.079 \approx 0.673
$$

LHS: 0.52, RHS: 0.673 → LHS < RHS.

Between -0.95 and -0.9, the equation crosses equality.

Refining further, we find the critical point near $x \approx -0.92$.

Determining the Smallest x with $F'(x) = 0$

Since the critical points exist only for $x < -0.8333$, and the approximate solution is around -0.92, this is the relevant critical point for the smallest x where the slope is zero.

Is this a minimum or maximum?

To classify the critical point, analyze the second derivative:

$$
F''(x) = \frac{d}{dx} \left(3 \cdot 2^x \ln 2 + 12x + 10 \right) = 3 \cdot 2^x (\ln 2)^2 + 12
$$

- Since $2^x > 0$ for all x, and $\ln 2 > 0$, both terms are positive.

- Therefore,

$$
F''(x) > 0 \quad \text{for all } x
$$

implying the critical point is a local minimum.

Final Answer and Significance

The smallest value of x when the function $F(x)= 3 \cdot 2^x + 6x^2 + 10x + 8$ has a slope of zero is approximately $\boxed{-0.92}$.

This critical point represents the lowest local point on the function, which can be vital in optimization problems or in understanding the behavior of the function across its domain.

Broader Implications and Applications

Identifying such critical points is essential in various fields:

- Economics: Optimizing profit or cost functions involving exponential growth and polynomial terms.
- Biology: Understanding population models where growth rates change.
- Engineering: Designing systems with extremal performance points.

Furthermore, the methodology outlined—derivative calculation, equation analysis, numerical approximation—serves as a template for tackling similar problems involving transcendental equations

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