What Is The Standard Form Of The Equation Of The Circle In The Graph? (x - 3)2 + (y + 1)2 = 2(x + 3)2

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Understanding the equation of a circle is fundamental in coordinate geometry, a branch of mathematics that deals with the study of geometric figures using algebraic equations. When graphing circles, one of the most crucial concepts is the standard form of their equations, which provides insight into the circle's center and radius. The given equation, $(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2$, appears complex at first glance, but with systematic algebraic manipulation and geometric interpretation, it can be transformed into a standard form that clearly reveals the circle's properties.

In this comprehensive article, we will explore the concept of the standard form of a circle's equation, analyze the provided equation step-by-step, and understand how to convert complex circle equations into their standard form. Whether you're a student preparing for exams or a mathematics enthusiast, this guide aims to clarify the process and deepen your understanding of circles in the coordinate plane.

Understanding the Equation of a Circle

Standard Form of a Circle's Equation

The most common and recognizable form of a circle's equation in a Cartesian coordinate system is the standard form:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:
- (h, k) is the center of the circle.
- r is the radius of the circle.

This form makes it straightforward to identify the circle's key features:
- The center directly from the (h, k) values.
- The radius by taking the square root of the right side of the equation.

For example, the circle with the equation $(x - 2)^2 + (y + 3)^2 = 16$ has a center at $(2, -3)$ and a radius of 4 units.

General Form of a Circle's Equation

In contrast, the general form of a circle's equation is:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

While this form is useful for algebraic manipulation, it doesn't directly reveal the circle's center or radius without completing the square.

Analyzing the Given Equation

The equation provided is:

$$
(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2
$$

At first glance, this looks like a combination of quadratic expressions involving both x and y, with an additional scaling factor on the right side. Our goal is to manipulate this into the standard form to clearly identify the circle's center and radius.

Step-by-Step Conversion to Standard Form

Step 1: Expand Both Sides

Let's expand both sides of the equation:

Left side:

$$
(x - 3)^2 = x^2 - 6x + 9
$$

$$
(y + 1)^2 = y^2 + 2y + 1
$$

Adding these:

$$
x^2 - 6x + 9 + y^2 + 2y + 1 = x^2 + y^2 - 6x + 2y + 10
$$

Right side:

$$
2(x + 3)^2 = 2(x^2 + 6x + 9) = 2x^2 + 12x + 18
$$

Updated equation:

$$
x^2 + y^2 - 6x + 2y + 10 = 2x^2 + 12x + 18
$$

Step 2: Bring All Terms to One Side

Subtract $2x^2 + 12x + 18$ from both sides:

$$
x^2 + y^2 - 6x + 2y + 10 - 2x^2 - 12x - 18 = 0
$$

Combine like terms:

$$
x^2 - 2x^2 = -x^2
$$

$$
-6x - 12x = -18x
$$

$$
10 - 18 = -8
$$

So, the simplified form is:

$$
- x^2 + y^2 - 18x + 2y - 8 = 0
$$

Multiplying through by -1 to make the quadratic terms positive:

$$
x^2 - y^2 + 18x - 2y + 8 = 0
$$

Step 3: Rearrange Terms

Group x and y terms:

$$
x^2 + 18x - y^2 - 2y + 8 = 0

\]

Now, we aim to complete the square for the x and y terms.

Step 4: Complete the Square

For the x terms:

$$
x^2 + 18x
$$

Complete the square:

$$
x^2 + 18x + \left(\frac{18}{2}\right)^2 = x^2 + 18x + 81
$$

We add 81, but to keep the equation balanced, we must subtract it later.

For the y terms:

$$
- y^2 - 2y
$$

Factor out -1:

\[
- (y^2 + 2y)
\]

Complete the square inside the parentheses:

\[
y^2 + 2y + 1 = (y + 1)^2
\]

Since we added 1 inside the parentheses, we must subtract it in the original equation:

\[
- (y^2 + 2y + 1) + 1 = - (y + 1)^2 + 1
\]

Now, rewriting the entire equation:

\[
(x^2 + 18x + 81) - (y + 1)^2 + 1 + 8 = 0
\]

Combine constants:

\[
81 + 1 + 8 = 90
\]

Thus, the equation becomes:

\[

(x + 9)^2 - (y + 1)^2 + 90 = 0

\]

Expressing the Equation in Standard Form

Rearranged:

\[

(x + 9)^2 - (y + 1)^2 = -90

\]

This is a difference of squares form, similar to the equation of a hyperbola, not a circle. Since the original question asks about the standard form of a circle, this suggests that the given equation does not represent a circle in its current form. Instead, it represents a hyperbola.

Important note: For an equation to represent a circle, the quadratic terms must be \(x^2 + y^2\) with equal coefficients and no subtraction between them.

Interpreting the Result

The key insight here is that the original equation:

\[

(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2

\]

does not directly conform to the standard form of a circle because, after algebraic manipulation, it reduces to a hyperbola's equation. This indicates that the initial equation describes a different conic section.

However, if the goal is to understand the general process of converting circle equations into standard form, the above steps demonstrate how to manipulate quadratic equations, complete the square, and analyze the form of the conic.

When Does an Equation Represent a Circle?

A quadratic equation in two variables represents a circle if:

- The quadratic terms x^2 and y^2 have the same coefficients.
- There are no cross-product terms like xy.
- The equation can be rearranged into the form $(x - h)^2 + (y - k)^2 = r^2$.

In the given equation, due to the subtraction of y^2, it resembles a hyperbola, not a circle.

Conclusion: What Is The Standard Form Of The Equation Of The Circle?

The standard form of a circle's equation is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:

- (h, k) is the center.
- r is the radius.

Transforming a quadratic equation into this form involves:

1. Expanding and simplifying the original equation.
2. Grouping x and y terms.
3. Completing the square for both variables.
4. Rearranging to isolate the perfect squares on one side.

In the case of the provided equation, after algebraic manipulation, it does not conform to the circle's standard form but instead represents a different conic section—a hyperbola.

Additional Tips for Recognizing Circle Equations

- Look for equations where the

Frequently Asked Questions

What is the standard form of the equation of a circle?

The standard form of a circle's equation is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is its radius.

How can I identify the center and radius from the equation $(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2$?

First, rewrite the equation in a form that consolidates all terms to one side. Then, expand and simplify to match the standard form, allowing you to identify the center (h, k) and radius r.

How do I convert the given equation $(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2$ into standard form?

Expand both sides: $(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2$. Then, expand the right side to $2(x^2 + 6x + 9)$. Simplify and rearrange all terms to complete the square, leading to the standard form.

What is the center and radius of the circle described by $(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2$?

After rewriting and simplifying, the center can be identified from the completed square form, and the radius is the square root of the constant term. The exact values depend on the simplified form.

Is the given equation $(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2$ a standard form of a circle?

No, it is not in standard form. It is an equation involving both squared terms on each side. It needs to be expanded and rearranged into $(x - h)^2 + (y - k)^2 = r^2$ for the standard form.

What steps are involved in converting the given equation into the standard circle form?

Expand both sides, collect like terms, move all to one side, complete the square for x and y terms if necessary, and simplify to get $(x - h)^2 + (y - k)^2 = r^2$.

Can the given equation represent a circle, or is it an ellipse or another conic?

Given the form involving squared terms of x and y, and assuming the equation simplifies to a form with only squared terms, it likely represents a circle. However, the specific form after simplification should be checked to confirm.

Additional Resources

Understanding the Standard Form of the Equation of a Circle in the Graph

When exploring the fascinating world of geometry and algebra, one of the fundamental shapes students and mathematicians encounter is the circle. The equation of a circle not only encapsulates its geometric essence but also offers a gateway into understanding symmetry, radius, and center points in a coordinate plane. In this article, we will take an in-depth look at the standard form of the equation of a circle, illustrated through the example:

$$
(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2
$$

Our goal is to decode this equation, interpret its components, and understand how it relates to the geometric properties of a circle. Think of this as a comprehensive guide that transforms a seemingly complex algebraic expression into a clear geometric picture.

What Is the Standard Form of a Circle's Equation?

The standard form of a circle's equation is a specific algebraic expression that directly reveals the circle’s center and radius. It is commonly written as:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:

- (h, k) are the coordinates of the center of the circle.
- r is the radius of the circle.

This form makes it straightforward to identify the key attributes of the circle without additional calculations. For example, if the equation is:

$$
(x - 2)^2 + (y + 4)^2 = 25
$$

then the circle has:

- Center at $(2, -4)$
- Radius $r = \sqrt{25} = 5$

Deciphering the Given Equation

Let's consider the specific equation:

\[
(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2
\]

At first glance, this resembles the general form of a circle's equation but contains an extra term on the right side, making it more complex. Our task is to manipulate this equation into the standard form that clearly identifies the circle's center and radius.

Step 1: Expand Both Sides

Begin by expanding both sides to simplify the equation:

Left Side:

\[
(x - 3)^2 + (y + 1)^2
\]

Expanding:

\[
(x^2 - 6x + 9) + (y^2 + 2y + 1)
\]

which simplifies to:

$$
x^2 - 6x + 9 + y^2 + 2y + 1
$$

or

$$
x^2 + y^2 - 6x + 2y + 10
$$

Right Side:

$$
2(x + 3)^2
$$

Expanding:

$$
2(x^2 + 6x + 9) = 2x^2 + 12x + 18
$$

Step 2: Bring All Terms to One Side

Set the equation to zero:

\[
x^2 + y^2 - 6x + 2y + 10 - (2x^2 + 12x + 18) = 0
\]

Simplify:

\[
x^2 + y^2 - 6x + 2y + 10 - 2x^2 - 12x - 18 = 0
\]

Combine like terms:

\[
(x^2 - 2x^2) + y^2 + (-6x - 12x) + 2y + (10 - 18) = 0
\]

which simplifies to:

\[
- x^2 + y^2 - 18x + 2y - 8 = 0
\]

Step 3: Rearrange and Prepare for Completing the Square

Rewrite the equation:

\[
- x^2 + y^2 - 18x + 2y = 8

\]

To make the equation resemble the standard circle form, multiply through by \(-1\) to make the quadratic terms positive:

\[
x^2 - y^2 + 18x - 2y = -8
\]

But notice the presence of \(- y^2 \). For a circle, the general quadratic form must have \(+ y^2\), not \(- y^2\). The \(- y^2\) indicates that the conic might not be a circle after all. However, let's proceed to verify.

Identifying the Type of Conic

The key is to analyze the quadratic terms:

\[
x^2 - y^2 + 18x - 2y = -8
\]

The presence of both \(x^2\) and \(- y^2\) suggests this is a hyperbola or other conic, not a circle, since a circle requires the coefficients of \(x^2\) and \(y^2\) to be equal and positive.

Conclusion: The given equation, after expansion and simplification, does not represent a circle but a hyperbola or another conic section. Thus, the original form:

\[

$$
(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2
$$

is an equation of a circle only if the right side is a perfect square of a single constant, which it isn't in this case.

Re-examining the Equation: Is It a Circle?

Given that the right side is $2(x + 3)^2$, unless this simplifies to a perfect square, it indicates that the initial equation is not in the standard circle form. To confirm, let's analyze the structure:

- The left side is a sum of two squared binomials.
- The right side is twice a squared binomial.

This suggests the equation is more complex and likely represents a general conic rather than a circle.

What Are the Conditions for a Circle?

For an equation to represent a circle in the coordinate plane:

1. It must be quadratic in both x and y.
2. The coefficients of x^2 and y^2 must be equal.
3. There should be no xy term (or it should be eliminable).

Our original equation does not satisfy these conditions, particularly because the right side is not a simple constant but a multiple of a squared binomial, complicating its classification.

Transforming the Equation Into a Recognized Form (If Possible)

Suppose the goal is to express the given equation in a form similar to the standard form of a circle, perhaps by rearranging or substituting.

Alternative Approach:

- Consider the original equation as an implicit relation and attempt to isolate terms.
- Recognize that the equation involves quadratic expressions on both sides, hinting at a more advanced conic.

Summary of Key Takeaways

- The standard form of a circle's equation is:

\[
(x - h)^2 + (y - k)^2 = r^2
\]

- The given equation:

\[
(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2

\]

does not conform to the standard circle form because the right side is not a constant but a quadratic expression involving x.

- Conclusion: The equation represents a non-circular conic—likely a hyperbola or parabola—due to the presence of quadratic terms on both sides and the structure of the equation.

Final Insights and Practical Implications

Understanding the standard form of the circle's equation is crucial for graphing, analyzing geometric properties, and solving related problems. When presented with complex equations like the one examined here, the key steps involve:

- Expanding and simplifying algebraic expressions.
- Recognizing the form of the equation—whether it depicts a circle, ellipse, parabola, or hyperbola.
- Converting the equation into a recognizable standard form via completing the square or algebraic manipulation.

In practical applications, this knowledge enables mathematicians, engineers, and students to quickly interpret the geometric nature of equations and accurately plot their graphs.

Conclusion

While the initial question was about the standard form of the equation of a circle, the specific example provided reveals the importance of analyzing and understanding the structure of quadratic equations. Not all equations with squared terms are circles; their coefficients and arrangement determine their classification.

In this case, the equation $(x - 3)^2 + (y + 1)^2 = 2(x + 3)^2$ does not represent a circle in the standard form. Instead, it exemplifies a more complex conic section, emphasizing the significance of algebraic manipulation and recognition in geometric analysis.

By mastering the process of transforming and identifying equations, students and professionals alike can deepen their comprehension of the beautiful interplay between algebra and geometry, unlocking the full potential of graphing and conic sections.

Key Takeaways Summary:

- Standard form of a circle: $\boxed{(x - h)^2 + (y - k)^2 = r^2}$
- The given

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