What Is The Sum Of Twice A Number And Five Is At Least Half The Difference Of Six And The Same Number

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Understanding algebraic expressions and inequalities can seem daunting at first, but breaking them down step-by-step makes them much more approachable. In this article, we will explore the problem: "What is the sum of twice a number and five is at least half the difference of six and the same number." We will analyze this statement, translate it into a mathematical inequality, and solve for the unknown number. By the end, you'll have a clear understanding of how to approach similar algebraic problems and interpret their solutions.

Deciphering the Problem Statement

Before jumping into calculations, it's crucial to understand the components of the problem:

- "Twice a number" suggests multiplying an unknown number, say x, by 2.
- "Sum of twice a number and five" indicates adding 5 to this doubled number, resulting in 2x + 5.
- "At least" signifies a greater than or equal to relationship.
- "Half the difference of six and the same number" involves calculating the difference between 6 and x, then halving that difference, which expresses as (6 - x)/2.

Putting it all together, the problem asks us to find the set of values for x such that:

> 2x + 5 ≥ (6 - x)/2

This inequality captures the essence of the problem, where the left side is "the sum of twice a number and five," and the right side is "half the difference of six and the same number." Our goal is to solve this inequality and interpret the solution.

Converting the Word Problem into an Algebraic Inequality

To analyze the inequality, let's formalize it:

Inequality:

$$ 2x + 5 \geq \frac{6 - x}{2} $$

This is a straightforward algebraic inequality involving a rational expression. To solve it, we need to eliminate the denominator and isolate x.

Step 1: Eliminate the Fraction

Multiply both sides of the inequality by 2 to clear the denominator:

$$ 2 \times (2x + 5) \geq 6 - x $$

which simplifies to:

$$ 4x + 10 \geq 6 - x $$

Step 2: Collect Like Terms

Bring all terms involving x to one side and constants to the other:

$$ 4x + x \geq 6 - 10 $$

$$ 5x \geq -4 $$

Step 3: Solve for x

Divide both sides by 5:

$$ x \geq -\frac{4}{5} $$

This is our solution set: all real numbers greater than or equal to $-\frac{4}{5}$.

Interpreting the Solution

The inequality $ x \geq -\frac{4}{5} $ implies that any real number $ x $ that is greater than or equal to -0.8 satisfies the original problem statement.

Visualizing the Solution

If we plot this on a number line:

- The boundary is at $-\frac{4}{5}$.
- The solution includes all points to the right of -0.8, including -0.8 itself.

Practical Implications

Suppose the unknown number x represents a certain quantity, like a score, measurement, or a variable in a real-world scenario. The solution indicates that as long as x is at least -0.8, the condition "the sum of twice the number and five is at least half the difference of six and the number" holds true.

Verification of the Solution

To ensure our solution is correct, let's verify with specific values of x:

- Test with $x = -1$:

$$2(-1) + 5 = -2 + 5 = 3$$

$$\frac{6 - (-1)}{2} = \frac{6 + 1}{2} = \frac{7}{2} = 3.5$$

Check if:

$$3 \geq 3.5$$?

No, so $x = -1$ does not satisfy the inequality, consistent with our solution that x must be ≥ -0.8.

- Test with $x = 0$:

$$2(0) + 5 = 0 + 5 = 5$$

$$\frac{6 - 0}{2} = \frac{6}{2} = 3$$

Check if:

$$5 \geq 3$$?

Yes, so $x=0$ satisfies the inequality.

- Test with $x = 1$:

$$2(1) + 5 = 2 + 5 = 7$$

$$\frac{6 - 1}{2} = \frac{5}{2} = 2.5$$

Check:

$$7 \geq 2.5$$?

Yes, confirming the solution set includes values greater than or equal to -0.8.

Summary of verification:

- Values $x \geq -0.8$ satisfy the inequality.

- Values less than -0.8 do not satisfy the inequality.

Broader Mathematical Context

This problem illustrates a common type of inequality involving linear expressions and rational terms. Understanding how to manipulate these expressions through algebraic operations—such as multiplying both sides by positive numbers, collecting like terms, and solving for the variable—is fundamental to algebra.

Key concepts involved include:

- Linear inequalities
- Rational expressions
- Solution sets and interval notation
- Verification of solutions

Real-World Applications of Similar Inequalities

Inequalities like this appear frequently in various fields:

- Economics: Determining acceptable ranges of investment, interest rates, or resource allocations.
- Physics: Establishing bounds for measurements or physical quantities.
- Statistics: Defining confidence intervals or thresholds.
- Engineering: Ensuring system parameters stay within safe operational limits.

Understanding how to set up and solve inequalities allows professionals to model constraints and make informed decisions based on variable relationships.

Conclusion

The problem "What is the sum of twice a number and five is at least half the difference of six and the same number" translates into the inequality:

$$ 2x + 5 \geq \frac{6 - x}{2} $$

which simplifies to:

$$ x \geq -\frac{4}{5} $$

The solution indicates that all real numbers greater than or equal to -0.8 satisfy the original statement. Mastering the process of translating word problems into algebraic inequalities, solving them step-by-step, and verifying the solutions is a vital skill in mathematics. It not only enhances problem-solving abilities but also prepares you to handle

complex real-world situations involving constraints and relationships between variables.

Remember: Always verify your solutions and interpret them within the context of the problem. With practice, solving inequalities becomes an intuitive and powerful tool in your mathematical toolkit.

Frequently Asked Questions

What is the main mathematical expression involved in the problem?

The problem involves the expression 'twice a number plus five' and relates it to 'at least half the difference of six and the same number.'

How can I translate the phrase 'twice a number' into an algebraic expression?

'Twice a number' can be expressed as 2x, where x represents the number.

What does the phrase 'at least' indicate in the inequality?

It indicates that the expression on one side is greater than or equal to the expression on the other side.

How do I interpret 'half the difference of six and the same number' algebraically?

The difference of six and the number is (6 - x), so half of that is (1/2)(6 - x).

What inequality do I set up based on the problem description?

You set up the inequality: $2x + 5 \geq (1/2)(6 - x)$.

How can I solve the inequality to find possible values of x?

Multiply both sides by 2 to clear the fraction, then solve the resulting linear inequality for x.

What is the solution set for x after solving the inequality?

Solving $2x + 5 \geq (6 - x)$ leads to $x \geq 1/3$, so all real x greater than or equal to 1/3 satisfy the

inequality.

What does this problem teach about translating word problems into inequalities?

It demonstrates how to identify key phrases, convert them into algebraic expressions, and set up inequalities to find solutions.

Additional Resources

Mathematical Expression Analysis

Introduction: Deciphering the Mathematical Puzzle

Mathematics often presents us with intricate puzzles that challenge our reasoning and problem-solving skills. One such intriguing problem is:

“What is the sum of twice a number and five is at least half the difference of six and the same number?”

At first glance, this sentence might seem straightforward, but it conceals a layered relationship between variables and constants that requires careful unpacking. As a mathematical expert, I will guide you through this problem's nuances, translating the words into algebraic expressions and solving the inequalities step-by-step.

This analysis will serve as a comprehensive review, not only clarifying the solution but also illustrating key concepts such as translating word problems into equations, understanding inequalities, and solving for variables.

Breaking Down the Problem: From Words to Variables

Understanding the Statement

The statement contains several components:

- "Twice a number" — This indicates multiplying a number, say x, by 2, giving 2x.

- "And five" — The number 5 is added to or involved with the previous expression.
- "Is at least" — This phrase signals an inequality, specifically "greater than or equal to."
- "Half the difference of six and the same number" — This refers to taking the difference between 6 and x, then halving that result.

In essence, the problem involves an inequality relating these components.

Identifying the Variables and Constants

- Variable: x, representing the unknown number.
- Constants: 5 and 6, fixed values in the problem.

Translating Words into an Algebraic Inequality

Step 1: Expressing “Twice a Number”

Mathematically, this is:
2x

Step 2: Adding Five

The phrase "the sum of twice a number and five" translates to:
2x + 5

Step 3: Understanding “At Least”

"At least" translates to the inequality symbol:
≥

Step 4: Expressing “Half the Difference of Six and the Same Number”

The difference of 6 and the number:
6 - x

Half of that difference:
(6 - x)/2

Step 5: Formulating the Inequality

Putting all parts together:
$2x + 5 \geq (6 - x)/2$

This inequality captures the original statement's meaning.

Solving the Inequality Step-by-Step

Step 1: Eliminate the Denominator

To simplify, multiply both sides by 2 (the denominator) to clear fractions:

$2 (2x + 5) \geq 2 [(6 - x)/2]$

This results in:

$4x + 10 \geq 6 - x$

Step 2: Bring Like Terms Together

Add x to both sides:

$4x + x + 10 \geq 6$
which simplifies to:
$5x + 10 \geq 6$

Subtract 10 from both sides:

$5x \geq 6 - 10$
which simplifies to:
$5x \geq -4$

Step 3: Isolate x

Divide both sides by 5:

$x \geq -4/5$

This is the solution to the inequality.

Interpreting the Result

The solution $x \geq -0.8$ indicates that any real number greater than or equal to -0.8 satisfies the original inequality.

From a practical perspective, this means:

- The number x can be exactly -0.8.
- Any number larger than -0.8 also satisfies the condition.
- Values less than -0.8 do not satisfy the inequality.

Visualizing the Solution Set

To better understand the solution, consider the number line:

- Mark the point -0.8 (which is -4/5).
- The solution set includes all points to the right of this point, including the point itself.
- In interval notation:
$[-4/5, \infty)$

This visual aid aids in understanding the range of solutions and their implications.

Additional Insights and Applications

Why Is This Important?

Understanding and solving such inequalities is fundamental in various real-world applications, including:

- Economics: Budget constraints often translate into inequalities.
- Engineering: Tolerance ranges for measurements.
- Statistics: Confidence intervals involve inequalities.

Mastering this process enhances critical thinking and analytical skills vital in scientific and technological fields.

Common Mistakes to Avoid

While solving inequalities, watch out for:

- Incorrectly multiplying or dividing by negative numbers: Remember to flip the inequality sign when multiplying/dividing by negatives.
- Neglecting to simplify: Always strive to reduce expressions to the simplest form before solving.
- Ignoring the solution set: Always interpret the algebraic solution in context, especially when dealing with real-world problems.

Summary of the Solution Process

To recap:

1. Translate the word problem into an algebraic inequality.
2. Eliminate fractions by multiplying through by the denominator.
3. Simplify and collect like terms.
4. Solve for the variable.
5. Interpret the solution in the context of the problem.
6. Visualize the solution set for clarity.

The final answer, $x \geq -4/5$, provides a comprehensive understanding of the original statement's meaning and the range of values satisfying the condition.

Final Thoughts: The Elegance of Algebra

This problem exemplifies how language and mathematics intertwine, transforming a simple sentence into an elegant inequality. It underscores the importance of careful translation, methodical solving, and insightful interpretation. Whether you're a student tackling algebra for the first time or a seasoned mathematician, such problems reinforce fundamental skills and deepen appreciation for the logical beauty of mathematics.

Mastering these steps not only solves the problem at hand but also equips you with tools to approach more complex situations involving inequalities, systems of equations, and real-world modeling with confidence and precision.

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