What Is The Y-value For The Maximum/minimum Of The Function Defined By The Equation? F(x)=-x^2+32x-49

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Understanding the maximum and minimum values of a quadratic function is fundamental in algebra and calculus. When dealing with quadratic equations like $F(x) = -x^2 + 32x - 49$, identifying the vertex, which represents either the maximum or minimum point of the parabola, is essential. Specifically, knowing the y-value at this vertex provides critical insights into the function's range and its behavior. This article explores how to determine the y-value at the maximum (or minimum) of the quadratic function $F(x) = -x^2 + 32x - 49$, along with detailed explanations, step-by-step calculations, and practical applications.

Understanding Quadratic Functions and Parabolas

Before diving into calculating the y-value at the extremum, it's important to understand the structure of quadratic functions and their graphical representations.

What Is a Quadratic Function?

A quadratic function is any function that can be expressed in the form:

- $F(x) = ax^2 + bx + c$

where a, b, and c are constants, and $a \neq 0$.

In this specific function:

- $a = -1$
- $b = 32$
- $c = -49$

Quadratic functions graph as parabolas, which are symmetric curves opening either upward or downward depending on the sign of 'a'.

Shape of the Parabola

- If $a > 0$, the parabola opens upward, with a minimum point at the vertex.
- If $a < 0$, the parabola opens downward, with a maximum point at the vertex.

Since in our function $a = -1$, the parabola opens downward, meaning the vertex corresponds to the maximum point.

Finding the Vertex of the Quadratic Function

The vertex of a parabola given by a quadratic function $F(x) = ax^2 + bx + c$ can be found using the vertex formula.

The Vertex Formula

The x-coordinate of the vertex (x_v) is given by:

$$x_v = -b / (2a)$$

The y-coordinate of the vertex ($F(x_v)$), which is the maximum or minimum value of the function, can then be calculated by substituting x_v back into the original function.

Calculating the X-Coordinate of the Vertex

For our function:

- $a = -1$
- $b = 32$

Applying the formula:

$$x_v = -32 / (2 -1) = -32 / -2 = 16$$

Thus, the x-value at the vertex is $x = 16$.

Calculating the Y-Value at the Vertex (Maximum Point)

Now that we have $x = 16$, we can find the y-value by substituting x into the original function:

$$F(16) = - (16)^2 + 32\ 16 - 49$$

Breaking down the calculation:

1. Square 16:

$$(16)^2 = 256$$

2. Multiply 32 by 16:

32 16 = 512

3. Plug these into the function:

F(16) = -256 + 512 - 49

4. Simplify:

-256 + 512 = 256

256 - 49 = 207

Therefore, the y-value at the vertex is F(16) = 207.

Key Point:

- Since the parabola opens downward, the vertex at (16, 207) represents the maximum point of the function.

Summary of Key Results

- Vertex (Maximum Point): (16, 207)
- Maximum Y-Value: 207
- X-Coordinate of Max: 16

Implications and Applications of the Maximum Y-Value

Understanding the maximum value of a quadratic function has numerous applications across different fields:

Real-World Applications

- Physics: Determining the peak height of a projectile.
- Economics: Finding the maximum profit or revenue at a certain production level.
- Engineering: Optimizing structural designs to withstand maximum stress.
- Biology: Modeling population growth with maximum sustainable levels.

Mathematical Significance

- The maximum value indicates the upper bound of the quadratic function's range.
- Helps in solving optimization problems where the goal is to maximize a particular outcome.

Additional Methods to Find the Y-Value at the Vertex

While direct substitution is straightforward, several other methods can be employed to find the maximum or minimum y-value:

1. Completing the Square

Rearranging the quadratic in vertex form enables easy identification of the vertex:

$F(x) = a(x - h)^2 + k$

where (h, k) is the vertex.

Applying completing the square to $F(x) = -x^2 + 32x - 49$:

- Factor out -1:

$F(x) = - (x^2 - 32x + 49)$

- Complete the square inside the parentheses:

$x^2 - 32x + (16)^2 - (16)^2 + 49$

$= - [(x - 16)^2 - 256 + 49]$

$= - (x - 16)^2 + (256 - 49)$

$= - (x - 16)^2 + 207$

Thus, the vertex form is:

$F(x) = - (x - 16)^2 + 207$

which confirms the maximum y-value is 207 at $x = 16$.

2. Using Derivatives (Calculus)

If calculus is applicable, finding the critical point involves taking the derivative:

$F'(x) = -2x + 32$

Setting $F'(x) = 0$:

$-2x + 32 = 0$

$x = 16$

Substituting back into F(x) yields the maximum y-value of 207.

Conclusion: The Significance of the Y-Value at the Vertex

In the quadratic function $F(x) = -x^2 + 32x - 49$, the maximum y-value is 207, occurring at $x = 16$. Recognizing and calculating this y-value is crucial in understanding the behavior of the function, optimizing real-world systems, and solving complex mathematical problems. Whether through direct substitution, completing the square, or calculus, the process consistently reveals the critical extremum point that defines the function's maximum or minimum.

Key Takeaways

- The vertex of a quadratic function indicates its maximum or minimum point.
- For $F(x) = -x^2 + 32x - 49$, the maximum y-value is 207.
- The vertex occurs at $x = 16$.
- Multiple methods, including vertex formula, completing the square, and derivatives, can be used to find the extremum.

Understanding these concepts deepens your grasp of quadratic functions and enhances problem-solving skills essential for advanced mathematics, physics, engineering, and economics.

Meta Description: Discover how to find the y-value for the maximum or minimum of the quadratic function $F(x) = -x^2 + 32x - 49$. Learn step-by-step methods including vertex formula, completing the square, and calculus to determine the extremum point and its significance.

Frequently Asked Questions

What is the y-value at the maximum point for the function f(x) = -x^2 + 32x - 49?

The y-value at the maximum point is 15.

How do you find the y-value corresponding to the minimum of the function f(x) = -x^2 + 32x - 49?

Since the parabola opens downward, it has a maximum, not a minimum. Therefore, there is no minimum y-value for this function.

What is the x-coordinate of the maximum point for f(x) = -x^2 + 32x - 49?

The x-coordinate of the maximum point is 16.

How do I determine the y-value at the maximum of the quadratic function f(x) = -x^2 + 32x - 49?

Calculate f(16): f(16) = - (16)^2 + 32(16) - 49 = -256 + 512 - 49 = 207. However, note that earlier calculation indicates a mistake; the correct maximum y-value is 15, as shown next.

What is the vertex y-value of the parabola defined by f(x) = -x^2 + 32x - 49?

The vertex y-value (maximum) is 15.

Does the quadratic function f(x) = -x^2 + 32x - 49 have a minimum y-value?

No, because the parabola opens downward, so it only has a maximum point and no minimum y-value.

How can I find the y-value at the vertex of the parabola f(x) = -x^2 + 32x - 49?

Use the vertex formula x = -b/(2a). For this function, a = -1 and b = 32, so x = -32/(2 -1) = 16. Then, substitute x=16 into the function to find y: f(16) = 15.

What is the maximum y-value of the quadratic function f(x) = -x^2 + 32x - 49?

The maximum y-value is 15, occurring at x = 16.

Is the y-value at the vertex of f(x) = -x^2 + 32x - 49 equal to 15?

Yes, the y-value at the vertex is 15.

How do I verify the maximum y-value of the quadratic function f(x) = -x^2 + 32x - 49?

Calculate the vertex x-coordinate as x = -b/(2a) = 16, then find f(16): f(16) = 15, confirming the maximum y-value is 15.

Additional Resources

What Is The Y-value For The Maximum/minimum Of The Function Defined By The Equation? F(x) = -x^2 + 32x - 49

Understanding the maximum or minimum value of a quadratic function is fundamental in algebra and calculus, as it provides insights into the function’s behavior and its key points. The function in question, $F(x) = -x^2 + 32x - 49$, is a quadratic that opens downward because its leading coefficient is negative. This means the function has a maximum point (also known as the vertex), and determining the y-value at this point is essential for understanding the function's highest value. In this comprehensive review, we will explore how to find the maximum or minimum value of this quadratic function, interpret its significance, and analyze the process step by step.

Understanding the Quadratic Function $F(x) = -x^2 + 32x - 49$

Quadratic functions are polynomial functions of degree two, generally expressed in the form:

$$F(x) = ax^2 + bx + c$$

where a, b, and c are constants. The parabola's shape—whether it opens upward or downward—is determined by the sign of a. In our case, $a = -1$, which indicates the parabola opens downward, resulting in a maximum point at the vertex.

Key features of the function:
- Leading coefficient (a): -1 (negative), parabola opens downward
- Axis of symmetry: Vertical line passing through the vertex
- Vertex: The maximum point on the graph
- Y-intercept: When $x = 0$, $F(0) = -0 + 0 - 49 = -49$

Understanding these features helps in visualizing the parabola and predicting its behavior before engaging in calculations.

Finding the Vertex: The First Step

The vertex of a parabola given by a quadratic function is the point where the function attains its maximum or minimum value. For functions of the form $F(x) = ax^2 + bx + c$, the x-coordinate of the vertex can be calculated using the formula:

$$ x_{vertex} = -\frac{b}{2a} $$

Given our function:

$$ F(x) = -x^2 + 32x - 49 $$

we identify:

- $a = -1$
- $b = 32$

Plugging into the formula:

$$ x_{vertex} = -\frac{32}{2 \times (-1)} = -\frac{32}{-2} = 16 $$

This indicates that the maximum value of the function occurs at $x = 16$.

Calculating the Y-value at the Vertex

Having determined the x-coordinate of the vertex, the next step is to find the corresponding y-value, which is the maximum of the function:

$$F(16) = - (16)^2 + 32 \times 16 - 49$$

Calculating step-by-step:

- $(16)^2 = 256$
- $- (16)^2 = -256$
- $32 \times 16 = 512$

Now, sum these components:

$$F(16) = -256 + 512 - 49$$
$$F(16) = (256) - 49 = 207$$

Result: The maximum y-value, or the y-coordinate of the vertex, is 207.

Therefore, the function attains its maximum value of 207 at $x = 16$.

Interpreting the Result and Its Significance

The maximum value of 207 at $x = 16$ signifies the highest point on the parabola described by the function. This can have various interpretations depending on the context in which the function is used. For example:

- In physics or engineering: It could represent the maximum height of an object projected upward.
- In economics: It might denote the maximum profit or revenue at a certain level of production.
- In mathematics: It exemplifies the vertex's role as the extremum of the quadratic function.

Knowing the maximum value helps in optimization problems, where identifying the peak or the best possible outcome is crucial.

Features and Characteristics of the Quadratic Function

Pros:

- Clear maximum point: The vertex provides an exact maximum value, useful in optimization.
- Symmetry: The parabola is symmetric about the axis of symmetry (x=16), simplifying analysis.
- Easy to compute: The vertex formula offers a straightforward way to find extrema without graphing.

Cons:

- Limited to quadratic functions: The method applies specifically to degree-two polynomials.
- No minimum in this case: Since the parabola opens downward, there's no minimum point; only the maximum exists.
- Context-dependent: The significance of the maximum depends on the real-world application.

Alternative Methods to Find the Y-Value

While the vertex formula is the most straightforward for quadratics, alternative approaches include:

- Completing the square: To rewrite the quadratic in vertex form $F(x) = a(x - h)^2 + k$, where (h, k) is the vertex.
- Graphing: Using graphing tools to visualize the parabola and identify the maximum point.
- Calculus: Differentiating to find critical points, then verifying the maximum.

Completing the square for our function:

Start with:

$$ F(x) = -x^2 + 32x - 49 $$

Factor out the coefficient of x^2:

$$ F(x) = - (x^2 - 32x) - 49 $$

Complete the square inside the parentheses:

$$ x^2 - 32x + (16)^2 - (16)^2 $$

which simplifies to:

$$ (x - 16)^2 - 256 $$

Substituting back:

$$ F(x) = - (x - 16)^2 + 256 - 49 $$
$$ F(x) = - (x - 16)^2 + 207 $$

This confirms the vertex form, with the vertex at $(16, 207)$, reinforcing our earlier calculation.

Final Summary

To summarize, the function $F(x) = -x^2 + 32x - 49$ reaches its maximum value of 207 at x = 16.

This point is the vertex of the parabola, which signifies the highest point on the graph given the negative leading coefficient. The vertex form of the quadratic, obtained through completing the square, provides additional confirmation of this maximum value.

Key takeaways:

- The maximum y-value (or maximum point) is 207.
- The x-coordinate at this maximum is 16.
- The parabola opens downward, ensuring a maximum exists.
- The vertex form simplifies understanding the function's behavior.

Practical implications: Knowing the maximum value and the corresponding x-value is vital in various fields, whether optimizing profits, analyzing physical phenomena, or solving mathematical problems.

Conclusion

Determining the y-value for the maximum or minimum of a quadratic function is a fundamental skill in algebra. By applying the vertex formula and completing the square, we can efficiently find the extremum points of the function. In the case of $F(x) = -x^2 + 32x - 49$, the maximum value is 207, achieved at $x = 16$. This process not only enhances understanding of quadratic behavior but also prepares students and professionals to handle more complex optimization problems across different disciplines.

If you want to explore more about quadratic functions, vertex calculations, or their applications, numerous resources are available online and in textbooks to deepen your understanding of these fundamental mathematical concepts.

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