When The Product Of 6 And The Square Of A Number Is Increased By 5 Times The Number, The Result Is 4.

When The Product Of 6 And The Square Of A Number Is Increased By 5 Times The Number, The Result Is 4. This statement presents a fascinating algebraic problem that involves translating a word problem into a mathematical equation and then solving it. Such problems are foundational in algebra and help develop problem-solving skills, logical reasoning, and understanding of quadratic equations. In this article, we will explore how to formulate the given statement as an algebraic expression, solve for the unknown number, analyze the solution, and understand the broader applications of such problems.

Understanding the Problem Statement

Before diving into the mathematical formulation, let's carefully analyze the problem statement:

- The phrase "Product of 6 and the square of a number" implies multiplication involving 6 and the square of an unknown number.
- "Increased by 5 times the number" indicates adding five times the same unknown number to the previous product.
- The "result is 4" shows the entire expression equals 4.

This problem often appears in algebra exercises, and understanding each component is vital for setting up the correct equation.

Translating the Word Problem into an Equation

The first step in solving algebraic problems is converting the word problem into a mathematical equation.

Step 1: Define the Variable

Let the unknown number be represented by:

- x

Step 2: Express Each Part Mathematically

- "Product of 6 and the square of a number" translates to: $6x^2$
- "5 times the number" translates to: $5x$

Step 3: Combine the Parts According to the Statement

Adding these together:

$$
6x^2 + 5x
$$

According to the problem, this sum equals 4:

$$
6x^2 + 5x = 4
$$

This is the core quadratic equation that needs to be solved.

Formulating the Equation

Based on the above translation, the algebraic equation is:

$$
6x^2 + 5x - 4 = 0
$$

This quadratic equation encapsulates the entire problem and provides the basis for finding the unknown number x.

Solving the Quadratic Equation

Quadratic equations can be solved through various methods, including factoring, completing the square, or applying the quadratic formula. In this case, the quadratic formula is most straightforward.

Quadratic Formula

For an equation of the form:

$$
ax^2 + bx + c = 0
$$

the solutions are:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Where:

- $a = 6$
- $b = 5$
- $c = -4$

Step-by-Step Solution

1. Calculate the discriminant:

$$
\Delta = b^2 - 4ac = 5^2 - 4 \times 6 \times (-4) = 25 + 96 = 121
$$

2. Compute the square root of the discriminant:

$$
\sqrt{\Delta} = \sqrt{121} = 11
$$

3. Find the two solutions:

$$
x = \frac{-5 \pm 11}{2 \times 6} = \frac{-5 \pm 11}{12}
$$

- First solution:

$$
x = \frac{-5 + 11}{12} = \frac{6}{12} = \frac{1}{2}
$$

- Second solution:

$$
x = \frac{-5 - 11}{12} = \frac{-16}{12} = -\frac{4}{3}
$$

Final solutions:

$$
x = \frac{1}{2} \quad \text{or} \quad x = -\frac{4}{3}
$$

Interpreting the Solutions

Now that we have two potential solutions, it is essential to analyze their validity in the context of the original problem.

Solution 1: $x = \frac{1}{2}$

- This is a positive value.
- Substituting back into the original components:

$$
6 \left(\frac{1}{2}\right)^2 + 5 \times \frac{1}{2} = 6 \times \frac{1}{4} + \frac{5}{2} = \frac{6}{4} + \frac{5}{2}
$$

- Simplify:

$$
\frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4
$$

- The sum is 4, confirming that $x = \frac{1}{2}$ is a valid solution.

Solution 2: $x = -\frac{4}{3}$

- Substituting into the original components:

$$
6 \left(-\frac{4}{3}\right)^2 + 5 \times \left(-\frac{4}{3}\right) = 6 \times \frac{16}{9} - \frac{20}{3}
$$

- Simplify:

$$
\frac{96}{9} - \frac{20}{3} = \frac{32}{3} - \frac{20}{3} = \frac{12}{3} = 4
$$

- The sum is also 4, confirming that $x = -\frac{4}{3}$ is a valid solution.

Conclusion: Both solutions satisfy the original problem and are valid solutions.

Applications and Significance of Solving Such Problems

Understanding how to translate complex word problems into algebraic equations is crucial in various fields. Here are some key applications:

- Mathematics Education: Enhances problem-solving skills and algebraic reasoning.
- Engineering: Used for modeling physical systems where relationships are quadratic.
- Economics and Finance: Helps in calculating profit maximization and cost analysis involving quadratic relationships.
- Science and Technology: Used in physics for motion equations, optics, and other quadratic phenomena.
- Data Analysis: Assists in curve fitting and regression analysis involving quadratic models.

Tips for Solving Similar Algebraic Word Problems

- Carefully read the problem: Identify key quantities, operations, and relationships.
- Define variables clearly: Use meaningful symbols.
- Translate words into algebra: Break down sentences into mathematical expressions.
- Formulate the equation: Combine all parts logically.
- Choose an appropriate solving method: Factoring, quadratic formula, completing the square.
- Verify solutions: Substitute solutions back into the original context.
- Consider the context: Some solutions may not be valid in real-world scenarios if they violate constraints.

Summary

This comprehensive guide has demonstrated how to approach the problem: "When the product of 6 and the square of a number is increased by 5 times the number, the result is 4." We translated the statement into the quadratic equation $6x^2 + 5x - 4 = 0$, solved it using the quadratic formula, and verified that both solutions $x = \frac{1}{2}$ and $x = -\frac{4}{3}$ satisfy the original problem. Mastering such problem-solving techniques is essential for students and professionals dealing with algebraic challenges across various disciplines.

Meta Description:
Learn how to solve the algebraic problem involving the product of 6 and the square of a number increased by 5 times the number equaling 4. Step-by-step guidance on translating words into equations and solving quadratic problems.

Keywords:
algebraic word problems, quadratic equations, solve for x, algebra tips, quadratic formula, math problem solving, quadratic solutions, algebra applications

Frequently Asked Questions

What is the algebraic equation representing the problem?

The equation is $6x^2 + 5x = 4$.

How do you solve for the value of the number in the equation?

By rearranging the equation into standard quadratic form and applying the quadratic formula or factoring methods.

What are the solutions to the quadratic equation $6x^2 + 5x - 4 = 0$?

The solutions are x = 0.5 and x = -rac{4}{3}.

What does the solution x = 0.5 represent in the context of the problem?

It represents the value of the number that satisfies the given condition in the problem.

Are both solutions valid for the problem's context?

Yes, unless the problem specifies that the number must be positive or within a certain range, both solutions are valid mathematically.

How can this problem be visualized graphically?

By graphing the quadratic function $y = 6x^2 + 5x - 4$ and identifying where it intersects the x-axis.

What real-world scenarios could this type of problem model?

It could model situations involving quadratic relationships, such as projectile motion, area calculations, or economic profit models.

What steps are involved in solving a quadratic equation like this one?

First, rewrite the equation in standard form, then factor or use the quadratic formula, and finally verify the solutions.

Additional Resources

Mathematical Equation Analysis

Understanding the relationships between variables in algebraic expressions is essential for solving real-world problems and enhancing problem-solving skills. Today, we delve into a fascinating algebraic problem:
"When the product of 6 and the square of a number is increased by 5 times the number, the result is 4."

This statement translates into a mathematical equation that, when dissected thoroughly, reveals interesting insights into the nature of algebraic expressions and their solutions. In this article, we will explore this problem step-by-step, analyze its components, and provide a comprehensive guide to solving it.

Deciphering the Problem Statement

Before diving into the solution, it's crucial to understand what the problem is asking:

- "Product of 6 and the square of a number": This phrase indicates we are multiplying 6 by the square of an unknown number, say x. Mathematically, this is expressed as $6x^2$.

- "Increased by 5 times the number": This means adding five times the same number x to the previous result, written as $5x$.

- "The result is 4": The sum of the above two expressions equals 4, creating an equation to solve.

Putting it all together, the problem statement translates into the algebraic equation:

$$
6x^2 + 5x = 4
$$

This concise equation encapsulates the entire problem. Our goal now is to analyze and solve this quadratic equation for x.

Formulating the Mathematical Model

Understanding the Components

Let's break down each part of the equation:

1. Quadratic term $6x^2$: Represents the product of 6 and the square of the number x. The quadratic nature indicates the relationship involves squared terms, leading to potential two solutions.

2. Linear term $5x$: Represents five times x, which is linear, adding complexity to the equation.

3. Constant term 4: The total sum or result of the sum of the previous terms.

Complete Equation

$$
6x^2 + 5x = 4
$$

To analyze and solve, it’s often easier to bring all terms to one side:

$$
6x^2 + 5x - 4 = 0
$$

This is a standard quadratic form:

$$
ax^2 + bx + c = 0
$$

where:

- $a = 6$
- $b = 5$
- $c = -4$

Having established the quadratic form, we are now prepared to explore methods for solving it.

Methods for Solving the Quadratic Equation

Quadratic equations can be solved through various methods, including:

- Factoring
- Completing the square
- Quadratic formula

Given the coefficients, the most straightforward and universal method here is the quadratic formula.

The Quadratic Formula

The quadratic formula provides solutions for any quadratic equation:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

This formula calculates the roots directly, provided the discriminant $D = b^2 - 4ac$ is known.

Step-by-Step Solution

Calculating the Discriminant

Given:

- $a = 6$,
- $b = 5$,
- $c = -4$,

The discriminant:

$$
D = b^2 - 4ac = (5)^2 - 4 \times 6 \times (-4) = 25 - (-96) = 25 + 96 = 121
$$

Since $D = 121$, which is positive and a perfect square, the solutions will be real and rational.

Applying the Quadratic Formula

$$
x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{121}}{2 \times 6} = \frac{-5 \pm 11}{12}
$$

Now, compute both solutions:

1. Using the positive root:

$$
x = \frac{-5 + 11}{12} = \frac{6}{12} = \frac{1}{2}
$$

2. Using the negative root:

$$
x = \frac{-5 - 11}{12} = \frac{-16}{12} = -\frac{4}{3}
$$

Interpreting the Solutions

The two solutions to the quadratic equation are:

$$
x = \frac{1}{2} \quad \text{and} \quad x = -\frac{4}{3}
$$

These solutions represent the values of the number x that satisfy the original problem statement.

Verifying the Solutions

It’s always prudent to verify solutions by substituting them back into the original statement:

$$
6x^2 + 5x = 4
$$

For $x = \frac{1}{2}$:

$$
6 \times \left(\frac{1}{2}\right)^2 + 5 \times \frac{1}{2} = 6 \times \frac{1}{4} + \frac{5}{2} = \frac{6}{4} + \frac{5}{2} = \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4
$$

For $x = -\frac{4}{3}$:

$$
6 \times \left(-\frac{4}{3}\right)^2 + 5 \times \left(-\frac{4}{3}\right) = 6 \times \frac{16}{9} - \frac{20}{3} = \frac{96}{9} - \frac{20}{3}
$$

Simplify:

$$
\frac{96}{9} = \frac{32}{3}
$$

and

\[
\frac{20}{3} \text{ remains as is}
\]

So,

\[
\frac{32}{3} - \frac{20}{3} = \frac{12}{3} = 4
\]

Both solutions satisfy the original equation, confirming their validity.

Implications and Real-World Context

This problem exemplifies how algebraic methods can be employed to find unknown quantities in various contexts. Such quadratic equations often arise in physics, engineering, finance, and even computer science, where relationships between variables need to be modeled and solved.

Possible real-world interpretations:

- Physics: Calculating the velocity of an object under acceleration, where the quadratic relates to kinematic equations.
- Economics: Determining the break-even points in cost-profit models.
- Engineering: Analyzing load distributions or stress-strain relationships.

Understanding how to translate word problems into algebraic equations and then solve them precisely allows for practical problem-solving in many disciplines.

Summary and Key Takeaways

- The problem statement translates into the quadratic equation \(6x^2 + 5x = 4 \).
- Rewriting as \(6x^2 + 5x - 4 = 0 \) makes it easier to apply algebraic solution methods.
- The discriminant \(D = 121 \) indicates two real solutions.
- Using the quadratic formula yields solutions \(x = \frac{1}{2} \) and \(x = -\frac{4}{3} \).
- Both solutions are verified by substitution into the original equation.
- Mastering quadratic equations is fundamental for tackling a wide array of mathematical and real-world problems.

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By approaching the problem methodically—breaking down the statement,
formulating the equation, solving systematically, and verifying solutions—you
develop a robust problem-solving framework applicable across many scenarios.
As algebra remains a cornerstone of analytical thinking, mastering these
techniques empowers you to navigate complex relationships with confidence.

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In conclusion, this exploration not only solves a specific quadratic problem
but also exemplifies best practices in algebraic reasoning, emphasizing
clarity, verification, and application. Whether for academic pursuits or
practical applications, understanding these principles enhances your
mathematical literacy and analytical prowess.
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