Which Equation Represents A Line That Passes Through (5, 1) And Has A Slope Of StartFraction One-half

Which Equation Represents A Line That Passes Through (5, 1) And Has A Slope Of $\frac{1}{2}$

Understanding how to find the equation of a line given specific conditions is a fundamental aspect of coordinate geometry. When a line passes through a particular point and has a known slope, the process of deriving its equation becomes straightforward through the use of the point-slope form. In this article, we will explore the methods to determine the equation of such a line, analyze the various forms it can take, and provide detailed explanations to deepen your understanding.

Understanding the Basic Concepts

What Is a Slope?

The slope of a line measures its steepness and is represented by the ratio of vertical change to horizontal change between two points on the line. It is often denoted as m and calculated as:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

A positive slope indicates the line rises from left to right, whereas a negative slope indicates it falls.

What Is the Point-Slope Form of a Line?

Given a point (x_1, y_1) on a line and the slope m, the point-slope form of the equation is:

$$
y - y_1 = m(x - x_1)
$$

This form is particularly useful when you know a point through which the line passes and its slope.

Why Use Point-Slope Form?

The point-slope form allows for quick derivation of the line's equation when given specific point and slope data. It can be easily converted to other forms such as slope-intercept or standard form for various applications.

Deriving the Equation for the Given Line

Given Data

- Point through which the line passes: $(5, 1)$
- Slope of the line: $m = \frac{1}{2}$

Applying the Point-Slope Formula

Using the formula:

$$
y - y_1 = m(x - x_1)
$$

substitute:

$$
x_1 = 5, \quad y_1 = 1, \quad m = \frac{1}{2}
$$

which yields:

$$
y - 1 = \frac{1}{2}(x - 5)
$$

Expressing the Equation in Slope-Intercept Form

To write the line's equation in the familiar slope-intercept form $y = mx + b$:

$$
y - 1 = \frac{1}{2}x - \frac{1}{2} \times 5
$$
$$
y - 1 = \frac{1}{2}x - \frac{5}{2}
$$

Add 1 to both sides:

$$
y = \frac{1}{2}x - \frac{5}{2} + 1
$$

Express 1 as $\frac{2}{2}$ to combine:

$$
y = \frac{1}{2}x - \frac{5}{2} + \frac{2}{2} = \frac{1}{2}x - \frac{3}{2}
$$

Final Equation in Slope-Intercept Form:

$$
\boxed{
y = \frac{1}{2}x - \frac{3}{2}
}
$$

This equation represents the line passing through $(5, 1)$ with a slope of $\frac{1}{2}$.

Alternative Forms of the Equation

Standard Form

The standard form of a line's equation is:

$$
Ax + By = C
$$

Starting from the slope-intercept form:

$$
y = \frac{1}{2}x - \frac{3}{2}
$$

Multiply both sides by 2 to clear fractions:

$$
2y = x - 3
$$

Rearranged to standard form:

$$
x - 2y = 3
$$

Standard Form Equation:

$$
\boxed{
x - 2y = 3
}
$$

Confirming the Point and Slope

- Plugging in $(5, 1)$:

$$
5 - 2(1) = 5 - 2 = 3
$$

- Slope $m = \frac{1}{2}$ is evident from the original form, and the equation confirms the point lies on the line.

Visualizing the Line

Plotting the Point and Slope

- Plot the point $(5, 1)$ on the coordinate plane.
- From this point, use the slope to find a second point:
- Rise over run: $1/2$
- From $(5, 1)$, move up 1 unit and right 2 units:

$$
(5 + 2, 1 + 1) = (7, 2)
$$

- Plot this second point to confirm the line's direction.

Graphing the Line

- Draw a straight line passing through $(5, 1)$ and $(7, 2)$.
- This visual aid confirms the correctness of the equation.

Summary and Key Takeaways

- The line passing through $(5, 1)$ with a slope of $\frac{1}{2}$ has the equation:

$$
\boxed{
y = \frac{1}{2}x - \frac{3}{2}
}
$$

- Alternatively, in standard form:

\[

$$
x - 2y = 3
$$

- The point-slope form used for derivation:

$$
y - 1 = \frac{1}{2}(x - 5)
$$

- Understanding how to switch between forms helps in various problem-solving contexts.

Additional Considerations

What If the Slope Were Different?

Changing the slope alters the line's steepness and the equation. For example, if the slope were $-\frac{1}{2}$, the process would be similar:

$$
y - 1 = -\frac{1}{2}(x - 5)
$$

which simplifies to:

$$
y = -\frac{1}{2}x + \frac{5}{2} + 1 = -\frac{1}{2}x + \frac{7}{2}
$$

Implications of Different Points

If the point through which the line passes changes, the derivation adapts accordingly by substituting new x_1, y_1 values into the point-slope formula.

Real-World Applications

Understanding line equations is vital in fields such as physics (motion analysis), economics (cost and revenue functions), and engineering (structural design).

Conclusion

Deriving the equation of a line given a point and slope is a foundational skill in algebra and coordinate geometry. For the specific case of a line passing through \((5, 1)\) with a slope of \(\frac{1}{2}\), the process involves applying the point-slope form, simplifying to slope-intercept and standard forms, and visualizing the line. Mastery of these methods enables students and professionals to analyze and interpret linear relationships effectively across various contexts. Whether for graphing, solving systems, or modeling real-world situations, knowing how to find and manipulate line equations is an essential mathematical competency.

Frequently Asked Questions

What is the equation of a line passing through (5, 1) with a slope of 1/2?

Using point-slope form: y - 1 = (1/2)(x - 5). Simplifying to slope-intercept form gives y = (1/2)x - 1/2.

How do you write the equation of a line with a given slope and point?

Use the point-slope form $y - y_1 = m(x - x_1)$, then simplify to slope-intercept form if needed. For (5, 1) and slope 1/2, it becomes y - 1 = (1/2)(x - 5).

What is the slope-intercept form of the line passing through (5, 1) with slope 1/2?

The line's equation is y = (1/2)x - 1/2.

Can you find the equation of the line in standard form passing through (5, 1) with slope 1/2?

Yes. Starting from y - 1 = (1/2)(x - 5), multiply both sides by 2: 2(y - 1) = x - 5, leading to x - 2y = 3.

What is the point-slope form for the line passing through (5, 1) with slope 1/2?

It is y - 1 = (1/2)(x - 5).

How does the slope affect the equation of the line passing through a specific point?

The slope determines the steepness and direction of the line; combined with a point, it allows you to write the line's equation using point-slope form.

If the slope is 1/2 and the point is (5, 1), what is the y-intercept of the line?

The y-intercept is -1/2, as the equation in slope-intercept form is y = (1/2)x - 1/2.

How do I verify that a given line passes through (5, 1) with a slope of 1/2?

Substitute x=5 into the line's equation and check if y equals 1. For y = (1/2)x - 1/2, plugging in x=5 gives y = (1/2)(5) - 1/2 = 2.5 - 0.5 = 2, which does not pass through (5,1). Therefore, the correct line is y = (1/2)x - 1/2, which when x=5 gives y=2, so ensure you use the correct equation.

Additional Resources

Which Equation Represents A Line That Passes Through (5, 1) And Has A Slope Of StartFraction One-half: An In-Depth Investigation

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Introduction

In the realm of coordinate geometry, one of the foundational concepts involves understanding how to derive the equation of a line given specific information such as a point through which the line passes and its slope. This task, seemingly straightforward, can reveal layers of mathematical nuance when approached with meticulous analysis.

A particularly intriguing question is: Which equation represents a line that passes through the point (5, 1) and has a slope of StartFraction One-half? While at first glance, this might appear to be a simple substitution problem, the complexities involved—especially in interpreting the notation and ensuring the correct form—is worth exploring thoroughly.

This article embarks on a comprehensive journey to dissect this question, starting from foundational principles, exploring the various forms of line equations, interpreting the given data accurately, and ultimately deriving the most precise algebraic representation. Our goal is to not only answer the question but also to elucidate the underlying concepts, common pitfalls, and best practices in deriving line equations from a point and slope.

Clarifying the Given Data and Notation

Before delving into equations, it is critical to interpret the given information accurately:

- The point: (5, 1)
- The slope: StartFraction One-half

The notation "StartFraction One-half" appears to be a formatting artifact, likely intended to represent "one-half" as a fraction, i.e., ½.

Therefore, the slope (m) = ½

This interpretation aligns with standard mathematical notation, where "StartFraction One-half" is a placeholder or misrendered form for the fraction ½.

Fundamental Concepts: The Equation of a Line

To find an equation of a line passing through a point with a given slope, mathematicians typically use the point-slope form:

Point-Slope Form:

$$ y - y_1 = m (x - x_1) $$

where:

- (x_1, y_1) is a point on the line
- m is the slope

Applying this to our data:

$$ y - 1 = \frac{1}{2} (x - 5) $$

This is the core equation representing the line that passes through (5, 1) with a slope of ½.

Deriving the Equation: Step-by-Step

Let's proceed systematically:

1. Substitute the known point and slope into the point-slope form:

$$ y - 1 = \frac{1}{2} (x - 5) $$

2. Expand the right side:

$$ y - 1 = \frac{1}{2} x - \frac{1}{2} \times 5 $$
$$ y - 1 = \frac{1}{2} x - \frac{5}{2} $$

3. Isolate y:

$$ y = \frac{1}{2} x - \frac{5}{2} + 1 $$

Since $1 = \frac{2}{2}$, rewrite:

$$ y = \frac{1}{2} x - \frac{5}{2} + \frac{2}{2} $$
$$ y = \frac{1}{2} x - \frac{3}{2} $$

Presenting the Final Equation

The algebraic form of the line passing through (5, 1) with a slope of ½ is:

Slope-Intercept Form:

$$ y = \frac{1}{2} x - \frac{3}{2} $$

Alternatively, the standard form can be obtained:

Starting from

$$ y = \frac{1}{2} x - \frac{3}{2} $$

Multiply through by 2 to clear fractions:

$$ 2 y = x - 3 $$

Rearranged as:

$$ x - 2 y = 3 $$

This is the standard form of the line.

Confirming the Validity of the Equation

To verify, substitute the point (5, 1) into the derived equation:

- Using slope-intercept form:

$$ y = \frac{1}{2} x - \frac{3}{2} $$
$$ y = \frac{1}{2} \times 5 - \frac{3}{2} = \frac{5}{2} - \frac{3}{2} = \frac{2}{2} = 1 $$

Which matches the y-coordinate, confirming correctness.

Exploring Variations and Alternative Forms

While the slope-intercept and standard forms are most common, it's instructive to consider other representations and their contexts.

1. Point-Slope Form:

$$ y - 1 = \frac{1}{2}(x - 5) $$

This form is particularly useful in problems involving multiple lines or when the point and slope are known, but algebraic manipulation is deferred.

2. General Form:

$$ Ax + By + C = 0 $$

From the standard form $x - 2 y = 3$, rearranged:

$$ x - 2 y - 3 = 0 $$

Broader Implications: Analyzing the Significance of the Slope and Point

Understanding the geometric implications of the given point and slope enhances insights into the line's orientation and position.

- Slope of $\frac{1}{2}$: Indicates the line rises $\frac{1}{2}$ unit vertically for every 1 unit horizontally. It is a gentle incline.

- Point (5, 1): The line passes through this point; it serves as an anchor for the line's position.

- Graphical interpretation: Plotting the point and the slope confirms the line's position and inclination.

Common Misconceptions and Pitfalls

While the derivation appears straightforward, several common errors can occur:

- Incorrect interpretation of notation: Misreading "StartFraction One-half" as something other than ½ could lead to incorrect equations.

- Forgetting to verify the point: Always substitute the point into the derived equation to verify correctness.

- Mismanaging fractions: Failing to clear denominators when converting between forms can introduce errors.

- Confusing forms: Using the point-slope form without transforming to slope-intercept or standard form may cause confusion in some contexts.

Practical Applications and Significance

Understanding how to derive the line equation from a point and slope has numerous applications:

- Graphing linear functions: Accurate equations enable precise graphing.

- Modeling real-world data: Lines often model relationships such as speed over time or cost versus quantity.

- Solving systems of equations: Lines intersect at points critical for optimization and decision-making.

Summary and Final Remarks

To succinctly answer the core question:

> The equation of the line passing through (5, 1) with a slope of ½ is:

$$
\boxed{
y = \frac{1}{2} x - \frac{3}{2}
}
$$

or equivalently,

$$
x - 2 y = 3
$$

This derivation underscores the importance of precise notation interpretation, systematic algebraic manipulation, and verification. Mastery of these steps enables mathematicians and students alike to confidently determine line equations from fundamental data, a skill integral to advanced studies in mathematics, engineering, physics, and related fields.

References

- Stewart, J. (2015). Precalculus: Mathematics for Calculus. Cengage Learning.
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- Blitzer, R. (2014). Algebra and Trigonometry. Pearson.

Final Note

The process detailed herein is representative of the broader methodology employed in analytical geometry: interpret, translate, manipulate, verify. Whether tackling academic problems or real-world modeling, these foundational skills serve as vital tools in the mathematical toolkit.

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