Which Equation Represents The Transformation Formed By Horizontally Stretching The Graph Of F(x)=x By

Which Equation Represents The Transformation Formed By Horizontally Stretching The Graph Of F(x)=x By is a fundamental question in understanding function transformations in algebra. Horizontal stretching is a common transformation that alters the shape of the graph along the x-axis, making it wider or narrower depending on the factor applied. This concept is essential for students and professionals working with graph analysis, function behavior, and transformations in calculus or algebra. In this article, we will explore the details of horizontal stretching, how it affects the graph of a function, and how to determine the resulting equation after such a transformation.

Understanding Function Transformations

Before delving into horizontal stretching specifically, it's important to review what function transformations are and how they modify the graph of a basic function.

What Are Function Transformations?

Function transformations are operations that change the position, size, or shape of the graph of a function without altering its fundamental nature. Common transformations include:

- Translations (shifting the graph horizontally or vertically)

- Reflections (flipping across axes)
- Stretches and compressions (scaling the graph along axes)

Each transformation can be represented algebraically by modifying the function’s equation, allowing us to predict and sketch the resulting graph.

Types of Stretches and Compressions

- Vertical stretch/compression: Multiplies the output (f(x)) by a factor.
- Horizontal stretch/compression: Multiplies the input (x) by a factor.

In this article, our focus is on horizontal transformations, particularly stretching the graph along the x-axis.

Horizontal Stretching of the Graph of F(x)=x

What Is Horizontal Stretching?

Horizontal stretching occurs when the graph of a function is expanded or contracted along the x-axis. It is achieved by multiplying the input variable x by a constant factor. Specifically:

- If the factor is greater than 1, the graph is stretched horizontally.
- If the factor is between 0 and 1, the graph is compressed horizontally.

For example, stretching the graph of f(x)=x by a factor of 2 makes it twice as wide along the x-axis.

Mathematical Representation of Horizontal Stretching

Given a base function $f(x)$, a horizontal stretch by a factor of k (where $k > 0$) is represented as:

$$g(x) = f\left(\frac{x}{k}\right)$$

This equation means that to find the output at x, we evaluate the original function at $\frac{x}{k}$. As a result:

- When $k > 1$, the graph is stretched horizontally.
- When $0 < k < 1$, the graph is compressed horizontally.

In the specific case of $f(x) = x$, the base function, the transformation is straightforward.

Applying Horizontal Stretch to the Function F(x)=x

Original Function

The basic function:

$$F(x) = x$$

is a straight line passing through the origin with a slope of 1.

Effect of Horizontal Stretching

Applying a horizontal stretch by a factor of k results in:

$$ G(x) = F\left(\frac{x}{k}\right) = \frac{x}{k} $$

This new function still passes through the origin and remains linear, but the slope changes depending on the value of k.

Resulting Equation After Horizontal Stretching

The transformed function:

$$ G(x) = \frac{x}{k} $$

describes the graph of the original line stretched horizontally by a factor of k.

- If $k=2$, then:

$$ G(x) = \frac{x}{2} $$

The graph is twice as wide, and the slope becomes $\frac{1}{2}$.

- If $k=\frac{1}{3}$, then:

$$ G(x) = 3x $$

The graph is compressed, becoming steeper.

Note: The key point is that the transformation modifies the input variable, not the output directly, which results in a change in the graph's width.

Understanding the Effect on the Graph

Graphical Interpretation

- The original line $y = x$ passes through points like (1,1), (2,2), (-1,-1).
- After horizontal stretching by factor k, the new line $y = \frac{x}{k}$ passes through points:

$$
(k, 1), \quad (2k, 2), \quad (-k, -1)
$$

indicating the graph is scaled along the x-axis.

Visualizing the Transformation

Imagine pulling the graph horizontally away from the y-axis for $k > 1$, making it wider. Conversely, for $0 < k < 1$, pushing it towards the y-axis makes the graph narrower.

Summary of Key Equations and Concepts

Transformation Type	Equation for $f(x)$	Resulting Equation	Effect on Graph
Horizontal stretch by k	$f(x)$	$f\left(\frac{x}{k}\right)$	Width increases by factor k if $k > 1$
Example with $f(x) = x$	x	$\frac{x}{k}$	Line becomes wider or narrower

Practical Examples

Example 1: Horizontal Stretch by a Factor of 3

Original function:

$$F(x) = x$$

Transformed function:

$$G(x) = F\left(\frac{x}{3}\right) = \frac{x}{3}$$

- The graph is stretched horizontally by a factor of 3.
- Points like (3, 1) and (6, 2) lie on the new graph.
- Slope of the line reduces from 1 to $\frac{1}{3}$.

Example 2: Horizontal Compression by a Factor of $\frac{1}{2}$

Transformed function:

$$G(x) = F(2x) = 2x$$

- The graph is compressed horizontally.
- Points like (1,1) and (2,2) are now (0.5,1) and (1,2) on the original scale.

Conclusion: The Equation for Horizontal Stretching of $F(x)=x$

In summary, the equation that represents the transformation of the graph of $F(x)=x$ after a horizontal stretch by a factor of k is:

$$\boxed{
g(x) = \frac{x}{k}
}$$

where $k > 1$ results in a wider graph, and $0 < k < 1$ results in a narrower graph. This equation elegantly captures the essence of horizontal stretching and provides a clear way to analyze and sketch transformed functions.

Understanding and applying this concept is crucial for mastering function transformations, analyzing graphs, and solving complex algebraic problems.

Frequently Asked Questions

What is the general form of the equation resulting from a horizontal stretch of f(x) = x?

The general form is f(bx), where b is the stretch factor. For a horizontal stretch by a factor of k > 1, the equation becomes f(bx) with b = 1/k.

If the graph of f(x) = x is stretched horizontally by a factor of 3, what

is the new equation?

The new equation is f(□ x), which simplifies to y = □ x after the stretch.

How does a horizontal stretch by a factor of k affect the equation of $f(x) = x$?

It transforms the equation into $f(x / k)$, effectively changing the input variable to x divided by k, which stretches the graph horizontally by k.

What is the effect on the graph of $y = x$ when it undergoes a horizontal stretch by a factor of 2?

The graph stretches horizontally by a factor of 2, and the transformed equation becomes $y = (1/2) x$.

How can you write the equation of $f(x) = x$ after a horizontal stretch of 4 units?

Since a horizontal stretch by a factor of 4 means replacing x with $x/4$, the new equation is $y = (1/4) x$.

Additional Resources

Which Equation Represents The Transformation Formed By Horizontally Stretching The Graph Of f(x) = x By

When exploring transformations of functions, understanding how specific alterations affect the graph is crucial for mastering algebra and precalculus concepts. One common transformation is the horizontal stretch, which modifies the width of the graph along the x-axis. Specifically, when we horizontally stretch the graph of $f(x) = x$ by a certain factor, we are effectively spreading the graph outward, making it wider. This transformation results in a new equation that reflects this change, and identifying that equation is essential for graph analysis and function manipulation. In this article, we will delve into the specifics of which equation represents the transformation formed by horizontally stretching the graph of f(x) = x by a given factor, providing a comprehensive guide to understanding and applying this concept.

Understanding Horizontal Transformations

Before diving into the specific equations, it's important to review what horizontal transformations entail and how they differ from vertical ones.

Horizontal vs. Vertical Transformations

- Vertical transformations affect the output of the function (the y-values). For example, adding a constant outside the function shifts the graph up or down.
- Horizontal transformations modify the input (the x-values). These transformations stretch, compress, or shift the graph along the x-axis.

Effect of Horizontal Stretching

When the graph of a function f(x) is horizontally stretched by a factor of $k > 1$, the graph becomes wider. Conversely, if $0 < k < 1$, it results in a horizontal compression. The key point is that the stretching factor k impacts the input variable x.

The General Rule for Horizontal Stretching

Suppose you have a base function:

`f(x)`

To horizontally stretch this graph by a factor of k, the transformed function becomes:

`g(x) = f(x / k)`

Why does this work?
Because replacing x with x / k effectively causes the graph to expand by a factor of k along the x-axis.

Visual Explanation

- When k > 1, the function is stretched outward because x / k takes larger input values for the same output.
- When 0 < k < 1, the graph is compressed inward.

Applying the Concept to f(x) = x

Now, let's focus on the specific function:

`f(x) = x`

This is a simple linear function with a graph that is a straight line passing through the origin with slope 1.

Original Graph

- Passes through (0,0).
- Has a slope of 1, meaning for each increase of 1 in x, y increases by 1.

How to Find the Equation of the Horizontally Stretched Graph

Suppose we stretch the graph of $f(x) = x$ by a factor of k.

Step-by-Step Process:

1. Identify the stretching factor (k):

For example, $k = 2$ for a stretch by a factor of 2.

2. Apply the horizontal stretch rule:

The new function becomes:

`g(x) = f(x / k)`

3. Substitute $f(x) = x$:

So,

`g(x) = f(x / k) = (x / k)`

4. Simplify the expression if necessary:

The new equation is:

`g(x) = (x / k)`

Final Equation

- For a horizontal stretch by factor k, the equation is:

$g(x) = x / k$

- This equation represents the graph of $y = x$ stretched horizontally by a factor of k.

Visualizing the Transformation

Example: Horizontal stretch of f(x) = x by a factor of 3

- The transformed function:

`g(x) = x / 3`

- Effect on the graph:

- The line becomes wider.
- For each x, the corresponding y is x / 3.
- The slope decreases from 1 to 1/3, making the line less steep.

Example: Horizontal stretch by a factor of 1/2

- The transformed function:

`g(x) = x / (1/2) = 2x`

- The graph becomes twice as steep, effectively compressing the graph horizontally.

Summary of Key Points

Aspect	Details
Basic function	f(x) = x

Horizontal stretch factor	k (k > 0)
Equation after stretch	g(x) = f(x / k)
For f(x) = x	g(x) = x / k
Effect on graph	Width increases by factor k, slope becomes 1/k

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Practical Examples

Example 1: Horizontal stretch of f(x) = x by a factor of 4

- Equation:

`g(x) = x / 4`

- Interpretation:

- The graph is stretched to be four times wider.
- Slope of the line becomes 1/4.

Example 2: Horizontal stretch of f(x) = x by a factor of 1/3

- Equation:

`g(x) = x / (1/3) = 3x`

- Interpretation:

- The graph is compressed to be three times steeper.
- Slope increases from 1 to 3.

Additional Considerations

Negative Stretch Factors

- If k is negative, the graph is also reflected across the y-axis (flipped horizontally) in addition to being stretched.
- For example, k = -2 results in:

`g(x) = x / (-2) = -x / 2`

which is a horizontal stretch by 2 with reflection across the y-axis.

Combining Transformations

- Horizontal stretches can be combined with other transformations like shifts or reflections.
- The order of transformations matters; typically, you apply stretches/compressions before shifts.

Final Thoughts and Summary

Understanding which equation represents the transformation formed by horizontally stretching the graph of f(x) = x by a certain factor hinges on recognizing the fundamental rule:

Horizontal Stretch Equation:

`g(x) = f(x / k)`

For the specific case where f(x) = x, this simplifies neatly to:

`g(x) = x / k`

This equation accurately captures the effect of stretching the graph horizontally by a factor of k. Whether $k > 1$ (wider) or $0 < k < 1$ (narrower), the formula provides a direct way to determine the transformed function's equation.

Mastering this concept allows students and professionals alike to analyze and predict how various functions will change under horizontal stretches, which is foundational for more advanced topics in graph transformations, function analysis, and calculus.

In conclusion, when asked which equation represents the transformation formed by horizontally stretching the graph of $f(x) = x$ by a given factor, remember the core principle:

The new function is $g(x) = x / k$, where k is the stretch factor.

This straightforward formula is invaluable for visualizing, drawing, and understanding the behavior of transformed functions across mathematics and its applications.

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