Write A System Of Equations To Describe The Situation Below, Solve Using Elimination, And Fill In The

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Introduction to the Problem

Understanding how to translate real-world situations into mathematical models is fundamental in algebra. When dealing with problems involving multiple unknowns, systems of equations serve as powerful tools to find solutions. In this article, we will explore a specific scenario, formulate a corresponding system of equations, and then demonstrate how to solve it using the elimination method. Finally, we will interpret the solution to fully understand the scenario.

Setting Up the Scenario

Imagine two friends, Alice and Bob, who are saving money for a trip. Alice deposits a certain amount of money into her savings account each month, and Bob does the same, but with a different monthly deposit. After a specific period, both have accumulated a certain total amount. Our goal is to determine how much each person deposits monthly, given some information about their total savings and the number of months they've been saving.

Details of the Situation

- Alice has been saving for 6 months.
- Bob has been saving for 8 months.
- After these periods, Alice's total savings amount to \$360.
- Bob's total savings amount to \$480.
- Each deposits a fixed, but different, amount every month.

Formulating the System of Equations

Let’s define variables to represent the unknowns:

- x = Alice's monthly deposit (in dollars)
- y = Bob's monthly deposit (in dollars)

Based on the information, we can write equations reflecting the total savings:

- Alice's total savings: $6x = 360$
- Bob's total savings: $8y = 480$

However, to make the problem more interesting and suitable for the elimination method, suppose there's an additional condition:

- The difference between their total savings is \$120.

Expressed mathematically, this becomes:

$$
|360 - 480| = 120
$$

which is already true, but for the purpose of solving a system using elimination, let's assume a different scenario where the total savings are related by a different condition, such as:

$$
\text{Alice's total savings} = \text{Bob's total savings} - 120
$$

or

$$
6x = 8y - 120
$$

which introduces a relationship between x and y. Alternatively, suppose we know that Alice's monthly deposit is \$10 less than Bob's:

$$
x = y - 10
$$

Now, our system becomes:

$$
\begin{cases}
6x = 360 \\
8y = 480 \\
x = y - 10
\end{cases}
$$

But since the first two equations directly give totals, and the third relates the deposits, we can combine the equations as follows:

Constructing the System of Equations

Based on the above, the most straightforward system to solve with elimination focuses on the two main equations:
\[
\begin{cases}
6x = 360 \quad \text{(Alice's total savings)} \\
8y = 480 \quad \text{(Bob's total savings)}
\end{cases}
\]
and the relationship:
\[
x = y - 10
\]
Thus, the system we will work with is:
\[
\begin{cases}
6x = 360 \\
8y = 480 \\
x = y - 10
\end{cases}
\]
However, since the first two equations can be solved directly, to illustrate elimination, let's modify the scenario slightly to involve equations where the variables are in the same terms and can be eliminated.

Revised System for Elimination

Suppose:

- Alice and Bob both save money for the same number of months, say 6 months.
- Alice's total savings after 6 months are \$300.
- Bob's total savings after 6 months are \$360.
- Alice deposits \(x \) dollars per month, and Bob deposits \(y \) dollars per month.

From this, we can formulate:
\[
\begin{cases}
6x = 300 \\
6y = 360
\end{cases}
\]
which simplifies to:
\[
\begin{cases}
x = 50 \\
y = 60
\end{cases}

\]
but again, solving directly is trivial. To showcase the elimination method, let's create a system where the total amounts depend on both variables in a combined way. For example:
$$
\begin{cases}
x + y = 110 \quad \text{(total deposit per month)} \\
3x + 2y = 250 \quad \text{(total savings over a certain period or another related total)}
\end{cases}
$$
This system now involves two equations with two variables, suitable for elimination.

Final System to Solve Using Elimination

We will focus on this system:
$$
\begin{cases}
x + y = 110 \quad \text{(Equation 1)} \\
3x + 2y = 250 \quad \text{(Equation 2)}
\end{cases}
$$
Our objective is to find the values of x and y that satisfy both equations, representing Alice’s and Bob’s monthly deposits.

Solving the System Using Elimination Method

Step 1: Align the equations

Write the equations:
$$
x + y = 110 \quad \text{(1)}
$$
$$
3x + 2y = 250 \quad \text{(2)}
$$

Step 2: Eliminate one variable

To eliminate y, we can manipulate the equations:
- Multiply Equation (1) by 2:
$$
2x + 2y = 220
$$
- Subtract this new equation from Equation (2):
$$
(3x + 2y) - (2x + 2y) = 250 - 220
$$

$$
(3x - 2x) + (2y - 2y) = 30
$$
$$
x = 30
$$

Step 3: Substitute back to find y

Using Equation (1):
$$
x + y = 110
$$
$$
30 + y = 110
$$
$$
y = 110 - 30
$$
$$
y = 80
$$

Interpreting the Results

Based on the calculations:

- Alice deposits \$30 per month.
- Bob deposits \$80 per month.

This solution satisfies both equations, meaning the scenario modeled by the system is consistent with these deposits.

Conclusion

By translating a real-world situation into a system of equations, we can apply the elimination method to find unknown values effectively. This process involves aligning equations, multiplying to match coefficients, subtracting to eliminate a variable, and then substituting back to find the remaining unknowns. Such techniques are invaluable in solving complex problems across various fields, including finance, engineering, and science. Remember, the key to successful elimination is carefully setting up the equations and manipulating them systematically to uncover the unknowns.

Frequently Asked Questions

What is the first step to write a system of equations for a word problem involving two quantities?

Identify the two unknown quantities and assign variables to them, then translate the given information into mathematical expressions involving those variables.

How do you set up a system of equations to describe a situation involving the total cost and individual prices?

Write one equation representing the total cost in terms of the quantities and prices, and a second equation based on the relationship between the quantities or the total number of items.

What is the elimination method for solving a system of equations?

The elimination method involves adding or subtracting the equations to eliminate one variable, allowing you to solve for the remaining variable more easily.

Can you give an example of writing a system of equations from a real-world problem?

Yes. For example, if a store sells pens and notebooks, and you know the total cost and the number of items bought, you can set up equations to find the price of each item.

How do you fill in a solution once you’ve used elimination to solve a system of equations?

After solving for one variable, substitute that value into one of the original equations to find the value of the second variable, then fill in both values into the context of the problem.

What common mistakes should be avoided when using elimination to solve systems?

Avoid mixing up equations, incorrectly adding or subtracting coefficients, and forgetting to check for extraneous solutions or valid solutions within the problem context.

How can you verify that your solution correctly describes the situation?

Substitute the found values back into the original equations and check if they satisfy all conditions of the problem.

What does it mean to 'fill in' in the context of solving systems of equations?

It refers to plugging the solutions for the variables back into the real-world context or original problem to interpret and confirm the results.

Why is it important to write a system of equations before solving it?

Writing the system helps clearly represent the problem mathematically, ensuring that your solution accurately models the situation and that no important detail is missed.

What are some tips for choosing the best method to solve a system of equations?

Consider the coefficients and constants; use elimination if coefficients are easily aligned to cancel out, or substitution if one variable is already isolated, to simplify the solving process.

Additional Resources

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Introduction

In the realm of algebra, systems of equations serve as powerful tools to model and solve real-world problems involving multiple variables. Whether it's business, engineering, or everyday scenarios, translating a verbal description into algebraic expressions allows for precise analysis and solutions. This article aims to provide a comprehensive guide on how to construct a system of equations from a given situation, employ the elimination method to solve it, and interpret the results effectively. Throughout, we will break down each step with clarity, ensuring a deep understanding of the process.

Understanding the Context: The Importance of Modeling Situations with Equations

Before diving into the mechanics of forming equations, it's crucial to appreciate why modeling real-world scenarios with mathematical expressions is valuable. When faced with a problem involving multiple unknowns—such as quantities, costs, or rates—creating equations helps in:

- Clarifying relationships: Making implicit connections explicit.
- Facilitating calculations: Allowing for systematic solving rather than guesswork.
- Predicting outcomes: Enabling analysis of how variables influence each other.
- Making informed decisions: Supporting data-driven choices.

In particular, the system of equations approach allows us to handle interconnected variables, especially when the problem involves two or more unknowns that are linked through multiple conditions.

Step 1: Interpreting the Situation and Identifying Variables

The first step in modeling any problem is to carefully read and understand the scenario. Typically, this involves:

- Extracting key information.
- Defining variables to represent unknown quantities.
- Recognizing the relationships between these quantities.

Example Scenario: Suppose you're managing a small business that sells two types of products—say, Product A and Product B. You know the following:

- The total number of units sold combined is 150.
- The total revenue generated from sales is $2,100.
- Each unit of Product A sells for $12.
- Each unit of Product B sells for $15.

Variables:

Let:

- x = number of units of Product A sold.
- y = number of units of Product B sold.

Step 2: Formulating the System of Equations

Based on the scenario, we now translate the information into algebraic expressions:

Equation 1 (Total Units):
Since the total units of both products sold is 150:

$$
x + y = 150
$$

Equation 2 (Total Revenue):
The revenue from Product A is $12x$ and from Product B is $15y$. The total revenue is $2,100:

$$
12x + 15y = 2100
$$

This pair of equations forms a system of linear equations representing the problem.

Step 3: Choosing the Method of Solution – Focus on Elimination

There are multiple methods to solve systems of equations, including substitution, elimination, and graphing. Here, we focus on the elimination method because it often simplifies the process when coefficients are aligned or can be easily manipulated to cancel out a variable.

Why Elimination?
Elimination involves adding or subtracting equations to eliminate one variable, making it straightforward to solve for the remaining variable(s).

Step 4: Preparing for Elimination

The key to elimination is to align coefficients of one variable so that adding or subtracting the equations cancels out that variable.

Given:

$$
\begin{cases}
x + y = 150 \quad \text{(Equation 1)} \\
12x + 15y = 2100 \quad \text{(Equation 2)}
\end{cases}
$$

To eliminate y, observe the coefficients: 1 and 15. To make these coefficients equal (or opposites), multiply Equation 1 by 15:

\[

$$
15(x + y) = 15 \times 150
$$

which simplifies to:

$$
15x + 15y = 2250
$$

Now, rewrite the system with the new equation:

$$
\begin{cases}
15x + 15y = 2250 \\
12x + 15y = 2100
\end{cases}
$$

Step 5: Applying Elimination

Subtract the second equation from the first:

$$
(15x + 15y) - (12x + 15y) = 2250 - 2100
$$

Simplify:

$$
(15x - 12x) + (15y - 15y) = 150
$$

$$
3x + 0 = 150
$$

$$
3x = 150
$$

Divide both sides by 3:

$$
x = \frac{150}{3} = 50
$$

Result:
The business sold 50 units of Product A.

Step 6: Solving for the Remaining Variable

Now, substitute $x = 50$ into Equation 1 to find y:

$$
x + y = 150
$$

$$
50 + y = 150
$$

Subtract 50 from both sides:

$$
y = 150 - 50 = 100
$$

Result:
The business sold 100 units of Product B.

Step 7: Verifying the Solution

It's important to verify the solution in the context of the original problem. Plugging the values into the revenue equation:

$$
12x + 15y = 2100
$$

$$
12 \times 50 + 15 \times 100 = 600 + 1500 = 2100
$$

Since the sum matches the total revenue, the solution is consistent and correct.

Step 8: Practical Interpretation and Insights

The solution indicates that the business sold:

- 50 units of Product A at $12 each.
- 100 units of Product B at $15 each.

This not only satisfies the total units sold but also matches the total revenue generated, confirming the model's accuracy.

Broader Analytical Considerations

The Significance of the Elimination Method

The elimination method is particularly effective when the coefficients of one variable are easily manipulated to cancel out. Its advantages include:

- Efficiency: For systems with aligned coefficients, it often requires fewer steps than substitution.
- Clarity: It directly shows how variables relate to each other.
- Scalability: It extends well to larger systems with more variables when combined with matrix methods.

Limitations and Alternatives

While elimination is powerful, it has limitations:

- It requires coefficients to be manipulated into compatible forms.
- Sometimes, substitution or graphical methods may be more straightforward, especially with specific coefficients or for visual understanding.

Step 9: Extending the Model to More Complex Situations

The example above is straightforward, but real-world problems can be more complex, involving:

- Additional variables (e.g., costs, profit margins).
- Nonlinear relationships.
- Constraints or inequalities.

In such cases, modeling may involve systems of inequalities, quadratic equations, or optimization techniques. Nonetheless, the fundamental principle remains: translate verbal descriptions into algebraic form and employ appropriate solving methods.

Conclusion

Constructing and solving systems of equations is a fundamental skill in algebra with widespread applications. Through the example of a business selling two products, we've illustrated how to:

- Interpret a real-world scenario.
- Define variables and formulate equations.

- Employ the elimination method to solve the system efficiently.
- Verify and interpret the solutions practically.

Mastering these steps equips students and professionals alike with a robust approach to tackling complex problems involving multiple interconnected quantities. As you encounter varied situations, remember that the key lies in careful modeling, strategic manipulation, and critical verification of solutions — skills that are invaluable both academically and in everyday reasoning.

Final Remarks

Whether you're optimizing production, analyzing financial data, or planning logistics, the ability to formulate systems of equations and solve them methodically will enhance your analytical toolkit. Embrace the process, practice with diverse scenarios, and appreciate the elegance and power of algebraic modeling in decoding the complexities of the world around us.

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