# Write An Augmented Matrix And Use Elementary Row Operations In Order To Solve The Following System Of

**Write An Augmented Matrix And Use Elementary Row Operations In Order To Solve The Following System Of** linear equations is a fundamental technique in algebra and linear algebra that simplifies the process of finding solutions to systems of equations. Whether you are a student learning about matrices or a professional solving complex systems, understanding how to construct augmented matrices and apply elementary row operations is essential. This method transforms the original system into a form that makes solutions readily apparent, often leading to quicker and more systematic problem solving.

In this comprehensive guide, we will explore the process step-by-step, from forming the augmented matrix to applying elementary row operations, and ultimately solving the system of equations. We will also include practical examples to illustrate each concept, ensuring you develop a clear understanding of this powerful technique.

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## Understanding the System of Equations

Before diving into matrices and row operations, it's crucial to understand the structure of the system of equations. Typically, a system consists of multiple linear equations involving the same set of variables.

### Example System of Equations

Suppose we have the following system:

1. $2x + 3y - z = 5$
2. $-x + 4y + 2z = 6$
3. $3x - y + z = 4$

This system involves three variables: x, y, and z, and three equations.

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# Forming the Augmented Matrix

The first step in solving the system via matrix methods is to write the augmented matrix, which consolidates the coefficients of the variables and the constants into a single matrix.

## What Is an Augmented Matrix?

An augmented matrix combines the coefficients of the variables from all equations into a matrix, with an additional column representing the constants from the right side of each equation.

## Constructing the Augmented Matrix

For the example above, the augmented matrix is:

$$
\begin{bmatrix}
2 & 3 & -1 & | & 5 \\
-1 & 4 & 2 & | & 6 \\
3 & -1 & 1 & | & 4 \\
\end{bmatrix}
$$

Note: The separator '|' is often used to distinguish between coefficients and constants, but in matrix notation, the constants are simply added as the last column.

In pure matrix form, it looks like:

$$
\left[\begin{array}{ccc|c}
2 & 3 & -1 & 5 \\
-1 & 4 & 2 & 6 \\
3 & -1 & 1 & 4 \\
\end{array}\right]
$$

---

# Elementary Row Operations

Elementary row operations are the tools used to manipulate the augmented matrix to reach a form that's easy to solve—most commonly the reduced row echelon form (RREF). There are three types of elementary row operations:

## Types of Elementary Row Operations

1. **Row swapping (Interchanging rows):** Swap the positions of two rows.

2. **Row scaling (Multiplying a row by a non-zero scalar):** Multiply all elements of a row by a non-zero scalar.

3. **Row addition (Adding a multiple of one row to another):** Add or subtract a multiple of one row from another row.

These operations do not change the solution set of the system and are used to simplify the augmented matrix step-by-step.

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# Applying Elementary Row Operations to Solve the System

The goal is to manipulate the augmented matrix into a form where solutions can be directly read or easily computed, such as the row echelon form or reduced row echelon form.

## Step-by-Step Process

1. Make the leading coefficient of the first row a 1 (if possible).
2. Use row operations to create zeros below this leading 1.
3. Repeat the process for subsequent rows, moving diagonally down the matrix (Gaussian elimination).
4. Once in row echelon form, perform back substitution to find the variable values.
5. Optionally, continue to reduce to RREF for direct solutions.

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# Practical Example: Solving the System

Let's go through an example with the earlier system.

Step 1: Write the augmented matrix

$$
\left[\begin{array}{ccc|c}
2 & 3 & -1 & 5 \\
-1 & 4 & 2 & 6 \\
3 & -1 & 1 & 4 \\
\end{array}\right]
$$

Step 2: Make the pivot of the first row a 1

Divide the first row by 2:

$$
R_1 \leftarrow \frac{1}{2} R_1
$$

$$
\left[\begin{array}{ccc|c}
1 & 1.5 & -0.5 & 2.5 \\
-1 & 4 & 2 & 6 \\
3 & -1 & 1 & 4 \\
\end{array}\right]
$$

Step 3: Eliminate below the first pivot

Add row 1 multiplied by 1 to row 2:

$$
R_2 \leftarrow R_2 + R_1
$$

$$
R_2 = (-1 + 1) \quad (4 + 1.5) \quad (2 - 0.5) \quad (6 + 2.5)
$$

$$
R_2 = 0 \quad 5.5 \quad 1.5 \quad 8.5
$$

Similarly, eliminate below in row 3:

Subtract 3 times row 1 from row 3:

\[
R_3 \leftarrow R_3 - 3 R_1
\]
\[
R_3 = (3 - 3 \times 1) \quad (-1 - 3 \times 1.5) \quad (1 - 3 \times -0.5) \quad (4 - 3 \times 2.5)
\]
\[
R_3 = 0 \quad -1 - 4.5 = -5.5 \quad 1 + 1.5 = 2.5 \quad 4 - 7.5 = -3.5
\]

Updated matrix:

\[
\left[\begin{array}{ccc|c}
1 & 1.5 & -0.5 & 2.5 \\
0 & 5.5 & 1.5 & 8.5 \\
0 & -5.5 & 2.5 & -3.5 \\
\end{array}\right]
\]

Step 4: Make the second pivot a 1

Divide row 2 by 5.5:

\[
R_2 \leftarrow \frac{1}{5.5} R_2
\]

\[
R_2 \approx 0 \quad 1 \quad \frac{1.5}{5.5} \approx 0.2727 \quad \frac{8.5}{5.5} \approx 1.5455
\]

Updated matrix:

\[
\left[\begin{array}{ccc|c}
1 & 1.5 & -0.5 & 2.5 \\
0 & 1 & 0.2727 & 1.5455 \\
0 & -5.5 & 2.5 & -3.5 \\

\end{array}\right]
\]

Step 5: Eliminate above and below the second pivot

Eliminate the 1.5 in row 1:

\[
R_1 \leftarrow R_1 - 1.5 R_2
\]
\[
R_1 = 1 \quad 1.5 - 1.5 \times 1 = 0 \quad -0.5 - 1.5 \times 0.2727 \approx -0.5 - 0.4091 = -0.9091 \quad 2.5 - 1.5 \times 1.5455 \approx 2.5 - 2.3182 = 0.1818
\]

Eliminate the -5.5 in row 3:

\[
R_3 \leftarrow R_3 + 5.5 R_2
\]
\[
R_3 = 0 \quad 0 \quad 2.5 + 5.5 \times 0.2727 \approx 2.5 + 1.5 = 4 \quad -3.5 + 5.5 \times 1.5455 \approx -3.5 + 8.5 = 5
\]

Updated matrix:

\[
\left[\begin{array}{ccc|c}
1 & 0 & -0.9091 & 0.1818 \\
0 & 1 & 0.2727 & 1.5455 \\
0 & 0 & 4 & 5 \\
\end{array}\right]
\]

Step 6: Solve for variables via back substitution

From the last row:

\[
4z = 5 \Right

# Frequently Asked Questions

## What is an augmented matrix in the context of solving a system of equations?

An augmented matrix is a matrix that combines the coefficients of the variables and the constants from a system of equations into a single matrix, typically written as [A|b], to facilitate solving the system using matrix operations.

## How do you write an augmented matrix for a system of three equations with three variables?

Write the coefficients of each variable in each equation as rows, and place the constants in the last column, resulting in a matrix of the form:
[
[$a_{11}$, $a_{12}$, $a_{13}$ | $b_1$],
[$a_{21}$, $a_{22}$, $a_{23}$ | $b_2$],
[$a_{31}$, $a_{32}$, $a_{33}$ | $b_3$]
].

## What are elementary row operations, and why are they used in solving systems via matrices?

Elementary row operations are operations that transform a matrix into a simpler form, such as row addition, row multiplication by a non-zero scalar, and row interchange. They are used to simplify the augmented matrix to row echelon or reduced row echelon form, making it easier to find the solutions of the system.

## Can you outline the steps to use elementary row operations to solve a system of linear equations?

Yes. First, write the augmented matrix of the system. Next, use elementary row operations to convert the matrix into row echelon form, where lower entries below pivots are zero. Then, perform back substitution to find the solutions. Alternatively, reduce to reduced row echelon form to directly read off solutions.

## What is the goal of applying elementary row operations when solving a system of equations?

The goal is to simplify the augmented matrix to a form where the solutions are easily obtainable, typically row echelon form or reduced row echelon form, which reveals the values of the variables directly or allows straightforward back substitution.

## How do you interpret the solutions obtained after row operations on an augmented matrix?

After applying row operations, the resulting matrix indicates the values of variables. If each variable has a leading 1 in its column with zeros elsewhere, the solution is unique. If there are rows with all zeros except in the constants column, it indicates infinitely many solutions or inconsistency, depending on the constants.

## What common mistakes should be avoided when performing elementary row operations?

Common mistakes include incorrect row additions or subtractions, forgetting to multiply a row by a scalar, swapping rows incorrectly, and making arithmetic errors. It's important to perform each operation carefully and double-check each step to ensure accuracy.

## Can elementary row operations be used to determine if a system has no solution, one solution, or infinitely many solutions?

Yes. By transforming the augmented matrix into row echelon form, you can identify inconsistent systems (no solutions) if a row reduces to [0 0 ... 0 | non-zero], or determine whether solutions are unique or infinite based on the presence and position of leading variables and free variables.

# Additional Resources

Write An Augmented Matrix And Use Elementary Row Operations In Order To Solve The Following System Of Equations – A Comprehensive Guide

Solving systems of linear equations is a fundamental skill in algebra, essential for fields ranging from engineering and computer science to economics and social sciences. One of the most effective and systematic methods for tackling such systems is through the use of augmented matrices and elementary row operations. This approach transforms the problem into a matrix manipulation task, allowing for a streamlined process to find solutions efficiently. In this article, we delve deeply into the concept of writing an augmented matrix, applying elementary row operations, and solving systems step-by-step, providing clarity and practical insights.

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# Understanding the Concept of Augmented Matrices

# What Is an Augmented Matrix?

An augmented matrix is a compact representation of a system of linear equations. It combines the coefficients of the variables and the constants into a single matrix, facilitating matrix operations that streamline solving the system.

For example, consider the system:

$$
\begin{cases}
2x + 3y - z = 5 \\
- x + 4y + 2z = 6 \\
3x - y + z = 4
\end{cases}
$$

The augmented matrix for this system is:

$$
\left[
\begin{array}{ccc|c}
2 & 3 & -1 & 5 \\
-1 & 4 & 2 & 6 \\
3 & -1 & 1 & 4
\end{array}
\right]
$$

This matrix encapsulates all the information needed to find the values of $x$, $y$, and $z$.

# Advantages of Using Augmented Matrices

- Compact Representation: Simplifies complex systems into manageable forms.
- Facilitates Systematic Operations: Enables the use of matrix algebra to solve equations efficiently.
- Compatibility with Computer Algorithms: Well-suited for implementation in computational software and calculators.
- Clear Visualization: Provides an organized view of the entire system.

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# Elementary Row Operations: The Core of Matrix Manipulation

## Types of Elementary Row Operations

Elementary row operations are the basic manipulations used to transform a matrix into a simpler form, typically row echelon form or reduced row echelon form. They are:

1. Row Swapping (Row Interchange): Swap two rows.
2. Row Multiplication: Multiply a row by a non-zero scalar.
3. Row Addition (Row Replacement): Add a multiple of one row to another row.

These operations are reversible and preserve the solution set of the system, making them ideal for solving equations systematically.

## Why Use Elementary Row Operations?

- They provide a methodical way to reduce complex systems.
- Enable the application of Gaussian elimination and Gauss-Jordan elimination methods.
- Help in identifying whether the system has a unique solution, infinitely many solutions, or no solution.

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# Step-by-Step Procedure for Solving the System

## 1. Write the Augmented Matrix

Begin by translating the system into its augmented matrix form, as shown in the earlier example. Ensure all coefficients and constants are correctly placed.

## 2. Apply Elementary Row Operations to Achieve Row Echelon Form

The goal is to create zeros below the leading coefficients (pivots) in each column.

- Select a pivot: Usually the first non-zero entry in the first row.

- Create zeros beneath the pivot: Use row addition/subtraction to eliminate variables.
- Move to the next column: Repeat the process for subsequent rows and columns.

## 3. Convert to Reduced Row Echelon Form

Once in row echelon form, further reduce the matrix so that each leading coefficient is 1, and zeros are above the pivots as well, facilitating easy back-substitution.

- Normalize each pivot row: Divide the entire row by the pivot element.
- Eliminate above pivots: Use row operations to create zeros above each leading 1.

## 4. Interpret the Matrix and Find the Solution

- Unique solution: When each variable has a leading 1 and the system is consistent.
- Infinite solutions: When some variables are free (columns without pivots).
- No solution: When a row reduces to form an inconsistent statement like \(0 = 1\).

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# Practical Example: Solving a System Using Augmented Matrices and Row Operations

Let's consider the following system:

\[
\begin{cases}
x + 2y + z = 4 \\
3x + 8y + 2z = 10 \\
-2x + y + 2z = -2
\end{cases}
\]

Step 1: Write the augmented matrix

\[
\left[
\begin{array}{ccc|c}

1 & 2 & 1 & 4 \\
3 & 8 & 2 & 10 \\
-2 & 1 & 2 & -2
\end{array}
\right]
\]

Step 2: Use row operations to reach row echelon form

- Keep Row 1 unchanged.
- Eliminate \(x\) from Row 2 and Row 3:

\[
R_2 \to R_2 - 3 R_1
\]
\[
R_3 \to R_3 + 2 R_1
\]

Calculations:

- \( R_2: (3,8,2,10) - 3 \times (1,2,1,4) = (0, 8 - 6, 2 - 3, 10 - 12) = (0, 2, -1, -2) \)
- \( R_3: (-2,1,2,-2) + 2 \times (1,2,1,4) = (0, 1 + 4, 2 + 2, -2 + 8) = (0, 5, 4, 6) \)

Updated matrix:

\[
\left[
\begin{array}{ccc|c}
1 & 2 & 1 & 4 \\
0 & 2 & -1 & -2 \\
0 & 5 & 4 & 6
\end{array}
\right]
\]

- Make the second pivot 1:

\[
R_2 \to \frac{1}{2} R_2
\]

\[

(0, 2, -1, -2) \to (0, 1, -\frac{1}{2}, -1)
\]

- Eliminate \(y\) from \(R_3\):

\[
R_3 \to R_3 - 5 R_2
\]

Calculations:

\[
(0, 5, 4, 6) - 5 \times (0, 1, -\frac{1}{2}, -1) = (0, 0, 4 - 5 \times (-\frac{1}{2}), 6 - 5 \times (-1))
\]

\[
= (0, 0, 4 + 2.5, 6 + 5) = (0, 0, 6.5, 11)
\]

Updated matrix:

\[
\left[
\begin{array}{ccc|c}
1 & 2 & 1 & 4 \\
0 & 1 & -\frac{1}{2} & -1 \\
0 & 0 & 6.5 & 11
\end{array}
\right]
\]

- Normalize the third row:

\[
R_3 \to \frac{1}{6.5} R_3
\]

\[
(0, 0, 6.5, 11) \to (0, 0, 1, \frac{11}{6.5}) = (0, 0, 1, \frac{22}{13})
\]

Step 3: Back-substitution to find solutions

- From \( R_3 \): \( z = \frac{22}{13} \)

- From \( R_2 \):

\[
y - \frac{1}{2} z = -1
\]
\[
y = -1 + \frac{1}{2} \times \frac{22}{13} = -1 + \frac{11}{13} = -\frac{13}{13} + \frac{11}{13} = -\frac{2}{13}
\]

- From \( R_1 \):

\[
x + 2 y + z = 4
\]
\[
x = 4 - 2 y - z
\]
\[
x = 4 - 2 \times \left(-\frac{2}{13}\right) - \frac{22}{13} = 4 + \frac{4}{13} - \frac{22}{13} = \frac{52}{13} + \frac{4}{13} - \frac{22}{13} = \frac{34}{13}
\]

Final solution:

\[
x = \frac{34}{13} \approx 2.615 \\
y = -\frac{

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