3. What Are The Intersection Points Of The Line Whose Equation Is Y=3x +3 And Thecircle Whose Equation

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Understanding the intersection points between a line and a circle is a fundamental concept in coordinate geometry. These intersection points are the solutions that satisfy both the line's and the circle's equations simultaneously. In this article, we will explore how to find the intersection points of the line given by the equation y = 3x + 3 and a circle with a specified equation. We'll walk through the mathematical process step-by-step, discuss different scenarios that may occur, and provide practical tips for solving such problems efficiently.

Understanding the Equations Involved

Before diving into the solution process, it is essential to understand the standard forms of the equations involved.

The Line Equation: y = 3x + 3

This is the equation of a straight line in slope-intercept form, where:

- **Slope (m):** 3
- **Y-intercept (b):** 3

This line crosses the y-axis at (0, 3) and has a steep slope of 3, indicating that for every increase of 1 in x, y increases by 3.

The Circle Equation

A general circle equation in the coordinate plane is:

\[(x - h)^2 + (y - k)^2 = r^2 \]

Where:

- **(h, k):** Center of the circle
- **r:** Radius of the circle

For this discussion, assume the circle's equation is:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

with specific values for h, k, and r, which should be provided to find exact intersection points.

Methodology to Find Intersection Points

Finding the intersection points involves solving the system of equations comprising the line and the circle. The general steps are:

1. Substitute the line's equation into the circle's equation to eliminate y.
2. Simplify the resulting quadratic equation in x.
3. Determine the nature of the solutions based on the discriminant.
4. Calculate the corresponding y-values for each valid x-value.

Let’s explore each step in detail.

Step-by-Step Solution Process

1. Substitution of y from the line into the circle equation

Given:

$$ y = 3x + 3 $$

and

$$ (x - h)^2 + (y - k)^2 = r^2 $$

Substitute y into the circle's equation:

$$ (x - h)^2 + ((3x + 3) - k)^2 = r^2 $$

This substitution transforms the problem into a quadratic equation in terms of x.

2. Simplify the quadratic equation

Expand the terms:

$$ (x - h)^2 + (3x + 3 - k)^2 = r^2 $$

which expands to:

$$ (x^2 - 2hx + h^2) + (9x^2 + 6x(1 - k) + (3 - k)^2) = r^2 $$

Combine like terms:

$$ x^2 - 2hx + h^2 + 9x^2 + 6x(1 - k) + (3 - k)^2 = r^2 $$

$$ (x^2 + 9x^2) + (-2hx + 6x(1 - k)) + [h^2 + (3 - k)^2] = r^2 $$

Simplify further:

$$ 10x^2 + x(-2h + 6(1 - k)) + (h^2 + (3 - k)^2 - r^2) = 0 $$

This is a quadratic in the standard form:

$$ Ax^2 + Bx + C = 0 $$

where:

- $A = 10$
- $B = -2h + 6(1 - k)$
- $C = h^2 + (3 - k)^2 - r^2$

3. Analyze the discriminant to determine the number of solutions

The discriminant D is:

$$ D = B^2 - 4AC $$

- If $D > 0$, there are two distinct real solutions; the line intersects the circle at two points.
- If $D = 0$, there is exactly one solution; the line is tangent to the circle.
- If $D < 0$, there are no real solutions; the line does not intersect the circle.

Calculate D with the known values of h, k, and r.

4. Find the x-coordinates of the intersection points

Use the quadratic formula:

$$ x = \frac{-B \pm \sqrt{D}}{2A} $$

Determine the x-values for each case.

5. Find the corresponding y-coordinates

Substitute each x-value into the line equation:

$$ y = 3x + 3 $$

to find the corresponding y-values.

Examples with Specific Circle Equations

To make the process clearer, let’s consider some specific examples.

Example 1: Circle with center at (0, 0) and radius 5

Equation:

$$ x^2 + y^2 = 25 $$

Substitute $y = 3x + 3$:

$$ x^2 + (3x + 3)^2 = 25 $$

Expand:

$$ x^2 + 9x^2 + 18x + 9 = 25 $$

Combine like terms:

$$ 10x^2 + 18x + 9 = 25 $$

Bring all to one side:

$$ 10x^2 + 18x + 9 - 25 = 0 $$

$$ 10x^2 + 18x - 16 = 0 $$

Divide through by 2:

$$ 5x^2 + 9x - 8 = 0 $$

Calculate the discriminant:

$$ D = 9^2 - 4 \times 5 \times (-8) = 81 + 160 = 241 $$

Since $D > 0$, two real solutions exist.

Calculate the solutions:

$$ x = \frac{-9 \pm \sqrt{241}}{2 \times 5} = \frac{-9 \pm \sqrt{241}}{10} $$

Approximate:

$$ \sqrt{241} \approx 15.52 $$

Thus,

$$ x_1 \approx \frac{-9 + 15.52}{10} = \frac{6.52}{10} = 0.652 $$

$$ x_2 \approx \frac{-9 - 15.52}{10} = \frac{-24.52}{10} = -2.452 $$

Find corresponding y-values:

$$ y_1 = 3(0.652) + 3 \approx 1.956 + 3 = 4.956 $$

$$ y_2 = 3(-2.452) + 3 \approx -7.356 + 3 = -4.356 $$

Intersection Points:

$$
\boxed{
(0.652, 4.956) \quad \text{and} \quad (-2.452, -4.356)
}
$$

Example 2: Tangent line to the circle

Suppose the circle is:

$$ (x - 1)^2 + (y - 2)^2 = 9 $$

and the line remains:

$$ y = 3x + 3 $$

Substituting:

$$ (x - 1)^2 + (3x + 3 - 2)^2 = 9 $$

Simplify:

$$ (x - 1)^2 + (3x + 1)^2 = 9 $$

Expand:

$$ x^2 - 2x + 1 + 9x^2 + 6x + 1 = 9 $$

Combine:

$$ 10x^2 + 4x + 2 = 9 $$

Bring all to one side:

$$ 10x^2 + 4x + 2 - 9 = 0 $$

$$ 10x^2 + 4x - 7 = 0 $$

Calculate discriminant:

$$ D = 4^2 - 4 \times 10 \times (-7) = 16 + 280 = 296 $$

Since $D > 0$, two solutions exist, indicating the line intersects the circle at two points, not tangent. To find a tangent, adjust the circle parameters to satisfy $D = 0$.

Practical Tips for Solving Intersection Problems

- Always verify the specific equation of the circle before solving.
- Use substitution to reduce the system to a quadratic in one variable.
- Calculate the discriminant carefully

Frequently Asked Questions

What is the general method to find the intersection points between the line y=3x+3 and a circle?

To find the intersection points, substitute y=3x+3 into the circle's equation, resulting in a quadratic in x. Solve the quadratic to find x-values, then use these to find corresponding y-values.

How do the coefficients of the line y=3x+3 influence its intersection points with a circle?

The slope (3) and y-intercept (3) determine the line's position and angle. These factors affect the number and location of intersection points with the circle, depending on whether the line intersects, is tangent, or misses the circle.

What indicates that the line is tangent to the circle in the intersection problem?

If substituting y=3x+3 into the circle's equation results in a quadratic with a discriminant of zero, it indicates the line is tangent to the circle, touching at exactly one point.

Can the line y=3x+3 intersect with the circle at two points? How is this determined?

Yes, if the quadratic obtained after substitution has a positive discriminant, it means the line intersects the circle at two distinct points.

What role does the circle's equation play in determining the intersection points with y=3x+3?

The circle's equation defines its center and radius. When combined with the line's equation, it allows solving for intersection points, which depend on the relative position of the line and circle (intersecting, tangent, or no intersection).

How can one visualize the intersection points of the line y=3x+3 and a circle for better understanding?

Plotting both the circle and the line on a coordinate plane helps visualize their positions; the points where they meet are the intersection points. Graphing tools or software can make this visualization clearer.

Additional Resources

Intersection Points of the Line \(y = 3x + 3 \) and a Circle: An In-Depth Analysis

Understanding the intersection points between a line and a circle is fundamental in coordinate geometry. It provides insights into how geometric figures relate, how to solve systems of equations graphically and algebraically, and has applications ranging from engineering to computer graphics. In this comprehensive exploration, we will examine the intersection points of the line \(y = 3x + 3 \) with a circle, analyzing the problem from multiple angles, including algebraic solutions, geometric interpretations, and special cases.

Introduction to the Problem

The problem involves determining the points where the straight line $y = 3x + 3$ intersects a circle defined by the general equation:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:
- (h, k) is the circle’s center,
- r is the radius.

The key goals are:
- To find the intersection points explicitly,
- To understand the conditions under which the line intersects the circle at zero, one, or two points,
- To interpret these intersections geometrically.

General Approach to Find Intersection Points

1. Substitution Method

Since the line is expressed explicitly as $y = 3x + 3$, the most straightforward approach is to substitute y into the circle's equation:

$$
(x - h)^2 + ((3x + 3) - k)^2 = r^2
$$

This substitution yields a quadratic equation in x, which can be solved using standard methods to find the intersection points.

2. Solving the Quadratic Equation

The quadratic in x will be of the form:

$$
A x^2 + B x + C = 0
$$

where:

- A, B, and C depend on h, k, and r,
- Discriminant $D = B^2 - 4AC$ determines the nature of the roots:
- $D > 0$: two distinct intersection points,
- $D = 0$: tangent (one point of intersection),
- $D < 0$: no real intersection points.

Once x is found, substitute back into $y = 3x + 3$ to get the y-coordinates.

Deriving the Intersection Equation

Let's formalize the process step-by-step, assuming a generic circle:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

and the line:

$$
y = 3x + 3
$$

Step 1: Substitute y into the circle's equation

$$
(x - h)^2 + ((3x + 3) - k)^2 = r^2
$$

Step 2: Expand and simplify

$$
(x - h)^2 + (3x + 3 - k)^2 = r^2
$$

Expanding each:

$$
(x^2 - 2 h x + h^2) + (9x^2 + 6x(3 - k) + (3 - k)^2) = r^2
$$

which simplifies to:

$$
x^2 - 2 h x + h^2 + 9x^2 + 6x(3 - k) + (3 - k)^2 = r^2
$$

Combine like terms:

$$
(1 + 9) x^2 + [-2 h + 6 (3 - k)] x + h^2 + (3 - k)^2 - r^2 = 0
$$

or:

$$
10 x^2 + \left(-2 h + 18 - 6k \right) x + \left(h^2 + (3 - k)^2 - r^2 \right) = 0
$$

This quadratic in x can be written as:

$$
A x^2 + B x + C = 0
$$

where:

- $A = 10$,
- $B = -2 h + 18 - 6 k$,
- $C = h^2 + (3 - k)^2 - r^2$.

Analyzing the Discriminant and Intersection Types

The discriminant:

$$
D = B^2 - 4 A C
$$

determines the nature of the intersection:

- Two points: $D > 0$,
- One point (tangent): $D = 0$,
- No points: $D < 0$.

This discriminant depends on the circle's parameters (h, k, r), which influence whether the line cuts, touches, or misses the circle.

Special Cases and Examples

1. When the Circle and Line are Tangent

The line touches the circle at exactly one point:

$$
D = 0
$$

This occurs when:

$$
B^2 = 4 A C
$$

which can be used to find specific configurations where the line is tangent to the circle.

Example:

Suppose the circle has center $(h, k) = (0, 0)$ and radius $r = 5$. The quadratic becomes:

$$
10 x^2 + (18 - 6 \times 0) x + 0 + (3 - 0)^2 - 25 = 0
$$
$$
10 x^2 + 18 x + 9 - 25 = 0
$$
$$
10 x^2 + 18 x - 16 = 0
$$

Discriminant:

$$
D = 18^2 - 4 \times 10 \times (-16) = 324 + 640 = 964 > 0
$$

Since the discriminant is positive, two intersection points exist. To find the tangent case, set $D = 0$:

$$
B^2 = 4AC
$$

which gives the condition for the circle's parameters to be tangent to the line.

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2. When the Line Does Not Intersect the Circle

If the discriminant:

$$
D < 0
$$

then the quadratic has no real solutions, indicating the line misses the circle entirely.

Implication:

- No intersection points,
- The line lies outside the circle.

---

3. When the Line Passes Through the Circle (Two Points)

For $D > 0$, the line intersects the circle at two distinct points. These points can be explicitly calculated:

$$
x_{1,2} = \frac{-B \pm \sqrt{D}}{2A}
$$

and

$$
y_{1,2} = 3 x_{1,2} + 3
$$

This provides the exact coordinates of the intersection points, essential in applications requiring precise location, such as collision detection or geometric constructions.

Explicit Calculation of Intersection Points

To find the exact points, follow these steps:

1. Calculate the discriminant D.

2. Find x-coordinates:

$$
x_{1} = \frac{-B + \sqrt{D}}{2A}
$$
$$
x_{2} = \frac{-B - \sqrt{D}}{2A}
$$

3. Calculate corresponding y-coordinates:

$$
y_{1} = 3x_{1} + 3
$$
$$
y_{2} = 3x_{2} + 3
$$

These points (x_1, y_1) and (x_2, y_2) are the intersection points.

Graphical Interpretation and Visualization

Visualizing the problem enhances understanding. The line $y = 3x + 3$ has a slope of 3, indicating a steep incline. The circle's position relative to this line determines the number of intersection points.

- When the circle is positioned such that the line cuts through it, two intersection points appear on the graph.
- If the circle is tangent, the line just touches the circle at a single point, visually a point of tangency.
- If the circle is outside the line’s path, no points of intersection are visible.

Plotting various configurations helps in grasping how shifts in the circle's center or radius affect the intersection pattern.

Special Considerations and Variations

1. Changing the Equation of the Line

If the line's equation is altered, the method remains similar but with different coefficients. For example, for $y = m x + c$:

- Substitute into the circle's equation,
- Derive the quadratic in \(x \),
- Analyze the discriminant.

2. Varying the Circle’s Parameters

Adjusting the circle's center \((h, k) \) or radius \(r \):

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