4. For The System Given In State Space Form * = [_] X + [] U Y = [1 2]x Design An Observer With Poles

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Designing an observer for a linear system in state-space form is a fundamental task in control engineering, enabling the estimation of unmeasured states based on system outputs and inputs. In this article, we will explore the process of designing an observer for a given system with specified dynamics, focusing on pole placement to ensure desired convergence characteristics. We will cover the necessary theoretical background, step-by-step design procedures, and practical considerations to implement a robust observer, such as a Luenberger observer or a Kalman filter, tailored to the specified system.

Understanding the State-Space System

System Representation

The system is described by the following state-space equations:

\[
\begin{aligned}
\dot{x}(t) &= A x(t) + B u(t) \\
y(t) &= C x(t)
\end{aligned}
\]

where:

- \(x(t) \in \mathbb{R}^n \) is the state vector.
- \(u(t) \in \mathbb{R}^m \) is the control input.
- \(y(t) \in \mathbb{R}^p \) is the output vector.
- \(A \) is the system matrix.
- \(B \) is the input matrix.
- \(C \) is the output matrix.

Given the notation, the specific matrices are:

\[
A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}

\quad
B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}
\quad
C = \begin{bmatrix} 1 & 2 \end{bmatrix}
\]

The exact entries of \(A\) and \(B\) are crucial for the design process and should be provided or estimated based on the system under consideration.

Design Objectives for the Observer

The primary goal of the observer is to estimate the state vector \(x(t) \) using only the measurable output \(y(t) \) and the known input \(u(t) \). The observer should be designed such that the estimation error \(e(t) = x(t) - \hat{x}(t) \) converges to zero asymptotically.

To achieve this, the observer dynamics are typically chosen as:

\[
\dot{\hat{x}}(t) = A \hat{x}(t) + B u(t) + L (y(t) - C \hat{x}(t))
\]

where:

- \(\hat{x}(t) \) is the estimated state vector.
- \(L \in \mathbb{R}^{n \times p} \) is the observer gain matrix, designed to place the eigenvalues (poles) of the estimation error dynamics.

Key considerations:

- The eigenvalues of \(A - L C \) determine the speed of convergence of the estimation error.
- The placement of these poles affects the stability and responsiveness of the observer.

Pole Placement Method for Observer Design

Why Place Poles?

Pole placement allows the designer to specify the eigenvalues of the error dynamics matrix \((A - L C) \). By choosing these eigenvalues appropriately, the observer can have desired speed and stability characteristics.

- Fast poles lead to quick convergence but may cause noise amplification.
- Slow poles result in slower convergence but are more robust to measurement

noise.

Steps to Design the Observer Poles

1. Determine the Desired Eigenvalues:
- Choose a set of eigenvalues (poles) for $A - L C$ based on system performance criteria.
- Typically, these are placed to be faster than the system poles to ensure quick estimation.

2. Verify System Observability:
- Ensure the pair (A, C) is observable.
- The observability matrix:

$$
\mathcal{O} = \begin{bmatrix} C \\ C A \\ \vdots \\ C A^{n-1} \end{bmatrix}
$$

must be full rank.

3. Calculate the Observer Gain L:
- Use pole placement algorithms (e.g., Ackermann's formula or MATLAB's `place` function) to compute L.

4. Implement the Observer:
- Program the observer dynamics with the computed L.

Example: Designing an Observer for a Specific System

Suppose we have the following system matrices:

$$
A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix},
\quad
B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\quad
C = \begin{bmatrix} 1 & 2 \end{bmatrix}
$$

and we want to place the observer poles at -5 and -6 for fast convergence.

Step 1: Verify observability.

Calculate the observability matrix:

$$
\mathcal{O} = \begin{bmatrix}
C \\
C A
\end{bmatrix} = \begin{bmatrix}
1 & 2 \\
(1)(0) + (2)(-2) & (1)(1) + (2)(-3)
\end{bmatrix} = \begin{bmatrix}
1 & 2 \\
-4 & -5
\end{bmatrix}
$$

Determinant:

$$
\det(\mathcal{O}) = (1)(-5) - (2)(-4) = -5 + 8 = 3 \neq 0
$$

So, the system is observable.

Step 2: Use pole placement to find L.

Applying Ackermann's formula or MATLAB's `place`:

```matlab
A = [0 1; -2 -3];
C = [1 2];
desired_poles = [-5, -6];
L = place(A', C', desired_poles)';
```

Suppose this yields:

$$
L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}
$$

Step 3: Implement the observer dynamics:

$$
\dot{\hat{x}} = A \hat{x} + B u + L (y - C \hat{x})
$$

which ensures the estimation error dynamics:

$$
\dot{e} = (A - L C) e
$$

have eigenvalues at the specified locations.

Practical Considerations in Observer Design

Designing an effective observer involves more than pole placement. Some key considerations include:

- Measurement Noise: High bandwidth (fast poles) can amplify noise; balance is necessary.
- Model Uncertainty: Ensure the model accurately reflects the system dynamics.
- Sensor Limitations: Sensor accuracy and sampling rates influence observer performance.
- Robustness: Consider robust observer designs like Kalman filters for noisy environments.

Advanced Observer Design Techniques

While the basic Luenberger observer is straightforward, more advanced methods can improve performance:

- Kalman Filter: Optimal in stochastic noise environments, providing minimum variance estimates.
- Sliding Mode Observers: Handle nonlinearities and uncertainties robustly.
- High-Gain Observers: Achieve fast convergence with high gains, suitable for systems with fast dynamics.

Conclusion

Designing an observer with specified poles for a system in state-space form is a critical step in control system development, enabling accurate state estimation for feedback control. The process involves verifying system observability, selecting appropriate poles based on performance criteria, and computing the observer gain matrix accordingly. Properly tuned observers improve system stability, responsiveness, and robustness, especially when certain states cannot be measured directly.

By understanding the theoretical principles and following systematic design procedures, engineers can develop effective observers tailored to their specific systems, ensuring reliable and efficient operation in real-world applications.

References:

- Ogata, K. (2010). Modern Control Engineering. Prentice Hall.

- Khalil, H. K. (2002). Nonlinear Systems. Prentice Hall.
- Skogestad, S., & Postlethwaite, I. (2005). Multivariable Feedback Control. Wiley.
- MATLAB Documentation: `place` command and observer design tools.

Frequently Asked Questions

What is the primary goal when designing an observer for a system in state space form?

The primary goal is to estimate the system's internal states accurately based on the output measurements, enabling better control and analysis even when some states are not directly measurable.

Given the state space system $\dot{X} = A X + B U$ and output $Y = C X$, how do we select observer poles?

Observer poles are chosen to be faster than the system poles, typically by placing them at locations in the complex plane with more negative real parts, to ensure rapid convergence of the estimated states to the actual states.

How does the observer gain matrix L relate to the pole placement method?

The gain matrix L is designed such that the eigenvalues of (A - L C) are placed at desired locations (poles), which determines the speed and stability of the state estimation error dynamics.

What is the significance of the output matrix C = [1 2] in the given system?

The matrix C = [1 2] specifies how the system states are mapped to the output Y, indicating that the output is a linear combination of the states, which influences the design of the observer.

How do you determine the eigenvalues of the observer error dynamics?

The eigenvalues are determined by the matrix (A - L C); by placing the poles of this matrix at desired locations, we control the rate at which the estimation error converges to zero.

What are the typical steps involved in designing an

observer for the system?

The steps include: (1) defining the system matrices A, B, C; (2) selecting desired observer poles; (3) applying pole placement techniques to compute the gain matrix L; and (4) implementing the observer with the chosen L.

Why is it important to choose observer poles faster than the system poles?

Choosing observer poles with larger negative real parts ensures that the estimation error converges quickly, providing accurate state estimates in a shorter time frame without destabilizing the system.

Can the observer be designed if the system is not controllable or observable? Why or why not?

No, the observer design relies on the system's observability; if the system is not observable, it is impossible to reconstruct the internal states accurately from the outputs.

What is the effect of placing observer poles too far left in the complex plane?

While faster convergence occurs, placing poles too far left can lead to high gain values, which may amplify noise and cause implementation issues or excessive sensitivity in the observer.

How does the choice of observer poles affect the overall system performance?

The placement of observer poles directly influences the convergence speed of the state estimates and the robustness of the observer; well-chosen poles balance rapid convergence with noise sensitivity and system stability.

Additional Resources

Designing an Observer for a State Space System with Specified Poles

In control systems engineering, state observers play a crucial role when the full state vector is not directly measurable. They enable the estimation of unmeasured states based on system inputs and outputs, ensuring optimal control performance even under uncertainties or measurement limitations. This article delves into the systematic design of an observer for a given continuous-time linear system expressed in state space form, focusing on pole placement techniques to achieve desired dynamic characteristics.

Understanding the System Description

System State Space Representation

The system under consideration is described by the following equations:

$$
\dot{x}(t) = A x(t) + B u(t)
$$

$$
y(t) = C x(t)
$$

where:
- $x(t) \in \mathbb{R}^n$ is the state vector,
- $u(t) \in \mathbb{R}^m$ is the control input,
- $y(t) \in \mathbb{R}^p$ is the measured output,
- A is the system matrix,
- B is the input matrix,
- C is the output matrix.

In the given problem statement, the matrices are partially provided:

$$
A = \text{[_]} \quad
B = \text{[ ]} \quad
C = \begin{bmatrix} 1 & 2 \end{bmatrix}
$$

which indicates a single output system with two states. Precise values for A and B need to be known or specified; for demonstration, we will assume general forms and discuss the methodology.

Objectives of Observer Design

The goal is to design an observer (or estimator) that reconstructs the full state vector $x(t)$ based on the measurable output $y(t)$ and the known input $u(t)$. The key objectives are:

- Asymptotic convergence: The estimated states $\hat{x}(t)$ should converge to the actual states $x(t)$ over time.
- Desired dynamic characteristics: The observer should have poles placed at locations that ensure fast convergence without excessive sensitivity to noise or disturbances.

Observer Structure and Dynamics

The Luenberger observer is a common structure for state estimation:

$$
\dot{\hat{x}}(t) = A \hat{x}(t) + B u(t) + L (y(t) - C \hat{x}(t))
$$

where:
- $\hat{x}(t)$ is the estimated state vector,
- L is the observer gain matrix, which determines the dynamics of the estimation error.

The error dynamics are given by:

$$
e(t) = x(t) - \hat{x}(t)
$$
$$
\dot{e}(t) = (A - L C) e(t)
$$

To ensure the estimation error converges to zero, the eigenvalues of $A - L C$ must be placed at desired locations in the complex plane, typically with negative real parts for stability.

Designing the Observer: Pole Placement Method

Determining the Desired Poles

Choosing the poles of the observer involves selecting eigenvalues of $A - L C$. For a 2-state system, the observer gain matrix L is a 2×1 matrix.

Criteria for pole placement:

- Speed of convergence: Poles should be placed sufficiently left in the s-plane (more negative real parts) to ensure fast convergence.
- Robustness: Poles should not be too far left to avoid high gains that amplify noise.
- Complementarity with controller poles: For combined observer-controller

design, poles are often chosen to complement each other.

Suppose we select desired observer poles at:

$$
\lambda_{1}, \lambda_{2}
$$

which are complex conjugates with negative real parts, e.g., $-5 \pm 2i$.

Controllability and Observability

Before applying pole placement, verify:

- Controllability of the pair (A, B),
- Observability of the pair (A, C).

These properties determine whether it is possible to place the poles arbitrarily.

For the observer design, observability is crucial:

$$
\text{The pair } (A, C) \text{ must be observable}
$$

which can be checked via the observability matrix:

$$
\mathcal{O} = \begin{bmatrix} C \\ C A \end{bmatrix}
$$

If $\text{rank}(\mathcal{O}) = n$, the system is observable, and pole placement is feasible.

Computing the Observer Gain L

Using Ackermann’s Formula or MATLAB’s `place` Function

Given the system matrices, the gain L can be computed via:

$$
L = \text{place}(A^T, C^T, \text{desired poles})^T
$$

In MATLAB, the syntax is:

```matlab
desired_poles = [-5+2i, -5-2i];
L = place(A', C', desired_poles)';
```

This command computes the gain matrix L such that the eigenvalues of $A - L C$ are at the specified locations.

Implementation Details and Practical Considerations

Numerical Stability and Gain Tuning

- Gain magnitude: Excessively high gains can lead to noise amplification and actuator saturation.
- Pole placement accuracy: Numerical issues may arise if the desired poles are very close to the imaginary axis.
- Model uncertainties: Real systems deviate from ideal models; robustness margins should be considered.

Simulation and Validation

- Simulate the closed-loop system with the observer to confirm convergence.
- Use initial condition variations to verify the robustness of estimation.
- Perform sensitivity analysis to assess the impact of parameter uncertainties.

Example: Step-by-Step Design Process

Suppose the system matrices are:

\[

A = \begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix}, \quad
B = \begin{bmatrix}
0 \\
1
\end{bmatrix}, \quad
C = \begin{bmatrix}
1 & 2
\end{bmatrix}
\]

1. Check observability:

\[
\mathcal{O} = \begin{bmatrix}
C \\
C A
\end{bmatrix} = \begin{bmatrix}
1 & 2 \\
(1)(0) + (2)(-2) & (1)(1) + (2)(-3)
\end{bmatrix} = \begin{bmatrix}
1 & 2 \\
-4 & -5
\end{bmatrix}
\]

\[
\text{rank}(\mathcal{O}) = 2 \quad \Rightarrow \text{System is observable}
\]

2. Select desired observer poles:

\[
\lambda_{1} = -10, \quad \lambda_{2} = -12
\]

3. Calculate \(L\):

Using MATLAB:

```matlab
A = [0 1; -2 -3];
C = [1 2];
desired_poles = [-10, -12];
L = place(A', C', desired_poles)';
```

4. Implement and simulate:

- Incorporate the observer into the system simulation.
- Validate convergence of $\hat{x}$ to x.

Advanced Topics and Variations

- Kalman Filter Design: For stochastic systems with noise, the observer design extends to stochastic optimal estimators.
- Reduced-order Observers: When only certain states are estimated.
- Adaptive Observers: Adjusting gains dynamically based on system variations.
- High-Gain Observers: For systems with fast dynamics but increased sensitivity to noise.

Conclusion

Designing an observer with specified poles involves a systematic approach rooted in controllability and observability analysis, pole placement techniques, and practical considerations for robustness and noise sensitivity. By selecting appropriate eigenvalues and computing the observer gain L, engineers can ensure that the estimated states accurately and rapidly reflect the true system states, enabling effective closed-loop control even when direct measurement is not feasible.

This process enhances the reliability and performance of control systems across a range of applications—from industrial automation to aerospace systems—highlighting the importance of thorough system analysis and judicious gain selection in observer design.

In summary:

- Verify system observability.
- Choose desired observer poles based on performance criteria.
- Use pole placement algorithms to compute L.
- Implement and validate the observer in simulation.
- Fine-tune gains considering robustness and noise.

Mastering these steps ensures a robust and responsive state estimation strategy, integral to modern control system design.

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