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Understanding the properties of circles, especially the lengths of their arcs, is a fundamental aspect of geometry. When given the lengths of the minor and major arcs, we can deduce various parameters such as the radius, central angles, and the relationship between these arcs. In this article, we will explore the problem where the length of the minor arc AB of a circle with center O is 2 cm, and the length of the corresponding major arc is 22 cm. Through detailed explanations, formulas, and step-by-step calculations, you'll gain a comprehensive understanding of how to analyze such scenarios in circle geometry.

Fundamentals of Circle Arcs

Before diving into the problem specifics, it's important to review some fundamental concepts related to circles and arcs.

What Is an Arc?

An arc of a circle is a continuous part of its circumference. It is defined by two points on the circle, with the arc being the segment of the circle's boundary between those points.

Minor and Major Arcs

- Minor arc: The shortest arc connecting two points on the circle, measuring less than 180° .
- Major arc: The longer arc connecting the same two points, measuring more than 180° .

In our case, arc AB is the minor arc, and its length is 2 cm, while the major arc AB has a length of 22 cm.

Key Concepts and Formulas

To analyze the problem effectively, let's review some important formulas related to circle arcs.

Arc Length Formula

The length of an arc (L) is related to its central angle (θ in degrees or radians) and the circle's radius (r):

- In degrees:

$$L = \frac{\theta}{360^\circ} \times 2\pi r$$

- In radians:

$$L = r \times \theta$$

where θ is in radians.

Since the circle's circumference is $(2\pi r)$, the length of any arc is proportional to its central angle.

Relationship Between Minor and Major Arcs

The sum of the measures of the minor and major arcs between two points on a circle is 360° , since they together form the entire circumference.

Analyzing the Given Data

Given:

- Length of minor arc $AB = 2$ cm
- Length of major arc $AB = 22$ cm

From the data, it's clear that the total length of the entire circumference of the circle is the sum of the minor and major arcs:

$$\begin{aligned} & \text{Circumference} = 2\pi r + 22\pi = 24\pi \end{aligned}$$

Using this, we can find the circle's radius and the central angles corresponding to each arc.

Calculating the Radius of the Circle

Since the total circumference (C) is:

$$C = 2\pi r$$

And we've established that:

$$C = 24\pi$$

we can solve for (r) :

$$\begin{aligned} 2\pi r &= 24\pi \\ r &= \frac{24}{2\pi} = \frac{12}{\pi} \text{ cm} \end{aligned}$$

Numerically:

$$r \approx \frac{12}{3.1416} \approx 3.82 \text{ cm}$$

This is the radius of the circle.

Determining the Central Angles

Next, we will find the measures of the central angles corresponding to the minor and major arcs.

Central Angle for the Minor Arc AB

Using the arc length formula in degrees:

$$L_{\text{minor}} = \frac{\theta_{\text{minor}}}{360^\circ} \times 2\pi r$$

Plugging in the known values:

$$2 = \frac{\theta_{\text{minor}}}{360^\circ} \times 2\pi \times \frac{12}{\pi}$$

Simplify:

$$2 = \frac{\theta_{\text{minor}}}{360^\circ} \times 2 \times 12$$
$$2 = \frac{\theta_{\text{minor}}}{360^\circ} \times 24$$

Solve for θ_{minor} :

$$\frac{\theta_{\text{minor}}}{360^\circ} = \frac{2}{24} = \frac{1}{12}$$

$$\theta_{\text{minor}} = \frac{1}{12} \times 360^\circ = 30^\circ$$

Thus, the central angle corresponding to the minor arc AB is 30° .

Central Angle for the Major Arc AB

Since the total of the minor and major arcs is 360° , the measure of the major arc's central angle is:

$$\begin{aligned} \backslash \\ \theta_{\text{major}} = 360^\circ - \theta_{\text{minor}} = 360^\circ - 30^\circ = 330^\circ \\ \backslash \end{aligned}$$

Summary:

- Minor arc AB: length = 2 cm, central angle = 30°
- Major arc AB: length = 22 cm, central angle = 330°

Understanding the Geometry and Its Implications

The calculations reveal that the minor arc AB subtends a 30° angle at the center O, while the major arc subtends a 330° angle. The large difference in arc lengths corresponds to the difference in central angles, consistent with the circle's total 360° .

Key observations:

- The minor arc is relatively small, indicating that points A and B are close together on the circle.
- The major arc covers nearly the entire circumference, excluding the minor arc's 30° , emphasizing the complementary nature of these arcs.

Practical Applications and Examples

Understanding arc lengths and their corresponding angles has numerous practical applications, including:

- Designing circular tracks or roads, where knowing the length of specific arcs helps in planning distances.
- Engineering and manufacturing, where precise measurements of circular components are crucial.
- Astronomy, for calculating angular separations between celestial objects.

Additional Problems and Exercises

To deepen your understanding, consider solving these related problems:

1. Find the radius of a circle if the minor arc AB measures 3 cm and subtends a 45° angle at the center.
2. Calculate the length of an arc that subtends a 120° angle in a circle with radius 5 cm.
3. Determine the measure of the central angle if the arc length is 10 cm in a circle with radius 4 cm.
4. Given a circle with a radius of 10 cm, find the lengths of the minor and major arcs if the central angle for the minor arc is 60° .

Summary and Key Takeaways

- The total circumference of the circle in our problem is 24 cm.
- The radius of the circle is approximately 3.82 cm.
- The minor arc AB measures 2 cm and subtends a 30° angle at the center.
- The major arc AB measures 22 cm and subtends a 330° angle.
- The sum of the minor and major arcs equals the total circumference, and their central angles sum to 360° .

Understanding the relationship between arc lengths, central angles, and radius is essential for solving many geometrical problems involving circles. Mastering these calculations enables precise analysis of circular figures in both academic and real-world contexts.

In conclusion, analyzing the lengths of minor and major arcs provides critical insights into the dimensions and properties of circles. Whether for academic purposes, engineering, or design, mastering these concepts is fundamental for accurate and efficient problem-solving in geometry.

Frequently Asked Questions

How is the length of the minor arc AB related to the central angle in a

circle?

The length of the minor arc AB is directly proportional to its central angle; specifically, if the circle has radius r and the central angle is θ (in degrees), then the arc length is $(\theta/360) \times 2\pi r$.

Given the minor arc AB length is 2 cm and the major arc is 22 cm, how can we find the radius of the circle?

Assuming the minor arc length corresponds to a central angle θ , we can set up the equation: $2 = (\theta/360) \times 2\pi r$. Using the total arc length (minor + major), which should sum to the full circle's circumference, helps determine r .

What is the measure of the central angle subtending the minor arc AB?

Using the arc length formula: $2 = (\theta/360) \times 2\pi r$. To find θ , we need the radius r , which can be calculated from the total arc lengths. Once r is known, $\theta = (2\pi r \times 360) / (2\pi r) = 360 \times (2 / (2\pi r))$.

How can we verify that the sum of the minor and major arcs equals the total circumference of the circle?

The total circumference is the sum of the minor and major arcs: $2 \text{ cm} + 22 \text{ cm} = 24 \text{ cm}$. If the circle's radius is known, the circumference should be $2\pi r$, which should equal 24 cm, confirming the arc lengths are consistent.

If the minor arc AB measures 2 cm and the major arc measures 22 cm, what is the central angle corresponding to the minor arc?

First, find the radius r using the total circumference: $2\pi r = 24 \text{ cm}$, so $r = 24 / (2\pi) \approx 3.82 \text{ cm}$. Then, the central angle θ for the minor arc is: $\theta = (\text{arc length} / \text{circumference}) \times 360^\circ = (2 / 24) \times 360^\circ = 30^\circ$.

What is the significance of knowing the length of minor and major arcs in circle geometry?

Knowing the lengths of minor and major arcs helps in determining angles, sector areas, and other properties of the circle, which are essential in various applications like design, engineering, and trigonometry.

Additional Resources

Understanding the Length of Minor and Major Arcs in a Circle: A Comprehensive Analysis of Arc AB

Introduction to Circles and Arcs

A circle is one of the most fundamental geometric shapes, characterized by all points in a plane equidistant from a fixed point called the center, denoted as O . Among the numerous properties of circles, the concepts of arcs—particularly minor and major arcs—are essential in understanding the geometry and measurements associated with circles.

An arc is a continuous segment of a circle's circumference. When we talk about an arc between two points, say A and B , on the circle, this arc can be classified as either minor or major, depending on its length relative to the entire circumference.

This detailed review aims to analyze the specific case where:

- The length of the minor arc AB is 2 cm.
- The length of the major arc AB is 22 cm.
- The circle's center is O .

Through this exploration, we will understand the relationships between arcs, central angles, and the circle's circumference, as well as how these measurements interrelate to form a coherent geometric picture.

Fundamental Concepts: Arcs, Central Angles, and Circumference

What Is an Arc?

- An arc is a portion of the circumference of a circle.
- The length of an arc depends on the central angle subtended at the circle's center.

Minor and Major Arcs

- Minor arc: The shorter arc between two points on a circle, subtended by a central angle less than 180° .
- Major arc: The longer arc between two points, subtended by a central angle greater than 180° .
- The two arcs complement each other such that their combined lengths equal the total circumference of the circle.

Central Angles and Arcs

- The measure of a central angle (in degrees) is directly proportional to the length of its corresponding arc.
- The relationship is given by:

$$\text{Arc length} = \frac{\theta}{360^\circ} \times \text{Circumference}$$

where θ is the measure of the central angle in degrees.

Analyzing the Given Data: Arc Lengths and Their Implications

Given:

- Length of minor arc AB = 2 cm
- Length of major arc AB = 22 cm

From this, we can deduce several critical points:

1. Total circumference of the circle

Since the minor and major arcs are parts of the same circle and together they complete the entire circumference:

$$\text{Circumference} = \text{Minor arc} + \text{Major arc} = 2\text{ cm} + 22\text{ cm} = 24\text{ cm}$$

2. Relationship between the arcs

- The minor arc is clearly less than half the circle (since its length is 2 cm out of 24 cm, which is only $\frac{2}{24} = \frac{1}{12}$ or approximately 8.33% of the circle).
- The major arc is correspondingly larger, occupying about 91.67% of the circle.

Calculating the Central Angles

The central angle subtended by an arc can be found using the proportion of the arc length relative to the total circumference:

$$\theta = \frac{\text{Arc length}}{\text{Circumference}} \times 360^\circ$$

Applying this to the minor and major arcs:

Minor Arc AB

$$\theta_{\text{minor}} = \frac{2\text{ cm}}{24\text{ cm}} \times 360^\circ = \frac{1}{12} \times 360^\circ = 30^\circ$$

Major Arc AB

$$\theta_{\text{major}} = \frac{22\text{ cm}}{24\text{ cm}} \times 360^\circ = \frac{11}{12} \times 360^\circ = 330^\circ$$

Key observations:

- The minor arc AB subtends a central angle of 30° .
- The major arc AB subtends a central angle of 330° .

This confirms that the minor arc is a small segment of the circle, while the major arc covers nearly the entire circle, just missing the minor arc by 30° .

Implications of the Central Angles and Arc Lengths

Understanding the central angles provides insight into the geometric relationships within the circle:

- The angle at the center (O) that subtends the minor arc AB is 30° .
- Conversely, the angle at the center that subtends the major arc AB is 330° .

Because the total angle around a point (the center O) is 360° , the sum of the minor and major arcs' central angles is:

$$\begin{aligned} & \backslash[\\ & 30^\circ + 330^\circ = 360^\circ \\ & \backslash] \end{aligned}$$

which aligns with the entire circle's angle measure.

Significance of the Arc Lengths in Geometric Constructions

Practical applications and geometric constructions stem from these measurements:

- Positioning points A and B:

Given the arc lengths, points A and B are separated by a 30° central angle, which can be constructed by drawing a circle with radius (r) , then marking points A and B such that the minor arc length between them is 2 cm.

- Determining the radius of the circle:

Since the circle's circumference is 24 cm, the radius (r) can be calculated using:

$$\begin{aligned} & \backslash[\\ & \text{Circumference} = 2 \pi r \\ & \backslash] \\ & \backslash[\\ & 24\text{ cm} = 2 \pi r \\ & \backslash] \\ & \backslash[\\ & r = \frac{24}{2\pi} = \frac{12}{\pi} \approx 3.82\text{ cm} \\ & \backslash] \end{aligned}$$

- Constructing the circle:

With the radius known, one can accurately draw the circle and mark points A and B such that the minor arc length between them is 2 cm.

Deeper Geometric Insights

Understanding the relationship between arc length, central angles, and radius offers profound geometric insights:

- The small central angle (30°) indicates a short arc and a correspondingly small chord connecting A and B.
- The long arc (major arc) of 330° is nearly the entire circle, and the corresponding chord AB spans a large portion of the circle's circumference.
- The chord length between points A and B can be computed using the radius and the central angle:

$$\text{Chord length} = 2r \sin\left(\frac{\theta}{2}\right)$$

Applying this to the minor arc:

$$\text{Chord length} = 2 \times \frac{12}{\pi} \times \sin(15^\circ)$$

since $(\theta_{\text{minor}} = 30^\circ)$, and $(\frac{\theta}{2} = 15^\circ)$.

Calculating:

$$\sin(15^\circ) \approx 0.2588$$
$$\text{Chord length} \approx 2 \times 3.82 \text{ cm} \times 0.2588 \approx 1.98 \text{ cm}$$

which aligns with the arc length of 2 cm, confirming the geometric consistency.

Real-World Applications and Problem-Solving Strategies

Understanding these relationships is critical in various fields:

- Engineering and Design:

Precise measurements of arcs are essential when designing circular parts, gears, or architectural structures.

- Navigation and Geography:

Calculating distances along curved paths on the Earth's surface involves similar principles.

- Mathematics and Education:

Teaching concepts of circle geometry, central angles, and arc lengths benefits from real-world context and problem-solving exercises.

Problem-solving strategies include:

- Using the known circumference to find central angles.
- Applying the arc length formula for different segments.
- Calculating chord lengths based on central angles and radius.
- Verifying geometric relationships by summing angles and arc lengths.

Summary and Key Takeaways

- The circle's total circumference in this scenario is 24 cm, derived from the sum of the minor and major arcs.
- The minor arc AB subtends a 30° central angle, with a length of 2 cm.
- The major arc AB subtends a 330° central angle, with a length of 22 cm.
- The radius of the circle is approximately 3.82 cm, calculated from the circumference.
- The chord length connecting points A and B is approximately 1.98 cm, consistent with the arc length.

This analysis demonstrates the interconnectedness of arc lengths, central angles, and radii, providing a comprehensive understanding of circle geometry.

Final Remarks

Mastering the relationship between minor and major arcs, their lengths, and central

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