6. Write The Parametric Equations Of The Line Through The Point P (-6, 4, 3), That Is Perpendicular To

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Understanding how to formulate the parametric equations of a line passing through a specific point and perpendicular to a given vector or plane is a fundamental concept in vector calculus and analytical geometry. This process involves identifying the point through which the line passes, determining the direction vector that is perpendicular to the specified geometric entity, and then constructing the parametric equations accordingly. This guide provides a comprehensive step-by-step approach to solving such problems, along with relevant examples and explanations to enhance understanding.

Understanding the Problem Statement

Before delving into the solution, it is essential to interpret the problem accurately. The problem asks for the parametric equations of a line that:

- Passes through a specified point, P(-6, 4, 3)
- Is perpendicular to a certain geometric entity (which could be a plane or a vector)

The key aspects involve:

- The point: P(-6, 4, 3)
- The direction of the line: perpendicular to a given plane or vector

Since the problem statement cuts off after "That Is Perpendicular To," it is common in such contexts that the line is perpendicular to either:

- A given vector (e.g., normal vector of a plane)
- A plane (the line is perpendicular to the plane, i.e., parallel to its normal vector)

In this comprehensive guide, we will explore both scenarios and the general methodology to find the parametric equations of such a line.

Fundamentals of Parametric Equations of a Line in 3D

A line in three-dimensional space can be represented parametrically as:

\[
\begin{cases}
x = x_0 + at \\
y = y_0 + bt \\
z = z_0 + ct \\
\end{cases}
\]

where:

- \((x_0, y_0, z_0)\) is a point through which the line passes (here, P(-6, 4, 3))
- \(\vec{d} = \langle a, b, c \rangle\) is the direction vector of the line
- \(t\) is a real parameter

The goal is to find \(\vec{d}\), given the perpendicularity condition.

Scenario 1: Line Perpendicular to a Given Vector

Suppose the line is perpendicular to a given vector \(\vec{n} = \langle n_x, n_y, n_z \rangle\). Then, the line's direction vector \(\vec{d}\) must be parallel to \(\vec{n}\), because the line is perpendicular to the vector \(\vec{n}\).

Step-by-step process:

1. Identify the normal vector: Determine \(\vec{n}\) from the problem statement. For example, if the line is perpendicular to a plane with normal vector \(\vec{n}\), then \(\vec{n}\) is given.

2. Set the direction vector: \(\vec{d} = \lambda \vec{n}\), where \(\lambda \neq 0\). For parametric equations, choosing \(\vec{d} = \vec{n}\) simplifies the representation.

3. Write the parametric equations: Using the point \(P(-6, 4, 3)\) and the direction vector \(\vec{d}\):

\[

\begin{cases}
x = -6 + n_x t \\
y = 4 + n_y t \\
z = 3 + n_z t \\
\end{cases}
\]

Example:

- Suppose the vector \(\vec{n} = \langle 2, -3, 5 \rangle\)

- The parametric equations are:

\[
\begin{cases}
x = -6 + 2 t \\
y = 4 - 3 t \\
z = 3 + 5 t \\
\end{cases}
\]

This line passes through P(-6, 4, 3) and is perpendicular to \(\vec{n}\).

Scenario 2: Line Perpendicular to a Plane

If the line is perpendicular to a plane, then:

- The line's direction vector \(\vec{d}\) is parallel to the plane's normal vector \(\vec{n}\).

Step-by-step process:

1. Identify the plane's normal vector: Usually given in the plane equation \(ax + by + cz + d = 0\), where \(\vec{n} = \langle a, b, c \rangle\).

2. Set the line's direction vector: \(\vec{d} = \vec{n}\).

3. Write the parametric equations:

\[
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t \\
\end{cases}
\]

where $(x_0, y_0, z_0) = P(-6, 4, 3)$.

Example:

- Given the plane equation $3x - 4y + 5z + 6 = 0$

- The normal vector is $\vec{n} = \langle 3, -4, 5 \rangle$

- The parametric equations of the line are:

$$
\begin{cases}
x = -6 + 3 t \\
y = 4 - 4 t \\
z = 3 + 5 t \\
\end{cases}
$$

General Approach for Formulating the Equations

When solving for the parametric equations of a line perpendicular to a given entity, follow this generalized process:

1. Determine the correct normal or direction vector:

- If given a vector $\vec{v}$, use $\vec{d} = \vec{v}$.
- If given a plane $ax + by + cz + d = 0$, use the normal vector $\vec{n} = \langle a, b, c \rangle$.

2. Use the point through which the line passes:

- The point $P(-6, 4, 3)$ is fixed.

3. Construct the parametric equations:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t \\
\end{cases}
$$

where $\langle a, b, c \rangle$ is the direction vector.

4. Optional: write symmetric equations:

If desired, the symmetric form can be written as:

\[
\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}
\]

provided \(a, b, c \neq 0\).

Additional Considerations and Special Cases

While the above scenarios cover most common cases, there are a few additional considerations:

- Zero Components in the Direction Vector: If any component of the direction vector is zero, the corresponding parametric equation simplifies accordingly.

- Line Parallel to Coordinate Axes: When the direction vector is along a coordinate axis, the parametric equations reduce to simple forms.

- Multiple Planes or Vectors: If the line must be perpendicular to multiple entities, the direction vector must satisfy the perpendicularity conditions simultaneously, often leading to solving a system of equations.

Summary of Steps to Find the Parametric Equations

To summarize, here are the concise steps to find the parametric equations of a line through a given point and perpendicular to a vector or plane:

1. Identify the point: \(P(x_0, y_0, z_0) = (-6, 4, 3)\).

2. Determine the normal or guiding vector:

- From a given vector \(\vec{v}\), set \(\vec{d} = \vec{v}\).
- From a plane's equation, extract \(\vec{n} = \langle a, b, c \rangle\).

3. Construct the parametric equations:

\[
\boxed{
\begin{cases}
x = x_0 + a t \\

y = y_0 + b t \\
z = z_0 + c t \\
\end{cases}
}
\]

4. Optional: write the symmetric form if needed.

Practical Example: Applying the Method

Suppose the problem states:

"Write the parametric equations of the line passing through P(-6, 4, 3) that is perpendicular to the plane \(2x + 3y - z + 7 = 0\)."

Solution:

1. The plane's normal vector:

\[
\vec{n} = \langle 2, 3, -1 \r

Frequently Asked Questions

How do you determine the parametric equations of a line passing through point P(-6, 4, 3) that is perpendicular to a given plane?

First, find the normal vector of the plane, which serves as the direction vector for the line. Then, use the point P as a point on the line and the normal vector as the direction vector to write the parametric equations: x = -6 + t n_x, y = 4 + t n_y, z = 3 + t n_z.

What is the significance of the normal vector in writing the parametric equations of a perpendicular line?

The normal vector of a plane is perpendicular to the plane itself, and when a line is perpendicular to the plane, its direction vector is parallel to this normal vector. Hence, it defines the line's direction in the parametric equations.

Can you provide an example of parametric equations for a line through P(-6, 4, 3) perpendicular to a specific plane, say 2x + 3y - z = 5?

Yes. The normal vector of the plane is (2, 3, -1). The parametric equations are: x = -6 + 2t, y = 4 + 3t, z = 3 - t, where t is a real parameter.

What steps are involved in deriving the parametric equations of a line perpendicular to a plane through a given point?

Identify the plane's normal vector, use that as the direction vector for the line, then substitute the given point into the parametric form: point + t direction vector, resulting in the equations for x, y, and z.

Why is it important to verify the perpendicularity of the line after deriving the parametric equations?

Verifying ensures that the line's direction vector is indeed orthogonal to the plane's normal vector, confirming that the line is correctly perpendicular to the plane.

How can understanding the parametric form of a perpendicular line aid in solving geometry problems?

It helps in visualizing the spatial relationship between objects, calculating distances, and finding intersections, making complex 3D problems more manageable and precise.

Additional Resources

Understanding the Parametric Equations of a Line Through a Point and Perpendicular to a Given Direction

In the realm of three-dimensional coordinate geometry, the problem of determining the parametric equations of a line passing through a specified point and perpendicular to a given vector or plane is both fundamental and intricate. This exploration focuses on the specific case where the line passes through the point P(-6, 4, 3), with an emphasis on deriving its parametric equations when the line is perpendicular to a certain given vector or plane. Such problems are prevalent in various fields—from physics and engineering to computer graphics—and understanding their solutions is crucial for both theoretical insight and practical application.

This article aims to dissect the process systematically, providing a comprehensive review suitable for educators, students, and researchers engaged in spatial analysis and geometric problem-solving.

Foundational Concepts in Spatial Geometry

Before delving into the specific problem, it is essential to clarify some foundational concepts that underpin the derivation of parametric equations for lines in three-dimensional space.

Points, Vectors, and Lines in 3D Space

- Point: An ordered triplet (x, y, z) representing a specific location in space.
- Vector: An entity with magnitude and direction, often represented as .
- Line: A set of points in space that extend infinitely in both directions, often described parametrically.

Parametric Equations of a Line

A line passing through a point $P_0(x_0, y_0, z_0)$ with direction vector d = can be expressed parametrically as:

- $x = x_0 + at$
- $y = y_0 + bt$
- $z = z_0 + ct$

where t is a real parameter.

Perpendicular Lines and Direction Vectors

- Two vectors are perpendicular (orthogonal) if their dot product equals zero: · = 0.
- For a line to be perpendicular to a plane, its direction vector must be normal (perpendicular) to the plane's surface.

Problem Specification: Writing The Parametric Equations of the Line

The core task involves determining the parametric equations of a line passing through the point P(-6, 4, 3) and perpendicular to a specific geometric object—either a vector or a plane.

Given the problem title, "Write The Parametric Equations Of The Line Through The Point P (-6, 4, 3), That Is Perpendicular To," the missing component is typically a vector or a plane normal vector. For the purpose of this review, we will analyze both scenarios:

- Scenario A: The line is perpendicular to a given vector.
- Scenario B: The line is perpendicular to a given plane.

The methodology in each case is similar, hinging on the properties of perpendicularity and the relationships between vectors and planes.

Scenario A: Line Perpendicular to a Given Vector

Suppose we are given a vector v = . The goal is to find the parametric equations of the line passing through P(-6, 4, 3) and perpendicular to v.

Step 1: Identify the Direction Vector

- Since the line must be perpendicular to v, its direction vector d must satisfy:

$d \cdot v = 0$

- The simplest choice is d = $v\perp$, the vector perpendicular to v. To find such a vector, we can select any vector that satisfies the dot product condition.

- Alternatively, if v is a known vector, then the line's direction vector d can be $v\perp$ or any scalar multiple thereof, provided $d \cdot v = 0$.

Example:

Suppose v = <2, 3, 1>.

- To find d, select any vector d = satisfying:

$2x + 3y + 1z = 0$

- For simplicity, choose $x = 3$, $y = -2$, $z = 0$:

$2(3) + 3(-2) + 1(0) = 6 - 6 + 0 = 0$

- Therefore, $d = <3, -2, 0>$.

Step 2: Write the Parametric Equations

- With P(-6, 4, 3) and $d = <3, -2, 0>$, the parametric equations are:

- $x = -6 + 3t$
- $y = 4 - 2t$
- $z = 3 + 0 \cdot t = 3$

This describes a line passing through P and perpendicular to v.

Summary for Scenario A

- Given: Point P(-6,4,3) and vector v.
- Find: A vector d such that $d \cdot v = 0$.
- Result: Parametric equations $x = x_0 + a\ t$, $y = y_0 + b\ t$, $z = z_0 + c\ t$, with = d.

Scenario B: Line Perpendicular to a Given Plane

Alternatively, if the line must be perpendicular to a plane, the process involves the plane's normal vector.

Step 1: Identify the Plane's Normal Vector

- Consider a plane with equation:

$A\ x + B\ y + C\ z + D = 0$

- The normal vector n = is perpendicular to the plane.

Step 2: Use the Normal Vector as the Direction Vector

- Since the line is perpendicular to the plane, its direction vector d must be parallel to n.

- Therefore, d = n = .

Step 3: Write the Parametric Equations

- Using P(-6, 4, 3) and d = , the parametric equations are:

- x = -6 + A t
- y = 4 + B t
- z = 3 + C t

This line is perpendicular to the plane, passing through the specified point.

Practical Applications and Implications

Understanding how to derive the parametric equations of such a line is crucial in various scientific and engineering contexts:

- Projection problems: Finding the shortest distance between a point and a plane involves a perpendicular line.
- Optics and acoustics: Modeling rays perpendicular to surfaces.
- Robotics: Planning trajectories orthogonal to surfaces or directions.
- Computer graphics: Rendering scenes involving perpendicular lines and planes.

Furthermore, these derivations underpin algorithms in CAD software and 3D modeling, where precise control over object orientations and intersections is vital.

Common Challenges and Considerations

While the methodology is straightforward, several challenges may arise during the calculation:

- Choosing the correct perpendicular vector: When multiple solutions exist, selecting the most appropriate vector depends on context.
- Ensuring non-degeneracy: The direction vector should not be the zero vector.

- Handling special cases: When the given vector or plane is aligned with coordinate axes, calculations may simplify or require special attention.

Conclusion

The task of deriving the parametric equations of a line passing through a specific point and perpendicular to a given vector or plane is foundational in three-dimensional geometry. It hinges on fundamental properties of vectors, dot products, and the geometric interpretations of perpendicularity.

Whether the goal is to find a line perpendicular to a vector or to a plane, the process involves:

- Identifying or constructing an appropriate direction vector satisfying the perpendicularity condition.
- Applying the point-direction form of the line's parametric equations.

Mastery of these concepts enables precise modeling of spatial relationships, vital across scientific disciplines. As demonstrated, the procedure is systematic, adaptable, and central to advanced geometric problem-solving.

In essence, understanding and accurately deriving the parametric equations of lines through a point and perpendicular to other geometrical entities form a cornerstone of spatial analysis, offering both theoretical insights and practical tools for complex three-dimensional problems.

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