7. A Ball On The End Of A String Is Revolved At A Uniform Rate In A Vertical Circle Of Radius 72.0 Cm.

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Understanding the dynamics of a ball swung in a vertical circle involves exploring the interplay of forces, motion, and energy. When a ball attached to a string is revolved at a constant angular velocity, the system exhibits fascinating behaviors governed by classical mechanics. This scenario not only illustrates fundamental principles like centripetal force and energy conservation but also provides insights into real-world applications such as amusement park rides, playground equipment, and engineering designs. In this article, we delve into the detailed analysis of a ball revolving in a vertical circle, examining the forces involved, the variations in tension during the motion, and the conditions necessary for maintaining uniform circular motion.

Understanding the Setup and Key Parameters

Basic Description of the System

The system consists of a ball attached to a string of length $r = 72.0\, \text{cm}$ (or 0.72 meters). The ball is swung in a vertical plane, completing revolutions at a uniform angular speed ω (radians per second). The key parameters include:

- Radius of the circle, $r = 0.72\, \text{m}$
- Mass of the ball, m (value to be specified or considered as a variable)
- Angular velocity, ω
- Period of revolution, T
- Speed of the ball at various positions, v

The primary goal is to analyze the forces acting on the ball at different points in the vertical circle, particularly at the lowest and highest points, and understand how the tension in the string varies during the motion.

Forces Acting on the Ball During Circular Motion

Fundamental Forces Involved

At any point during the motion, two main forces act on the ball:

- **Gravitational force (mg):** Acts vertically downward throughout the motion.
- **Tension in the string (T):** Acts along the string, directed towards the center of the circle.

Since the ball moves in a vertical circle, the tension varies depending on the position of the ball in the circle, especially between the top and bottom points.

Conditions for Uniform Circular Motion

For the motion to be uniform (constant angular velocity), the net radial force must provide the necessary centripetal force:

$$
T + mg \cos \theta = \frac{mv^2}{r}
$$

where:

- θ is the angle between the vertical and the string at the ball's current position (with $\theta = 0$ at the bottom, $\theta = 180^\circ$ at the top),
- v is the speed of the ball at that position.

At the bottom of the circle ($\theta = 0^\circ$), the tension must counteract gravity and provide the centripetal force:

$$
T_{bottom} = \frac{mv_{bottom}^2}{r} - mg
$$

At the top of the circle ($\theta = 180^\circ$):

$$
T_{top} = \frac{mv_{top}^2}{r} + mg
$$

Understanding these relationships is key to analyzing the system's behavior.

Energy Considerations and Velocity at Different Points

Conservation of Mechanical Energy

Assuming negligible air resistance and friction, the total mechanical energy remains constant throughout the motion:

$$
\text{Potential Energy} + \text{Kinetic Energy} = \text{Constant}
$$

Choosing the bottom point as the reference level where potential energy is zero:

$$
PE_{bottom} = 0
$$
$$
KE_{bottom} = \frac{1}{2} m v_{bottom}^2
$$
$$
PE_{top} = 2 r m g
$$
$$
KE_{top} = \frac{1}{2} m v_{top}^2
$$

Applying energy conservation between bottom and top points:

$$
\frac{1}{2} m v_{bottom}^2 = \frac{1}{2} m v_{top}^2 + 2 r m g
$$

Simplifying:

$$
v_{bottom}^2 = v_{top}^2 + 4 r g
$$

This relation allows calculation of velocities at different points once one velocity is known.

Implications for the Tension in the String

Using the velocities, the tension at the top and bottom can be expressed as:

$$
T_{bottom} = m \left(\frac{v_{bottom}^2}{r} \right) - mg
$$
$$
T_{top} = m \left(\frac{v_{top}^2}{r} \right) + mg
$$

These tensions are critical in ensuring the string remains taut; if the tension drops to zero, the string would slack, and the motion would no longer be uniform circular motion.

Analyzing the Conditions for Uniform Circular Motion

Minimum Speed at the Top of the Circle

To prevent the string from slackening at the top of the circle, the tension must be at least zero:

$$
T_{top} \geq 0
$$

Using the expression for tension at the top:

$$
0 = m \left(\frac{v_{top}^2}{r} \right) + mg
$$

which yields:

$$
v_{top}^2 = - r g
$$

Since v_{top}^2 cannot be negative, the minimum speed at the top to keep the string taut is:

$$
v_{top} = \sqrt{r g}
$$

Numerically:

$$
v_{top} = \sqrt{0.72\, \text{m} \times 9.8\, \text{m/s}^2} \approx \sqrt{7.056} \approx 2.66\, \text{m/s}
$$

This is the critical speed; any lower, and the string would slack at the top, causing the motion to cease being uniform circular motion.

Minimum Speed at the Bottom of the Circle

Using energy conservation, the minimum velocity at the bottom corresponds to the critical top speed:

$$
v_{bottom}^2 = v_{top}^2 + 4 r g = (2.66)^2 + 4 \times 0.72 \times 9.8
$$
$$
v_{bottom}^2 = 7.056 + 4 \times 0.72 \times 9.8
$$
$$
v_{bottom}^2 = 7.056 + 4 \times 7.056
$$
$$
v_{bottom}^2 = 7.056 + 28.224 = 35.28
$$
$$
v_{bottom} \approx \sqrt{35.28} \approx 5.94\, \text{m/s}
$$

Therefore, to keep the ball moving in a vertical circle without the string slackening, the speed at the bottom must be at least approximately 5.94 m/s.

Calculating Tension at Different Positions

Tension at the Bottom

Using the speed at the bottom:

$$
T_{bottom} = m \left(\frac{v_{bottom}^2}{r} \right) - mg
$$

Assuming a specific mass m, say 0.5 kg for illustration:

$$
T_{bottom} = 0.5 \left(\frac{(5.94)^2}{0.72} \right) - 0.5 \times 9.8
$$

Compute numerator:

$$
(5.94)^2 = 35.28
$$

Then:

$$
T_{bottom} = 0.5 \times \frac{35.28}{0.72} - 4.9 \approx 0.5 \times 49.00 - 4.9 = 24.50 - 4.9 = 19.6\, \text{N}
$$

Thus, the tension at the bottom is approximately 19.6 N.

Tension at the Top

Using the minimum top speed:

$$
T_{top} = 0.5 \left(\frac{(2.66)^2}{0.72} \right) + 4.9
$$

Calculate:

$$
(2.66)^2 = 7.06
$$

$$
T_{top} = 0.5 \times \frac{7.06}{0.72} + 4.9 \approx 0.5 \times 9.81 + 4.9 \approx 4.905 + 4.9 = 9.805\, \text{N}
$$

Tension at the top is roughly 9.8 N, indicating that tension remains positive at the minimum speeds, ensuring the motion stays uniform.

Practical Consider

Frequently Asked Questions

What are the key forces acting on the ball when it is revolved in a vertical circle?

The main forces are the tension in the string and the gravitational force (weight) of the ball. These

forces vary in magnitude and direction at different points in the circle.

How does the tension in the string vary at the top and bottom of the vertical circle?

At the bottom, tension is greater because it must support the weight and provide the centripetal force, resulting in higher tension. At the top, tension is less and may even become zero if the speed is just sufficient to maintain circular motion.

What is the minimum speed needed at the top of the circle to keep the ball moving in a vertical circle?

The minimum speed at the top occurs when the tension in the string is zero, and it can be calculated using the relation $v = \sqrt{(g R)}$, where g is acceleration due to gravity and R is the radius of the circle.

How does the radius of the circle affect the required speed for uniform circular motion?

The larger the radius, the greater the minimum speed needed at the top to maintain circular motion, since $v = \sqrt{(g R)}$. An increased radius

means a higher minimum speed.

What is the significance of uniform rate in the context of this vertical circle?

A uniform rate means the ball has constant angular velocity, so the linear speed remains constant during the motion, simplifying the analysis of forces and tension at different points.

How can the maximum tension in the string be calculated when the ball is at the bottom of the circle?

The maximum tension is found using $T = m(v^2 / R) + mg$, where m is the mass of the ball, v is the speed at the bottom, R is the radius, and g is acceleration due to gravity.

Additional Resources

Vertical Circular Motion of a Ball on a String: An In-Depth Analysis

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Introduction

Imagine holding a ball attached to a string, spinning it around in a perfect vertical circle. This seemingly simple activity involves intricate physics principles that govern the motion of the ball at various positions within the circle. The scenario of a ball on the end of a string revolving at a uniform rate in a vertical circle — with a radius of 72.0 centimeters — presents a fascinating exploration into forces, energy conservation, and dynamics in rotational motion.

This article aims to dissect this scenario thoroughly, examining the physics involved, the critical parameters, and the implications on the ball's motion at different points in the circle. Whether you are a physics enthusiast, an educator, or a student, understanding this problem deepens your appreciation of rotational dynamics and the complexities of circular motion.

The Setting and Key Parameters

Before delving into the physics, let's establish the scenario and key parameters:

- Radius of the vertical circle (r): 72.0 cm = 0.72 m
- Mass of the ball (m): Not specified, but essential

for force calculations
- Angular velocity (ω): The rate at which the ball completes each revolution, assumed constant in magnitude
- Linear (tangential) speed (v): Related to angular velocity as $v = r\omega$
- Position in the circle: Top, bottom, and intermediate points

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Fundamental Concepts in Vertical Circular Motion

1. Uniform Circular Motion vs. Vertical Circular Motion

- Uniform Circular Motion: The object moves with constant angular speed around a circle. The magnitude of velocity remains constant, but the direction changes continuously.
- Vertical Circular Motion: The object moves along a vertical circle, experiencing changes in gravitational potential energy and tension in the string as it moves from top to bottom and vice versa.

2. Forces Acting on the Ball

At any point in the circle, the forces acting on the ball are:

- Gravity (weight): mg, acting vertically downward
- Tension in the string (T): Acts along the string, directed towards the center of the circle

3. Energy Conservation

Since the system is assumed to be ideal (no friction or air resistance), the total mechanical energy remains constant:

$$
\text{Kinetic Energy} + \text{Potential Energy} = \text{Constant}
$$

Analyzing the Motion at Key Positions

The motion in a vertical circle is characterized by variations in tension and speed at different points, primarily at the top and bottom positions.

1. Motion at the Bottom of the Circle

- Position: The ball is at the lowest point in the circle.
- Forces: Tension (T_b) acts upward, gravity acts downward.

Force Balance:

$$
T_b - mg = \frac{mv_b^2}{r}
$$

Where:
- v_b: Speed at the bottom
- T_b: Tension at the bottom

Energy Considerations:

Assuming the potential energy is zero at the bottom:

$$
\text{Total energy at bottom} = \frac{1}{2} m v_b^2
$$

---

2. Motion at the Top of the Circle

- Position: The ball reaches the highest point.
- Forces: Tension (T_t) acts downward, gravity acts downward.

Force Balance:

**$$
mg + T_t = \frac{mv_t^2}{r}
$$**

Where:
- v_t: Speed at the top
- T_t: Tension at the top (which can be zero or positive, depending on the speed)

Energy Considerations:

Potential energy at the top:

$$
PE_{top} = 2r \times mg
$$

Total energy at the top:

$$
E_{top} = \frac{1}{2} m v_t^2 + 2 r m g
$$

Determining the Conditions for Motion

To ensure the ball completes the vertical circle without the string going slack, specific conditions on velocity and tension must be met.

1. Minimum Speed at the Top

If the tension T_t drops to zero, the tension just disappears, and the only force providing the necessary centripetal acceleration is gravity.

$$
T_t = 0 \Rightarrow mg = \frac{m v_{t, min}^2}{r}
$$

Thus:

$$
v_{t, min} = \sqrt{r g}
$$

Plugging in the numbers:

$$
v_{t, min} = \sqrt{0.72 \times 9.8} \approx \sqrt{7.056} \approx 2.66\, \text{m/s}
$$

This is the minimum speed at the top to prevent the string from slackening.

2. Corresponding Speed at the Bottom

Using energy conservation:

$$
\frac{1}{2} m v_b^2 = \frac{1}{2} m v_t^2 + 2 r m g
$$

$$
v_b^2 = v_t^2 + 4 r g
$$

Substitute $v_t = 2.66\, \text{m/s}$:

$$
v_b^2 = (2.66)^2 + 4 \times 0.72 \times 9.8
$$

$$
v_b^2 = 7.07 + 4 \times 0.72 \times 9.8
$$

Calculate:

$$
4 \times 0.72 \times 9.8 = 4 \times 7.056 = 28.224
$$

So:

$$
v_b^2 = 7.07 + 28.224 = 35.294
$$

$$
v_b \approx \sqrt{35.294} \approx 5.94\,
\text{m/s}
$$

Calculating Tension at the Bottom and Top for a Given Speed

Suppose the ball is spun at a certain constant angular velocity ω, leading to a linear speed:

$$
v = r \omega
$$

For example, if the linear speed at the bottom is known (say 6 m/s), the tensions are:

- At the bottom:

$$
T_b = \frac{m v_b^2}{r} + mg
$$

- At the top:

$$

$$
T_t = \frac{m v_t^2}{r} - mg
$$

(assuming v_t is sufficient to keep tension positive)

Practical Implications and Safety Considerations

Understanding these forces and energy relationships is crucial when designing or performing activities involving vertical circular motion, such as:

- Amusement park rides: Ensuring safety through appropriate speeds to prevent slack or excessive tension.
- Physics demonstrations: Visualizing the change in tension and energy throughout the motion.
- Engineering applications: Designing rotating machinery or components subjected to similar forces.

Summary of Key Points

- The radius of the circle (72.0 cm) significantly influences the minimum speeds and tensions

required.
- Energy conservation provides a straightforward method to relate speeds at different points.
- The minimum speed at the top to maintain tension is $v_{t, min} = \sqrt{r g} \approx 2.66\, \text{m/s}$.
- The speed at the bottom corresponding to this minimum is approximately 5.94 m/s.
- Tensions vary throughout the motion: maximum at the bottom and minimum at the top, with the possibility of slack if speeds are too low at the top.

Final Thoughts

The motion of a ball on a string in a vertical circle is a compelling demonstration of physics principles, combining forces, energy, and motion in a cohesive system. The key takeaway is that constant angular velocity doesn't imply constant tension; instead, the tension fluctuates depending on the position within the circle and the speed of rotation.

Understanding these dynamics not only enriches one's grasp of physics but also informs practical applications where safety and design depend on precise force calculations. Whether for academic

pursuits or real-world engineering, the elegant dance of forces in a vertical circle continues to fascinate and instruct.

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