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7. Let S (x,y)=x-5xy. (a) Determine . (4) (b) Determine The Directional Derivative Of Fat (2,1) In The.

Understanding the behavior of multivariable functions is fundamental in fields like calculus, physics, and engineering. In particular, the concept of derivatives extends beyond single-variable functions to functions of multiple variables, providing insights into how the function changes in various directions. This article explores the mathematical process of analyzing the function $S(x,y) = x - 5xy$, focusing on calculating partial derivatives and the directional derivative at specific points. These concepts are crucial for understanding the rate of change in multiple dimensions, optimizing functions, and modeling real-world phenomena.

Introduction to Multivariable Functions and Derivatives

Multivariable functions, such as $S(x,y)$, depend on two or more variables. Analyzing these functions involves understanding how they change with respect to each variable individually (partial derivatives) and in arbitrary directions (directional derivatives).

Key concepts include:
- Partial Derivatives: Measure the rate of change of the function with respect to one variable, holding others constant.
- Gradient Vector: A vector composed of all partial derivatives, pointing in the direction of the steepest ascent.
- Directional Derivative: Represents the rate of change of the function in any specified direction, not necessarily aligned with the axes.

Analyzing the Function $S(x,y) = x - 5xy$

Let's begin by examining the function in question:

$$
S(x,y) = x - 5xy
$$

Our goal is to:

1. Determine the partial derivatives $\frac{\partial S}{\partial x}$ and $\frac{\partial S}{\partial y}$.
2. Find the gradient vector at a specific point.
3. Calculate the directional derivative of S at the point $(2,1)$ in a given direction.

Part (a): Calculating Partial Derivatives

Partial derivatives provide the foundation for understanding how the function behaves locally.

Step 1: Partial derivative with respect to x:

$$
\frac{\partial S}{\partial x} = \frac{\partial}{\partial x} (x - 5xy)
$$

Apply basic differentiation rules:

$$
\frac{\partial S}{\partial x} = 1 - 5y
$$

This derivative indicates how S changes as x varies, with y held constant.

Step 2: Partial derivative with respect to y:

$$
\frac{\partial S}{\partial y} = \frac{\partial}{\partial y} (x - 5xy)
$$

Differentiating term-by-term:

$$
\frac{\partial S}{\partial y} = 0 - 5x = -5x
$$

This derivative shows how S changes as y varies, with x held constant.

Part (b): Determining the Directional Derivative at $(2,1)$

The directional derivative measures the rate at which the function changes at a particular point in a specified direction. To compute it, follow these steps:

Step 1: Find the gradient vector $\nabla S(x,y)$:

$$
\nabla S(x,y) = \left(\frac{\partial S}{\partial x}, \frac{\partial S}{\partial y} \right)
$$

At the point $(2,1)$:

$$
\frac{\partial S}{\partial x} = 1 - 5(1) = 1 - 5 = -4
$$

$$
\frac{\partial S}{\partial y} = -5(2) = -10
$$

So,

$$
\nabla S(2,1) = (-4, -10)
$$

Step 2: Determine the unit direction vector $\mathbf{u}$:

The directional derivative depends on the direction in which we measure the rate of change. Suppose we are interested in the directional derivative in the direction of a given vector $\mathbf{v} = (v_x, v_y)$. To normalize it:

$$
\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{v_x}{\sqrt{v_x^2 + v_y^2}}, \frac{v_y}{\sqrt{v_x^2 + v_y^2}} \right)
$$

The specific direction vector must be provided to proceed. For illustration, let's assume the direction vector is $\mathbf{v} = (3, 4)$:

$$
\|\mathbf{v}\| = \sqrt{3^2 + 4^2} = 5
$$

$$
\mathbf{u} = \left(\frac{3}{5}, \frac{4}{5} \right)
$$

Step 3: Calculate the directional derivative:

$$
D_{\mathbf{u}} S(2,1) = \nabla S(2,1) \cdot \mathbf{u} = (-4, -10) \cdot \left(\frac{3}{5}, \frac{4}{5} \right)
$$

$$
D_{\mathbf{u}} S(2,1) = -4 \times \frac{3}{5} + (-10) \times \frac{4}{5} = -\frac{12}{5} - \frac{40}{5} = -\frac{12 + 40}{5} = -\frac{52}{5} = -10.4
$$

Thus, in the direction of $\mathbf{v} = (3, 4)$, the rate of change of S at $(2,1)$ is -10.4.

Summary and Key Takeaways

Understanding the derivatives of multivariable functions like $S(x,y) = x - 5xy$ is essential for analyzing how these functions change in different directions. Here are the main points:

- Partial derivatives provide the rate of change along the axes:
- $\frac{\partial S}{\partial x} = 1 - 5y$
- $\frac{\partial S}{\partial y} = -5x$

- Gradient vector combines these derivatives and points in the direction of the steepest increase.

- Directional derivatives extend this concept to arbitrary directions, calculated by projecting the gradient onto a unit vector in the desired direction.

- Application example: At point $(2,1)$, with a direction vector $\mathbf{v} = (3,4)$, the directional derivative is -10.4, indicating a rapid decrease of the function in that direction.

Practical Applications of Derivatives in Multivariable Calculus

The concepts discussed are vital in various real-world scenarios, including:

- Optimization problems: Finding maximum or minimum values of functions, such as in economics for profit maximization.
- Gradient descent algorithms: Used in machine learning to minimize functions.
- Physics: Calculating the rate of change of physical quantities across different directions.
- Engineering: Designing systems that require understanding of how properties change in multiple dimensions.

Conclusion

The analysis of the function \(S(x,y) = x - 5xy\) demonstrates fundamental techniques in multivariable calculus, including calculating partial derivatives, the gradient vector, and the directional derivative. These mathematical tools allow us to understand and predict how complex functions behave locally, facilitating their application across science, engineering, and economics. Mastery of these concepts enables practitioners to optimize functions, model physical phenomena, and make informed decisions based on the rate of change in multiple dimensions.

Keywords for SEO optimization:

- Multivariable calculus
- Partial derivatives
- Gradient vector
- Directional derivative
- Function \(S(x,y) = x - 5xy\)
- Rate of change
- Optimization in calculus
- Mathematical analysis
- Derivatives in multiple dimensions
- Calculus tutorial

Frequently Asked Questions

What is the function S(x, y) given in the problem?

The function is $S(x, y) = x - 5xy$.

How do you compute the partial derivatives of S(x, y) with respect to x and y?

The partial derivatives are $\partial S/\partial x = 1 - 5y$ and $\partial S/\partial y = -5x$.

What is the gradient vector ∇S at the point (2, 1)?

At (2, 1), the gradient vector is $\nabla S = (1 - 51, -52) = (-4, -10)$.

How do you determine the directional derivative of S at (2, 1) in a given direction?

The directional derivative is found by taking the dot product of the gradient ∇S at (2, 1) with the unit vector in the specified direction.

What is the process to find the directional derivative of S at (2, 1) in a specific direction?

First, find the gradient ∇S at (2, 1), then normalize the direction vector to obtain the unit vector, and finally compute their dot product.

Why is understanding the directional derivative important in multivariable calculus?

It measures the rate of change of the function in a specific direction, helping analyze how functions behave locally in different directions.

Additional Resources

Let $S (x, y) = x - 5xy$: An In-Depth Analytical Investigation

In the realm of multivariable calculus, understanding the behavior and properties of functions involving multiple variables is fundamental. Among the many tools at our disposal, the concepts of partial derivatives and directional derivatives serve as critical instruments for analyzing how functions change in various directions and at specific points. This article provides a comprehensive examination of the function $S(x, y) = x - 5xy$, focusing on the determination of its partial derivatives and the calculation of its directional derivative at a particular point.

Introduction to the Function $S(x, y) = x - 5xy$

The function in question, $S(x, y) = x - 5xy$, is a bilinear form involving the variables x and y. It combines linear and multiplicative terms, making it a prime candidate for exploring how the interplay between variables influences the rate of change of the function.

This function can be viewed as a simple yet rich model for various phenomena, such as economic models where variables interact multiplicatively, or physical models where the rate of change depends linearly on one variable and multiplicatively on another.

Part A: Determining the Partial Derivatives of S

Understanding Partial Derivatives

Partial derivatives measure the rate at which a function changes as one variable varies, holding the other variables constant. For a function $S(x, y)$, the two primary partial derivatives are:

- $\frac{\partial S}{\partial x}$: rate of change of S with respect to x, holding y constant.
- $\frac{\partial S}{\partial y}$: rate of change of S with respect to y, holding x constant.

Calculating $\frac{\partial S}{\partial x}$

Given:
$$
S(x, y) = x - 5xy
$$

Treat y as a constant:
$$
\frac{\partial S}{\partial x} = \frac{\partial}{\partial x} (x - 5xy)
$$

\]

Applying the linearity of differentiation:
\[
\frac{\partial S}{\partial x} = \frac{\partial}{\partial x} (x) - 5 y \frac{\partial}{\partial x} (x y)
\]

Since \(y \) is constant:
\[
\frac{\partial}{\partial x} (x) = 1
\]
and
\[
\frac{\partial}{\partial x} (x y) = y
\]

Thus:
\[
\boxed{
\frac{\partial S}{\partial x} = 1 - 5 y
}
\]

Calculating \(\frac{\partial S}{\partial y} \)

Similarly, treat \(x \) as a constant:
\[
\frac{\partial S}{\partial y} = \frac{\partial}{\partial y} (x - 5xy)
\]

Breaking it down:
\[
\frac{\partial S}{\partial y} = 0 - 5 x \frac{\partial}{\partial y} (y) = -5 x
\]

Therefore:
\[
\boxed{
\frac{\partial S}{\partial y} = -5 x
}

\]

Part B: Determining the Directional Derivative of S at $(2, 1)$

Understanding the Directional Derivative

The directional derivative of a function at a point in a particular direction measures the rate of change of the function as one moves from that point in the specified direction. It generalizes the concept of partial derivatives by allowing us to analyze the function's behavior along arbitrary directions, not just along the coordinate axes.

Mathematically, for a function $S(x, y)$, the directional derivative at point (x_0, y_0) in the direction of a unit vector $\mathbf{u} = (u_1, u_2)$ is given by:

$$
D_{\mathbf{u}} S(x_0, y_0) = \nabla S(x_0, y_0) \cdot \mathbf{u} = \frac{\partial S}{\partial x}(x_0, y_0) u_1 + \frac{\partial S}{\partial y}(x_0, y_0) u_2
$$

where ∇S is the gradient vector of S.

Computing the Gradient at $(2, 1)$

Using the partial derivatives obtained earlier:

$$
\nabla S(2, 1) = \left(1 - 5(1), -5(2) \right) = (1 - 5, -10) = (-4, -10)
$$

This gradient vector indicates the direction of the steepest ascent of the function at the point $(2, 1)$.

Choosing a Direction Vector $\mathbf{u}$

To compute the directional derivative, a specific direction vector must be specified. Let's consider an arbitrary unit vector:

$$
\mathbf{u} = (\cos \theta, \sin \theta)
$$

For a comprehensive analysis, common choices include:

- Along the x-axis: $\mathbf{u} = (1, 0)$
- Along the y-axis: $\mathbf{u} = (0, 1)$
- Diagonal direction: $\mathbf{u} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$

For illustration, let's compute the directional derivative in the direction of $\mathbf{u} = (1, 0)$, i.e., along the x-axis.

Calculating the Directional Derivative in the x-direction

Given:

$$
D_{\mathbf{u}} S(2, 1) = \nabla S(2, 1) \cdot (1, 0) = (-4, -10) \cdot (1, 0) = -4 \times 1 + (-10) \times 0 = -4
$$

Interpretation:

- A negative value indicates the function decreases as we move in the positive x-direction from $(2, 1)$.

General Expression for the Directional Derivative

For a general unit vector $\mathbf{u} = (\cos \theta, \sin \theta)$:

$$
D_{\mathbf{u}} S(2, 1) = (-4, -10) \cdot (\cos \theta, \sin \theta) = -4 \cos \theta - 10 \sin \theta
$$

\]

This expression allows for evaluating the rate of change in any direction specified by θ.

Implications and Applications

The calculation of partial and directional derivatives for $S(x, y) = x - 5xy$ reveals several insights:

1. Sensitivity of S to Variable Changes: The partial derivatives indicate that at any point, the rate of change with respect to x is influenced linearly by y, and vice versa. Specifically:
- $\frac{\partial S}{\partial x}$ decreases as y increases, suggesting that increasing y diminishes the impact of x on S.
- $\frac{\partial S}{\partial y}$ depends linearly on x, with a negative sign indicating an inverse relationship.

2. Anisotropic Behavior: The directional derivatives demonstrate that the function's rate of change varies with direction, not just along coordinate axes. Directional derivatives provide a nuanced understanding of the function's local behavior.

3. Gradient as Steepest Ascent: The gradient vector at $(2, 1)$, $(-4, -10)$, points in the direction of maximum increase of S. Moving opposite to this vector would decrease S most rapidly.

4. Practical Applications:
- In optimization problems, knowing the gradient helps in designing algorithms for finding maxima or minima.
- In physics or economics, understanding the directional derivative guides how a system responds to changes in multiple variables simultaneously.

Conclusion

The function $S(x, y) = x - 5xy$ offers an illustrative example of multivariable calculus principles. The explicit calculation of its partial derivatives reveals how the function responds locally to changes in each variable, while the exploration of the directional derivative at a specific point underscores the importance of directionality in understanding real-world phenomena modeled by such functions.

By dissecting the gradient and various directional derivatives, we gain a comprehensive perspective on the local behavior of \(S \), enabling more informed analytical and practical decision-making. Whether applied in theoretical mathematics, engineering, economics, or physical sciences, mastering these concepts equips

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